A Soliton Solution to the Klein-Gordon Equation

My preon foundation of the Standard Model and mass-generalized Maxwell's equations hint at and are consistent with this soliton solution [1][2][3][4][5].

A soliton is a solitary travelling wave. Generally it does not change or changes little or gradually as it travels.

The simplest elementary particles in the preon model of the Standard Model are the first order objects (including electrons and their neutrinos). Recall that a first order object may be described by (R,R,R) (where R represents either a E or a B component).

$e^- = e(1) = \overline{(E^1,E^2,E^3)}_1$	$\mu^- = e(2) = \overline{(E^1,E^2,E^3)}_2$	$\tau^- = e(3) = \overline{(E^1,E^2,E^3)}_3$
$\nu_e = \nu(1) = (B^1,B^2,B^3)_1$	$\nu_\mu = \nu(2) = (B^1,B^2,B^3)_2$	$\nu_\tau = \nu(3) = (B^1,B^2,B^3)_3$
$u_R = u_1(1) = (B^1,E^2,E^3)_1$	$c_R = u_1(2) = (B^1,E^2,E^3)_2$	$t_R = u_1(3) = (B^1,E^2,E^3)_3$
$u_G = u_2(1) = (E^1,B^2,E^3)_1$	$c_G = u_2(2) = (E^1,B^2,E^3)_2$	$t_G = u_2(3) = (E^1,B^2,E^3)_3$
$u_B = u_3(1) = (E^1,E^2,B^3)_1$	$c_B = u_3(2) = (E^1,E^2,B^3)_2$	$t_B = u_3(3) = (E^1,E^2,B^3)_3$
$d_R = d_1(1) = \overline{(E^1,B^2,B^3)}_1$	$s_R = d_1(2) = \overline{(E^1,B^2,B^3)}_2$	$b_R = d_1(3) = \overline{(E^1,B^2,B^3)}_3$
$d_G = d_2(1) = \overline{(B^1,E^2,B^3)}_1$	$s_G = d_2(2) = \overline{(B^1,E^2,B^3)}_2$	$b_G = d_2(3) = \overline{(B^1,E^2,B^3)}_3$
$d_B = d_3(1) = \overline{(B^1,B^2,E^3)}_1$	$s_B = d_3(2) = \overline{(B^1,B^2,E^3)}_2$	$b_B = d_3(3) = \overline{(B^1,B^2,E^3)}_3$

The Helmholtzian factorization [6] generalization of the d'Alembertian [7] factorization is a compact expression of the mass-generalized Maxwell's equations expanded, for re-retrieval.

d'Alembertian factorization:

$$J \equiv \begin{pmatrix} J^1 \\ J^2 \\ J^3 \\ J^0 \end{pmatrix} \equiv \begin{pmatrix} \Box A^1 \\ \Box A^2 \\ \Box A^3 \\ \Box A^0 \end{pmatrix} = \Box \mathbf{A}$$

$$= \begin{pmatrix} \partial_0 & \partial_3 & -\partial_2 & \partial_1 \\ -\partial_3 & \partial_0 & \partial_1 & \partial_2 \\ \partial_2 & -\partial_1 & \partial_0 & \partial_3 \\ \partial_1 & \partial_2 & \partial_3 & -\partial_0 \end{pmatrix} \begin{pmatrix} \partial_0 & -\partial_3 & \partial_2 & \partial_1 \\ \partial_3 & \partial_0 & -\partial_1 & \partial_2 \\ -\partial_2 & \partial_1 & \partial_0 & \partial_3 \\ \partial_1 & \partial_2 & \partial_3 & -\partial_0 \end{pmatrix} \begin{pmatrix} A^1 \\ A^2 \\ A^3 \\ A^0 \end{pmatrix}$$

$$= \begin{pmatrix} -\partial_0 & \partial_3 & -\partial_2 & -\partial_1 \\ -\partial_3 & -\partial_0 & \partial_1 & -\partial_2 \\ \partial_2 & -\partial_1 & -\partial_0 & -\partial_3 \\ -\partial_1 & -\partial_2 & -\partial_3 & \partial_0 \end{pmatrix} \begin{pmatrix} -\partial_0 & -\partial_3 & \partial_2 & -\partial_1 \\ \partial_3 & -\partial_0 & -\partial_1 & -\partial_2 \\ -\partial_2 & \partial_1 & -\partial_0 & -\partial_3 \\ -\partial_1 & -\partial_2 & -\partial_3 & \partial_0 \end{pmatrix} \begin{pmatrix} A^1 \\ A^2 \\ A^3 \\ A^0 \end{pmatrix}$$

$$= \begin{pmatrix} [\partial_2 E^3 - \partial_3 E^2 + \partial_0 B^1] - [\partial_2 B^3 - \partial_3 B^2 + \partial_0 E^1] + \partial_1(\nabla \cdot \mathbf{f}^*) \\ [-\partial_1 E^3 + \partial_3 E^1 + \partial_0 B^2] - [-\partial_1 B^3 + \partial_3 B^1 + \partial_0 E^2] + \partial_2(\nabla \cdot \mathbf{f}^*) \\ [\partial_1 E^2 - \partial_2 E^1 + \partial_0 B^3] - [\partial_1 B^2 - \partial_2 B^1 + \partial_0 E^3] + \partial_3(\nabla \cdot \mathbf{f}^*) \\ [\partial_1 B^1 + \partial_2 B^2 - \partial_3 B^3] - [\partial_1 E^1 + \partial_2 E^2 - \partial_3 E^3] - \partial_0(\nabla \cdot \mathbf{f}^*) \end{pmatrix}$$

Note the left brackets are identically zero (the Homogeneous Maxwell's equations).

the right brackets are the terms of the Inhomogeneous Maxwell's equations.

Thus, this factorization reveals a symmetry/assymetry.

Switch the **E**'s & **B**'s and you get their respective charge densities (for appropriate gauge).

Helmholtzian factorization:

$$\mathbf{J} \equiv \begin{pmatrix} J^1 \\ J^2 \\ J^3 \\ J^0 \end{pmatrix} = \begin{pmatrix} (\Box - |m|^2)A^1 \\ (\Box - |m|^2)A^2 \\ (\Box - |m|^2)A^3 \\ (\Box - |m|^2)A^0 \end{pmatrix} = (\Box - |m|^2)\mathbf{A}$$

$$= \begin{pmatrix} D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & D_1 \\ -D_3^{\Leftrightarrow} & D_0 & D_1^{\Leftrightarrow} & D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & D_0 & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & D_1 \\ D_3^{\Leftrightarrow} & D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & D_0^{\Updownarrow} & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0 \end{pmatrix} \begin{pmatrix} A^1 \\ A^2 \\ A^3 \\ A^0 \end{pmatrix}$$

$$= \begin{pmatrix} -D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & -D_1 \\ -D_3^{\Leftrightarrow} & -D_0 & D_1^{\Leftrightarrow} & -D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} -D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\ D_3^{\Leftrightarrow} & -D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & -D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & -D_0^{\Updownarrow} & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix} \begin{pmatrix} A^1 \\ A^2 \\ A^3 \\ A^0 \end{pmatrix}$$

$$\vdots$$

where:

$$D_i^+ \equiv (\partial_i + m_i) \quad , \quad D_i^- \equiv (\partial_i - m_i)$$

$$D_i \equiv \begin{pmatrix} D_i^+ & 0 \\ 0 & D_i^- \end{pmatrix} \quad , \quad D_i^{\Updownarrow} \equiv \begin{pmatrix} D_i^- & 0 \\ 0 & D_i^+ \end{pmatrix} \quad ,$$

$$D_i^{\Leftrightarrow} \equiv \begin{pmatrix} 0 & D_i^- \\ D_i^+ & 0 \end{pmatrix} \quad , \quad D_i^{\Leftrightarrow\Updownarrow} \equiv \begin{pmatrix} 0 & D_i^+ \\ D_i^- & 0 \end{pmatrix}$$

$$J^1 \equiv \begin{pmatrix} J_+^i \\ J_-^i \end{pmatrix} \quad , \quad A^1 \equiv \begin{pmatrix} A_+^i \\ A_-^i \end{pmatrix}$$

$$\mathbf{R} \equiv \begin{pmatrix} \mathbf{R}_+ \\ \mathbf{R}_- \end{pmatrix} \quad , \quad \mathbf{R}_{\Updownarrow} \equiv \begin{pmatrix} \mathbf{R}_- \\ \mathbf{R}_+ \end{pmatrix}$$

$$\mathbf{E} = \mathbf{w}^{4;1}\left(-D_0^{\Updownarrow}A^1 - D_1A^0\right) + \mathbf{w}^{4;2}\left(-D_0^{\Updownarrow}A^2 - D_2A^0\right) + \mathbf{w}^{4;3}\left(-D_0^{\Updownarrow}A^3 - D_3A^0\right)$$

$$\mathbf{B} = \mathbf{w}^{4;1}(D_2A^3 - D_3A^2) + \mathbf{w}^{4;2}(-D_1A^3 + D_3A^1) + \mathbf{w}^{4;3}(D_1A^2 - D_2A^1)$$

$$\mathbf{E}_{\Updownarrow} = \mathbf{w}^{4;1}\left(-D_0^{\Leftrightarrow\Updownarrow}A^1 - D_1^{\Leftrightarrow}A^0\right) + \mathbf{w}^{4;2}\left(-D_0^{\Leftrightarrow\Updownarrow}A^2 - D_2^{\Leftrightarrow}A^0\right) + \mathbf{w}^{4;3}\left(-D_0^{\Leftrightarrow\Updownarrow}A^3 - D_3^{\Leftrightarrow}A^0\right.$$

$$\mathbf{B}_{\Updownarrow} = \mathbf{w}^{4;1}(D_2^{\Leftrightarrow}A^3 - D_3^{\Leftrightarrow}A^2) + \mathbf{w}^{4;2}(-D_1^{\Leftrightarrow}A^3 + D_3^{\Leftrightarrow}A^1) + \mathbf{w}^{4;3}(D_1^{\Leftrightarrow}A^2 - D_2^{\Leftrightarrow}A^1)$$

So, these expand to:

$$\mathbf{E} = \begin{pmatrix} \mathbf{E}_+ \\ \mathbf{E}_- \end{pmatrix} = \mathbf{w}^{4;1}\begin{pmatrix} -(\partial_0 - m_0)A_+^1 - (\partial_1 + m_1)A_+^0 \\ -(\partial_0 + m_0)A_-^1 - (\partial_1 - m_1)A_-^0 \end{pmatrix} +$$
$$+ \mathbf{w}^{4;2}\begin{pmatrix} -(\partial_0 - m_0)A_+^2 - (\partial_2 + m_2)A_+^0 \\ -(\partial_0 + m_0)A_-^2 - (\partial_2 - m_2)A_-^0 \end{pmatrix} +$$
$$+ \mathbf{w}^{4;3}\begin{pmatrix} -(\partial_0 - m_0)A_+^3 - (\partial_3 + m_3)A_+^0 \\ -(\partial_0 + m_0)A_-^3 - (\partial_3 - m_3)A_-^0 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} \mathbf{B}_+ \\ \mathbf{B}_- \end{pmatrix} = \mathbf{w}^{4;1}\begin{pmatrix} (\partial_2 + m_2)A_+^3 - (\partial_3 + m_3)A_+^2 \\ (\partial_2 - m_2)A_-^3 - (\partial_3 - m_3)A_-^2 \end{pmatrix} +$$

$$+\,\mathbf{w}^{4;2}\begin{pmatrix} -(\partial_1+m_1)A_+^3+(\partial_3+m_3)A_+^1 \\ -(\partial_1-m_1)A_-^3+(\partial_3-m_3)A_-^1 \end{pmatrix}+$$
$$+\,\mathbf{w}^{4;3}\begin{pmatrix} (\partial_1+m_1)A_+^2-(\partial_2+m_2)A_+^1 \\ (\partial_1-m_1)A_-^2-(\partial_2-m_2)A_-^1 \end{pmatrix}$$

$$\mathbf{E}_{\Updownarrow} = \begin{pmatrix} \mathbf{E}_- \\ \mathbf{E}_+ \end{pmatrix} = \mathbf{w}^{4;1}\begin{pmatrix} -(\partial_0+m_0)A_-^1-(\partial_1-m_1)A_-^0 \\ -(\partial_0-m_0)A_+^1-(\partial_1+m_1)A_+^0 \end{pmatrix}+$$
$$+\,\mathbf{w}^{4;2}\begin{pmatrix} -(\partial_0+m_0)A_-^2-(\partial_2-m_2)A_-^0 \\ -(\partial_0-m_0)A_+^2-(\partial_2+m_2)A_+^0 \end{pmatrix}+$$
$$+\,\mathbf{w}^{4;3}\begin{pmatrix} -(\partial_0+m_0)A_-^3-(\partial_3-m_3)A_-^0 \\ -(\partial_0-m_0)A_+^3-(\partial_3+m_3)A_+^0 \end{pmatrix} \qquad \checkmark$$

$$\mathbf{B}_{\Updownarrow} = \begin{pmatrix} \mathbf{B}_- \\ \mathbf{B}_+ \end{pmatrix} = \mathbf{w}^{4;1}\begin{pmatrix} (\partial_2-m_2)A_-^3-(\partial_3-m_3)A_-^2 \\ (\partial_2+m_2)A_+^3-(\partial_3+m_3)A_+^2 \end{pmatrix}+$$
$$+\,\mathbf{w}^{4;2}\begin{pmatrix} -(\partial_1-m_1)A_-^3+(\partial_3-m_3)A_-^1 \\ -(\partial_1+m_1)A_+^3+(\partial_3+m_3)A_+^1 \end{pmatrix}+$$
$$+\,\mathbf{w}^{4;3}\begin{pmatrix} (\partial_1-m_1)A_-^2-(\partial_2-m_2)A_-^1 \\ (\partial_1+m_1)A_+^2-(\partial_2+m_2)A_+^1 \end{pmatrix} \qquad \checkmark$$

and:

$$\mathbf{J} \equiv \begin{pmatrix} J^1 \\ J^2 \\ J^3 \\ J^0 \end{pmatrix} = \begin{pmatrix} (\Box-|m|^2)A^1 \\ (\Box-|m|^2)A^2 \\ (\Box-|m|^2)A^3 \\ (\Box-|m|^2)A^0 \end{pmatrix} = (\Box-|m|^2)\mathbf{A}$$
$$= \begin{pmatrix} D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & D_1 \\ -D_3^{\Leftrightarrow} & D_0 & D_1^{\Leftrightarrow} & D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & D_0 & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0^{\Updownarrow} \end{pmatrix}\begin{pmatrix} D_0^{\Updownarrow}A^1+D_1A^0-D_3^{\Leftrightarrow}A^2+D_2^{\Leftrightarrow}A^3 \\ D_0^{\Updownarrow}A^2+D_2A^0-D_1^{\Leftrightarrow}A^3+D_3^{\Leftrightarrow}A^1 \\ D_0^{\Updownarrow}A^3+D_3A^0-D_2^{\Leftrightarrow}A^1+D_1^{\Leftrightarrow}A^2 \\ D_1^{\Updownarrow}A^1+D_2^{\Updownarrow}A^2+D_3^{\Updownarrow}A^3-D_0A^3 \end{pmatrix}$$

$$
= \begin{pmatrix} D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & D_1 \\ -D_3^{\Leftrightarrow} & D_0 & D_1^{\Leftrightarrow} & D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & D_0 & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} -E^1 + B^1_{\Updownarrow} \\ -E^2 + B^2_{\Updownarrow} \\ -E^3 + B^3_{\Updownarrow} \\ \overset{m}{\boldsymbol{\nabla}}_{\Updownarrow} \cdot \mathbf{A}^* \end{pmatrix}
$$

$$
= \begin{pmatrix} D_0\left(B^1_{\Updownarrow} - E^1\right) + D_3^{\Leftrightarrow}\left(B^2_{\Updownarrow} - E^2\right) - D_2^{\Leftrightarrow}\left(B^3_{\Updownarrow} - E^3\right) + D_1\left(\overset{m}{\boldsymbol{\nabla}}_{\Updownarrow} \cdot \mathbf{A}^*\right) \\ -D_3^{\Leftrightarrow}\left(B^1_{\Updownarrow} - E^1\right) + D_0\left(B^2_{\Updownarrow} - E^2\right) + D_1^{\Leftrightarrow}\left(B^3_{\Updownarrow} - E^3\right) + D_2\left(\overset{m}{\boldsymbol{\nabla}}_{\Updownarrow} \cdot \mathbf{A}^*\right) \\ D_2^{\Leftrightarrow}\left(B^1_{\Updownarrow} - E^1\right) - D_1^{\Leftrightarrow}\left(B^2_{\Updownarrow} - E^2\right) + D_0\left(B^3_{\Updownarrow} - E^3\right) + D_3\left(\overset{m}{\boldsymbol{\nabla}}_{\Updownarrow} \cdot \mathbf{A}^*\right) \\ D_1^{\Updownarrow}\left(B^1_{\Updownarrow} - E^1\right) + D_2^{\Updownarrow}\left(B^2_{\Updownarrow} - E^2\right) + D_3^{\Updownarrow}\left(B^3_{\Updownarrow} - E^3\right) - D_0^{\Updownarrow}\left(\overset{m}{\boldsymbol{\nabla}}_{\Updownarrow} \cdot \mathbf{A}^*\right) \end{pmatrix}
$$

$$
= \begin{pmatrix} \left[D_2^{\Leftrightarrow}E^3 - D_3^{\Leftrightarrow}E^2 + D_0 B^1_{\Updownarrow}\right] - \left[D_2^{\Leftrightarrow}B^3_{\Updownarrow} - D_3^{\Leftrightarrow}B^2_{\Updownarrow} + D_0 E^1\right] + D_1\left(\overset{m}{\boldsymbol{\nabla}}_{\Updownarrow} \cdot \mathbf{A}^*\right) \\ \left[-D_1^{\Leftrightarrow}E^3 + D_3^{\Leftrightarrow}E^1 + D_0 B^2_{\Updownarrow}\right] - \left[-D_1^{\Leftrightarrow}B^3_{\Updownarrow} + D_3^{\Leftrightarrow}B^1_{\Updownarrow} + D_0 E^2\right] + D_2\left(\overset{m}{\boldsymbol{\nabla}}_{\Updownarrow} \cdot \mathbf{A}^*\right) \\ \left[D_1^{\Leftrightarrow}E^2 - D_2^{\Leftrightarrow}E^1 + D_0 B^3_{\Updownarrow}\right] - \left[D_1^{\Leftrightarrow}B^2_{\Updownarrow} - D_2^{\Leftrightarrow}B^1_{\Updownarrow} + D_0 E^3\right] + D_3\left(\overset{m}{\boldsymbol{\nabla}}_{\Updownarrow} \cdot \mathbf{A}^*\right) \\ \left[D_1^{\Updownarrow}B^1_{\Updownarrow} + D_2^{\Updownarrow}B^2_{\Updownarrow} + D_3^{\Updownarrow}B^3_{\Updownarrow}\right] - \left[D_1^{\Updownarrow}E^1 + D_2^{\Updownarrow}E^2 + D_3^{\Updownarrow}E^3\right] - D_0^{\Updownarrow}\left(\overset{m}{\boldsymbol{\nabla}}_{\Updownarrow} \cdot \mathbf{A}^*\right) \end{pmatrix}
$$

Working these out just as the ones above were worked out confirms that these are the homogeneous and inhomogeneous mass-generalized Maxwell's equations (plus the usual but generalized gauge terms).

These and the other factorization are worked out in more detail in [8] (including the various gauges).

The mass-generalized Maxwell's equations may be written (as I first did in this form in 2001):

$$
(\vec{\boldsymbol{\nabla}} + \vec{\mathbf{m}}) \cdot \vec{\mathbf{B}} = 0, \qquad (\vec{\boldsymbol{\nabla}} - \vec{\mathbf{m}}) \times \vec{\mathbf{B}} = (\frac{\partial}{\partial t} + m_0)\vec{\mathbf{E}} + \vec{\mathbf{J}}
$$

$$
(\vec{\boldsymbol{\nabla}} - \vec{\mathbf{m}}) \cdot \vec{\mathbf{E}} = \rho, \qquad (\vec{\boldsymbol{\nabla}} + \vec{\mathbf{m}}) \times \vec{\mathbf{E}} = -(\frac{\partial}{\partial t} - m_0)\vec{\mathbf{B}}
$$

$$
\vec{\mathbf{B}} \equiv (\vec{\boldsymbol{\nabla}} + \vec{\mathbf{m}}) \times \vec{\mathbf{A}} \;, \qquad \vec{\mathbf{E}} \equiv -(\frac{\partial}{\partial t} - m_0)\vec{\mathbf{A}} - (\vec{\boldsymbol{\nabla}} + \vec{\mathbf{m}})A^0
$$

As you can see, this form summarizes the above equations in matrix form, where the $\mathbf{R}_+/\mathbf{R}_-$ parts are substituted for the corresponding $\vec{\mathbf{R}}$ along with the corresponding sign of the mass.

The merging process of all fundamental objects (first or second order) (particle) with their antiobject (antiparticle) is slightly more complicated than simple addition.

The object is a doublet with a single non-zero entry.

At merge each is multiplied by merge factor $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, essentially converting the doublet/spin-$\frac{1}{2}$ object into a spin-1 object, by a projection operator flattening the space as follows:

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\mathbf{R}_{+} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} R \\ 0 \end{pmatrix} = \begin{pmatrix} R \\ R \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\mathbf{R}_{-} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ R \end{pmatrix} = \begin{pmatrix} R \\ R \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\vec{\mathbf{R}} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}(\mathbf{R}_{+} + \mathbf{R}_{-}) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} R_{+} \\ R_{-} \end{pmatrix} = \begin{pmatrix} R_{+} + R_{-} \\ R_{+} + R_{-} \end{pmatrix}$$

A free doublet naturally has two degrees of freedom (dimensions).

The above projection/flattening operator drops one degree of freedom to one (like a shadow or a flat mirror reflection).

The spin of a particle is the reciprocal of the dimension of the particle space it is a member of.

So, the spin of a fermion (first or second order object) is the reciprocal of the dimension of the fermion space:: $\frac{1}{2}$, and, likewise, the spin of a photon is: 1 .

The equations of a thus merged first order object with it’s anti-object become:

$$0 + 0 = (\vec{\boldsymbol{\nabla}} + \vec{\mathbf{m}}) \cdot \vec{\mathbf{B}} + (\vec{\boldsymbol{\nabla}} - \vec{\mathbf{m}}) \cdot \vec{\mathbf{B}} = \Big[(\vec{\boldsymbol{\nabla}} + \vec{\mathbf{m}}) + (\vec{\boldsymbol{\nabla}} - \vec{\mathbf{m}}) \Big] \cdot \vec{\mathbf{B}} = 2\,\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{B}}$$

$$0 = \rho - \rho = (\vec{\boldsymbol{\nabla}} - \vec{\mathbf{m}}) \cdot \vec{\mathbf{E}} + (\vec{\boldsymbol{\nabla}} + \vec{\mathbf{m}}) \cdot \vec{\mathbf{E}} = \Big[(\vec{\boldsymbol{\nabla}} - \vec{\mathbf{m}}) + (\vec{\boldsymbol{\nabla}} + \vec{\mathbf{m}}) \Big] \cdot \vec{\mathbf{E}} = 2\,\vec{\boldsymbol{\nabla}} \cdot$$

$$\begin{aligned}
\mathbf{0} &= \vec{\mathbf{J}} - \vec{\mathbf{J}} = (\vec{\boldsymbol{\nabla}} - \vec{\mathbf{m}}) \times \vec{\mathbf{B}} - (\frac{\partial}{\partial t} + m_0)\vec{\mathbf{E}} + (\vec{\boldsymbol{\nabla}} - \vec{\mathbf{m}}) \times \vec{\mathbf{B}} - (\frac{\partial}{\partial t} + m_0)\vec{\mathbf{E}} \\
&= (\vec{\boldsymbol{\nabla}} - \vec{\mathbf{m}}) \times \vec{\mathbf{B}} + (\vec{\boldsymbol{\nabla}} - \vec{\mathbf{m}}) \times \vec{\mathbf{B}} - (\frac{\partial}{\partial t} + m_0)\vec{\mathbf{E}} - (\frac{\partial}{\partial t} + m_0)\vec{\mathbf{E}} \\
&= \Big[(\vec{\boldsymbol{\nabla}} - \vec{\mathbf{m}}) + (\vec{\boldsymbol{\nabla}} - \vec{\mathbf{m}}) \Big] \times \vec{\mathbf{B}} - \Big[(\frac{\partial}{\partial t} + m_0)\vec{\mathbf{E}} + (\frac{\partial}{\partial t} + m_0) \Big]\vec{\mathbf{E}} \\
&= 2\Big[\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}} - \frac{\partial}{\partial t}\vec{\mathbf{E}} \Big]
\end{aligned}$$

$$\begin{aligned}
\mathbf{0} &= (\vec{\boldsymbol{\nabla}} + \vec{\mathbf{m}}) \times \vec{\mathbf{E}} + (\frac{\partial}{\partial t} - m_0)\vec{\mathbf{B}} + (\vec{\boldsymbol{\nabla}} - \vec{\mathbf{m}}) \times \vec{\mathbf{E}} + (\frac{\partial}{\partial t} + m_0)\vec{\mathbf{B}} \\
&= \Big[(\vec{\boldsymbol{\nabla}} + \vec{\mathbf{m}}) + (\vec{\boldsymbol{\nabla}} - \vec{\mathbf{m}}) \Big] \times \vec{\mathbf{E}} + \Big[(\frac{\partial}{\partial t} - m_0) + (\frac{\partial}{\partial t} + m_0) \Big]\vec{\mathbf{B}} \\
&= 2\Big[\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}} + \frac{\partial}{\partial t}\vec{\mathbf{B}} \Big]
\end{aligned}$$

And the potentials satisfy:

$$\vec{\mathbf{B}} + \vec{\mathbf{B}} \equiv (\vec{\nabla} + \vec{\mathbf{m}}) \times \vec{\mathbf{A}} + (\vec{\nabla} - \vec{\mathbf{m}}) \times \vec{\mathbf{A}} = \Big[(\vec{\nabla} + \vec{\mathbf{m}}) + (\vec{\nabla} - \vec{\mathbf{m}}) \Big] \times \vec{\mathbf{A}} = 2\,\vec{\nabla} \times \vec{\mathbf{A}}$$

$$\begin{aligned}
\vec{\mathbf{E}} + \vec{\mathbf{E}} &\equiv -\left(\frac{\partial}{\partial t} - m_0\right)\vec{\mathbf{A}} - (\vec{\nabla} + \vec{\mathbf{m}})A^0 - \left(\frac{\partial}{\partial t} + m_0\right)\vec{\mathbf{A}} - (\vec{\nabla} - \vec{\mathbf{m}})A^0 \\
&= \Big[-\left(\frac{\partial}{\partial t} - m_0\right) - \left(\frac{\partial}{\partial t} + m_0\right) \Big]\vec{\mathbf{A}} + \Big[-(\vec{\nabla} + \vec{\mathbf{m}}) - (\vec{\nabla} - \vec{\mathbf{m}}) \Big]A^0 \\
&= 2\Big[-\frac{\partial}{\partial t}\vec{\mathbf{A}} - \vec{\nabla}A^0 \Big]
\end{aligned}$$

These are the equations of light, each of $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$, as well as $\vec{\mathbf{A}}$ and $\phi = A^0$ satisfying the wave equation.

The second order objects (quarks) merge, likewise. Since via the self-fermion interaction the merging of second order object with it's anti-object then becomes the merger of first order object with it's anti-object.

Dealing with the mass-generalized Maxwell's equations for fundamental objects is not like dealing with a single equation. Each fermion is specified by a non-zero triplet. Each component of the triplet may be an E or a B, and each is either $+$ or $-$. Four possible states for each component comes to twelve. Since three are non-zero, nine must vanish. The A^i_+ & A^i_- must satisfy the pair of homogeneous and pair of inhomogeneous equations, specified by the architecture of a fermion.

$$\begin{aligned}
E^i_+ &\equiv -(\partial_0 - m_0)A^i_+ - (\partial_i + m_i)A^0_+ \\
B^i_+ &\equiv (-1)^{i+1}\Big[(\partial_{2-T_0(i)} + m_{2-T_0(i)})A_+^{3-T_0(i-1)} - (\partial_{3-T_0(i-1)} + m_{3-T_0(i-1)})A_+^{2-T_0(i)} \Big] \\
E^i_- &\equiv -(\partial_0 + m_0)A^i_- - (\partial_i - m_i)A^0_- \\
B^i_- &\equiv (-1)^{i+1}[(\partial_{2-T_0(i)} - m_{2-T_0(i)})A_-^{3-T_0(i-1)} - (\partial_{3-T_0(i-1)} - m_{3-T_0(i-1)})A_-^{2-T_0(i)}]
\end{aligned}$$
$$(i \in \{1,2,3\})$$

Some initial definitions may facilitate initial analysis of fundamental objects.

Let:

$$R(m,\sigma,i) \equiv \begin{cases} E^i_\sigma & ,m = 0 \\ B^i_\sigma & ,m = 1 \end{cases} \quad [m \in \{0,1\}, \sigma \in \{+,-\}, i \in \mathbb{N}]$$

$$X_h(m_1,m_2,m_3,\sigma) \equiv (R(m_1,\sigma,1), R(m_2,\sigma,2), R(m_3,\sigma,3))_h$$

$$\overline{\sigma} \equiv \begin{cases} - & ,\sigma = + \\ + & ,\sigma = - \end{cases} \quad , \quad s(\sigma) \equiv \begin{cases} 1 & ,\sigma = + \\ -1 & ,\sigma = - \end{cases} \quad [\sigma \in \{+,-\}]$$

This notation may rather efficiently express a fundamental object fermion of generation h.

The polarity σ efficiently expresses when all the opposing field strengths vanish.

For fermion expressed by: $X_h(m_1,m_2,m_3,\sigma)$:

$$(R(m,\overline{\sigma},1), R(m,\overline{\sigma},2), R(m,\overline{\sigma},3))_h = (0,0,0)_h \quad , \quad \forall m \in \{0,1\}$$

and thus:

$$0 = E^i_{\overline{\sigma}} \equiv -(\partial_0 + s(\overline{\overline{\sigma}})m_0)A^i_{\overline{\sigma}} - (\partial_i + s(\overline{\sigma})m_i)A^0_{\overline{\sigma}}$$

$$0 = B^i_{\overline{\sigma}} \equiv (-1)^{i+1}\Big[(\partial_{2-T_0(i)} + s(\overline{\sigma})m_{2-T_0(i)})A^{3-T_0(i-1)}_{\overline{\sigma}} - (\partial_{3-T_0(i-1)} + s(\overline{\sigma})m_{3-T_0(i-1)})A^{2-T_0(i)}_{\overline{\sigma}} \Big]$$

So, regardless of what the fermion is, these will be satisfied for it.

Of course, vanishing of all the field potentials $A^i_{\overline{\sigma}}$ for a given σ will satisfy these, but a more general solution may be obtained (including this) by analyzing all these equations.

Note first, that: $e^{-\int Pdx}\left(ye^{\int Pdx}\right)' = y' + Py = \left(\frac{d}{dx} + P\right)y$

So, we may write: $X_h(m_1, m_2, m_3, \sigma)$:

$$\Rightarrow \begin{cases} 0 = E^i_{\overline{\sigma}} \equiv -e^{-s(\overline{\overline{\sigma}})m_0x^0}\partial_0\left(A^i_{\overline{\sigma}}e^{s(\overline{\overline{\sigma}})m_0x^0}\right) - e^{-s(\overline{\sigma})m_ix^i}\partial_i\left(A^0_{\overline{\sigma}}e^{s(\overline{\sigma})m_ix^i}\right) \\ 0 = B^i_{\overline{\sigma}} \equiv (-1)^{i+1}\Big[e^{-s(\overline{\sigma})m_{2-T_0(i)}x^{2-T_0(i)}}\partial_{2-T_0(i)}\left(A^{3-T_0(i-1)}_{\overline{\sigma}}e^{s(\overline{\sigma})m_{2-T_0(i)}x^{2-T_0(i)}}\right) + \\ \qquad - e^{-s(\overline{\sigma})m_{3-T_0(i)}x^{3-T_0(i)}}\partial_{3-T_0(i)}\left(A^{2-T_0(i-1)}_{\overline{\sigma}}e^{s(\overline{\sigma})m_{3-T_0(i)}x^{3-T_0(i)}}\right)\Big] \end{cases}$$

$$\Rightarrow \begin{cases} \partial_i\left(A^0_{\overline{\sigma}}e^{s(\overline{\sigma})m_ix^i}\right) = -e^{s(\overline{\sigma})m_ix^i - s(\overline{\overline{\sigma}})m_0x^0}\partial_0\left(A^i_{\overline{\sigma}}e^{s(\overline{\overline{\sigma}})m_0x^0}\right) \\ \partial_{2-T_0(i)}\left(A^{3-T_0(i-1)}_{\overline{\sigma}}e^{s(\overline{\sigma})m_{2-T_0(i)}x^{2-T_0(i)}}\right) = \\ \qquad = e^{s(\overline{\sigma})m_{2-T_0(i)}x^{2-T_0(i)} - s(\overline{\sigma})m_{3-T_0(i)}x^{3-T_0(i)}}\partial_{3-T_0(i)}\left(A^{2-T_0(i-1)}_{\overline{\sigma}}e^{s(\overline{\sigma})m_{3-T_0(i)}x^{3-T_0(i)}}\right)\Big] \end{cases}$$

So:

$$\begin{cases} \partial_1\left(A^0_{\overline{\sigma}}e^{s(\overline{\sigma})m_1x^1}\right) = -e^{s(\overline{\sigma})m_1x^1 - s(\overline{\overline{\sigma}})m_0x^0}\partial_0\left(A^1_{\overline{\sigma}}e^{s(\overline{\overline{\sigma}})m_0x^0}\right) \\ \partial_2\left(A^0_{\overline{\sigma}}e^{s(\overline{\sigma})m_2x^2}\right) = -e^{s(\overline{\sigma})m_2x^2 - s(\overline{\overline{\sigma}})m_0x^0}\partial_0\left(A^2_{\overline{\sigma}}e^{s(\overline{\overline{\sigma}})m_0x^0}\right) \\ \partial_3\left(A^0_{\overline{\sigma}}e^{s(\overline{\sigma})m_3x^3}\right) = -e^{s(\overline{\sigma})m_3x^3 - s(\overline{\overline{\sigma}})m_0x^0}\partial_0\left(A^3_{\overline{\sigma}}e^{s(\overline{\overline{\sigma}})m_0x^0}\right) \end{cases}$$

$$\Rightarrow \begin{cases} \partial_1\left(A^0_{\overline{\sigma}}e^{s(\overline{\sigma})m_1x^1 + s(\overline{\sigma})m_2x^2 + s(\overline{\sigma})m_3x^3 + s(\overline{\overline{\sigma}})m_0x^0}\right) = -\partial_0\left(A^1_{\overline{\sigma}}e^{s(\overline{\sigma})m_1x^1 + s(\overline{\sigma})m_2x^2 + s(\overline{\sigma})m_3x^3 + s(\overline{\overline{\sigma}})m_0x^0}\right) \\ \partial_2\left(A^0_{\overline{\sigma}}e^{s(\overline{\sigma})m_1x^1 + s(\overline{\sigma})m_2x^2 + s(\overline{\sigma})m_3x^3 + s(\overline{\overline{\sigma}})m_0x^0}\right) = -\partial_0\left(A^2_{\overline{\sigma}}e^{s(\overline{\sigma})m_1x^1 + s(\overline{\sigma})m_2x^2 + s(\overline{\sigma})m_3x^3 + s(\overline{\overline{\sigma}})m_0x^0}\right) \\ \partial_3\left(A^0_{\overline{\sigma}}e^{s(\overline{\sigma})m_1x^1 + s(\overline{\sigma})m_2x^2 + s(\overline{\sigma})m_3x^3 + s(\overline{\overline{\sigma}})m_0x^0}\right) = -\partial_0\left(A^3_{\overline{\sigma}}e^{s(\overline{\sigma})m_1x^1 + s(\overline{\sigma})m_2x^2 + s(\overline{\sigma})m_3x^3 + s(\overline{\overline{\sigma}})m_0x^0}\right) \end{cases}$$

$$\Rightarrow \begin{cases} \partial_1\left(A^0_{\overline{\sigma}}e^{s(\overline{\sigma})m_1x^1 + s(\overline{\sigma})m_2x^2 + s(\overline{\sigma})m_3x^3 + s(\overline{\overline{\sigma}})m_0x^0}\right) = -\partial_0\left(A^1_{\overline{\sigma}}e^{s(\overline{\sigma})m_1x^1 + s(\overline{\sigma})m_2x^2 + s(\overline{\sigma})m_3x^3 + s(\overline{\overline{\sigma}})m_0x^0}\right) \\ \partial_1\left(A^2_{\overline{\sigma}}e^{s(\overline{\sigma})m_1x^1 + s(\overline{\sigma})m_2x^2 + s(\overline{\sigma})m_3x^3 + s(\overline{\overline{\sigma}})m_0x^0}\right) = \partial_2\left(A^1_{\overline{\sigma}}e^{s(\overline{\sigma})m_1x^1 + s(\overline{\sigma})m_2x^2 + s(\overline{\sigma})m_3x^3 + s(\overline{\overline{\sigma}})m_0x^0}\right) \\ \qquad + \varphi_{012}(x^1, x^2, x^3) \\ \partial_1\left(A^3_{\overline{\sigma}}e^{s(\overline{\sigma})m_1x^1 + s(\overline{\sigma})m_2x^2 + s(\overline{\sigma})m_3x^3 + s(\overline{\overline{\sigma}})m_0x^0}\right) = \partial_3\left(A^1_{\overline{\sigma}}e^{s(\overline{\sigma})m_1x^1 + s(\overline{\sigma})m_2x^2 + s(\overline{\sigma})m_3x^3 + s(\overline{\overline{\sigma}})m_0x^0}\right) \\ \qquad + \varphi_{013}(x^1, x^2, x^3) \\ \partial_2\left(A^3_{\overline{\sigma}}e^{s(\overline{\sigma})m_1x^1 + s(\overline{\sigma})m_2x^2 + s(\overline{\sigma})m_3x^3 + s(\overline{\overline{\sigma}})m_0x^0}\right) = \partial_3\left(A^2_{\overline{\sigma}}e^{s(\overline{\sigma})m_1x^1 + s(\overline{\sigma})m_2x^2 + s(\overline{\sigma})m_3x^3 + s(\overline{\overline{\sigma}})m_0x^0}\right) \\ \qquad + \varphi_{023}(x^1, x^2, x^3) \end{cases}$$

Likewise:

$$\Rightarrow \begin{cases} \partial_i\left(A^0_{\overline{\sigma}} e^{s(\overline{\sigma})m_i x^i}\right) = -e^{s(\overline{\sigma})m_i x^i - s(\overline{\overline{\sigma}})m_0 x^0}\partial_0\left(A^i_{\overline{\sigma}} e^{s(\overline{\overline{\sigma}})m_0 x^0}\right) \\ \partial_{2-T_0(i)}\left(A^{3-T_0(i-1)}_{\overline{\sigma}} e^{s(\overline{\sigma})m_{2-T_0(i)}x^{2-T_0(i)}}\right) = \\ \qquad = e^{s(\overline{\sigma})m_{2-T_0(i)}x^{2-T_0(i)} - s(\overline{\sigma})m_{3-T_0(i)}x^{3-T_0(i)}}\partial_{3-T_0(i)}\left(A^{2-T_0(i-1)}_{\overline{\sigma}} e^{s(\overline{\sigma})m_{3-T_0(i)}x^{3-T_0(i)}}\right)\Big] \end{cases}$$

So:

$$\begin{cases} \partial_2\left(A^3_{\overline{\sigma}} e^{s(\overline{\sigma})m_2 x^2}\right) = e^{s(\overline{\sigma})m_2 x^2 - s(\overline{\sigma})m_3 x^3}\partial_3\left(A^2_{\overline{\sigma}} e^{s(\overline{\sigma})m_3 x^3}\right) \\ \partial_1\left(A^3_{\overline{\sigma}} e^{s(\overline{\sigma})m_1 x^1}\right) = e^{s(\overline{\sigma})m_1 x^1 - s(\overline{\sigma})m_3 x^3}\partial_3\left(A^1_{\overline{\sigma}} e^{s(\overline{\sigma})m_3 x^3}\right) \\ \partial_1\left(A^2_{\overline{\sigma}} e^{s(\overline{\sigma})m_1 x^1}\right) = e^{s(\overline{\sigma})m_1 x^1 - s(\overline{\sigma})m_2 x^2}\partial_2\left(A^1_{\overline{\sigma}} e^{s(\overline{\sigma})m_2 x^2}\right) \end{cases}$$

$$\Rightarrow \begin{cases} \partial_1\left(A^2_{\overline{\sigma}} e^{s(\overline{\sigma})m_1 x^1 + s(\overline{\sigma})m_2 x^2 + s(\overline{\sigma})m_3 x^3 + s(\overline{\overline{\sigma}})m_0 x^0}\right) = \partial_2\left(A^1_{\overline{\sigma}} e^{s(\overline{\sigma})m_1 x^1 + s(\overline{\sigma})m_2 x^2 + s(\overline{\sigma})m_3 x^3 + s(\overline{\overline{\sigma}})m_0 x^0}\right) \\ \partial_1\left(A^3_{\overline{\sigma}} e^{s(\overline{\sigma})m_1 x^1 + s(\overline{\sigma})m_2 x^2 + s(\overline{\sigma})m_3 x^3 + s(\overline{\overline{\sigma}})m_0 x^0}\right) = \partial_3\left(A^1_{\overline{\sigma}} e^{s(\overline{\sigma})m_1 x^1 + s(\overline{\sigma})m_2 x^2 + s(\overline{\sigma})m_3 x^3 + s(\overline{\overline{\sigma}})m_0 x^0}\right) \\ \partial_2\left(A^3_{\overline{\sigma}} e^{s(\overline{\sigma})m_1 x^1 + s(\overline{\sigma})m_2 x^2 + s(\overline{\sigma})m_3 x^3 + s(\overline{\overline{\sigma}})m_0 x^0}\right) = \partial_3\left(A^2_{\overline{\sigma}} e^{s(\overline{\sigma})m_1 x^1 + s(\overline{\sigma})m_2 x^2 + s(\overline{\sigma})m_3 x^3 + s(\overline{\overline{\sigma}})m_0 x^0}\right) \end{cases}$$

The third equation is satisfied identically when the first two are satisfied.
This shows that both sets of equations have three equations in common.
Like the above, except not including:

$$\partial_1\left(A^0_{\overline{\sigma}} e^{s(\overline{\sigma})m_1 x^1 + s(\overline{\sigma})m_2 x^2 + s(\overline{\sigma})m_3 x^3 + s(\overline{\overline{\sigma}})m_0 x^0}\right) = -\partial_0\left(A^1_{\overline{\sigma}} e^{s(\overline{\sigma})m_1 x^1 + s(\overline{\sigma})m_2 x^2 + s(\overline{\sigma})m_3 x^3 + s(\overline{\overline{\sigma}})m_0 x^0}\right)$$

and: $\varphi_{0ij}(x^1, x^2, x^3) = 0$

This is quite interesting. And that all six equations reduce to four minus one (since one is satisfied identically by the other two) which is less than the number of potentials, and then there will be arbitrary functions in the solutions as well.

Now, half of the equations for all fermions have been established; and for each fermion three are subject to densities, while three vanish established as above.

Note before going on, that these equations are satisfied by:

$$\begin{cases} A^i_{\overline{\sigma}} = \Psi(x^0)e^{-[s(\overline{\sigma})m_1 x^1 + s(\overline{\sigma})m_2 x^2 + s(\overline{\sigma})m_3 x^3 + s(\overline{\overline{\sigma}})m_0 x^0]} \\ A^0_{\overline{\sigma}} = \Psi(x^0)(-x^1 - x^2 - x^3)e^{-[s(\overline{\sigma})m_1 x^1 + s(\overline{\sigma})m_2 x^2 + s(\overline{\sigma})m_3 x^3 + s(\overline{\overline{\sigma}})m_0 x^0]} \end{cases}, \quad (i \in \{1,2,3\})$$

but depending on the signs of $s(\overline{\sigma})m_i$
one side or the other of the component would have infinities.

Alternatively, a construction of potentials may be as follows.

If: $A^i_{\overline{\sigma}} = \Psi_i(x^0, x^1, x^2, x^3)e^{-|\mathbf{k}\cdot\mathbf{r}|}$ (**B** scenario)

$$\begin{cases} 0 = B^1_{\overline{\sigma}} = (\partial_2 + s(\overline{\sigma})m_2)A^3_{\overline{\sigma}} - (\partial_3 + s(\overline{\sigma})m_3)A^2_{\overline{\sigma}} \Rightarrow (\partial_3 + s(\overline{\sigma})m_3)A^2_{\overline{\sigma}} = (\partial_2 + s(\overline{\sigma})m_2)A^3_{\overline{\sigma}} \\ 0 = B^2_{\overline{\sigma}} = -(\partial_1 + s(\overline{\sigma})m_1)A^3_{\overline{\sigma}} + (\partial_3 + s(\overline{\sigma})m_3)A^1_{\overline{\sigma}} \Rightarrow (\partial_3 + s(\overline{\sigma})m_3)A^1_{\overline{\sigma}} = (\partial_1 + s(\overline{\sigma})m_1)A^3_{\overline{\sigma}} \\ 0 = B^3_{\overline{\sigma}} = (\partial_1 + s(\overline{\sigma})m_1)A^2_{\overline{\sigma}} - (\partial_2 + s(\overline{\sigma})m_2)A^1_{\overline{\sigma}} \Rightarrow (\partial_2 + s(\overline{\sigma})m_2)A^1_{\overline{\sigma}} = (\partial_1 + s(\overline{\sigma})m_1)A^2_{\overline{\sigma}} \end{cases}$$

$$\Rightarrow \begin{cases} \left[\partial_3\Psi_2 + \left(-\frac{\mathbf{k}\cdot\mathbf{r}}{|\mathbf{k}\cdot\mathbf{r}|}k_3 + s(\overline{\sigma})m_3\right)\Psi_2\right]e^{-|\mathbf{k}\cdot\mathbf{r}|} = \left[\partial_2\Psi_3 + \left(-\frac{\mathbf{k}\cdot\mathbf{r}}{|\mathbf{k}\cdot\mathbf{r}|}k_2 + s(\overline{\sigma})m_2\right)\Psi_3\right]e^{-|\mathbf{k}\cdot\mathbf{r}|} \\ \left[\partial_3\Psi_1 + \left(-\frac{\mathbf{k}\cdot\mathbf{r}}{|\mathbf{k}\cdot\mathbf{r}|}k_3 + s(\overline{\sigma})m_3\right)\Psi_1\right]e^{-|\mathbf{k}\cdot\mathbf{r}|} = \left[\partial_1\Psi_3 + \left(-\frac{\mathbf{k}\cdot\mathbf{r}}{|\mathbf{k}\cdot\mathbf{r}|}k_1 + s(\overline{\sigma})m_1\right)\Psi_3\right]e^{-|\mathbf{k}\cdot\mathbf{r}|} \\ \left[\partial_2\Psi_1 + \left(-\frac{\mathbf{k}\cdot\mathbf{r}}{|\mathbf{k}\cdot\mathbf{r}|}k_2 + s(\overline{\sigma})m_2\right)\Psi_1\right]e^{-|\mathbf{k}\cdot\mathbf{r}|} = \left[\partial_1\Psi_2 + \left(-\frac{\mathbf{k}\cdot\mathbf{r}}{|\mathbf{k}\cdot\mathbf{r}|}k_1 + s(\overline{\sigma})m_1\right)\Psi_2\right]e^{-|\mathbf{k}\cdot\mathbf{r}|} \end{cases}$$

$$\Rightarrow \left\{ \begin{array}{|c|c|c|} \hline \partial_3\Psi_2 = \partial_2\Psi_3 & -\frac{\mathbf{k}\cdot\mathbf{r}}{|\mathbf{k}\cdot\mathbf{r}|}k_3\Psi_2 = -\frac{\mathbf{k}\cdot\mathbf{r}}{|\mathbf{k}\cdot\mathbf{r}|}k_2\Psi_3 & s(\overline{\sigma})m_3\Psi_2 = s(\overline{\sigma})m_2\Psi_3 \\ \hline \partial_3\Psi_1 = \partial_1\Psi_3 & -\frac{\mathbf{k}\cdot\mathbf{r}}{|\mathbf{k}\cdot\mathbf{r}|}k_3\Psi_1 = -\frac{\mathbf{k}\cdot\mathbf{r}}{|\mathbf{k}\cdot\mathbf{r}|}k_1\Psi_3 & s(\overline{\sigma})m_3\Psi_1 = s(\overline{\sigma})m_1\Psi_3 \\ \hline \partial_2\Psi_1 = \partial_1\Psi_2 & -\frac{\mathbf{k}\cdot\mathbf{r}}{|\mathbf{k}\cdot\mathbf{r}|}k_2\Psi_1 = -\frac{\mathbf{k}\cdot\mathbf{r}}{|\mathbf{k}\cdot\mathbf{r}|}k_1\Psi_2 & s(\overline{\sigma})m_2\Psi_1 = s(\overline{\sigma})m_1\Psi_2 \\ \hline \end{array} \right|$$

$$\Rightarrow \left\{ \begin{array}{|c|c|c|} \hline \partial_3\Psi_2 = \partial_2\Psi_3 & \Psi_3 = \frac{k_3}{k_2}\Psi_2 & \Psi_3 = \frac{m_3}{m_2}\Psi_2 \\ \hline \partial_3\Psi_1 = \partial_1\Psi_3 & \Psi_3 = \frac{k_3}{k_1}\Psi_1 & \Psi_3 = \frac{m_3}{m_1}\Psi_1 \\ \hline \partial_2\Psi_1 = \partial_1\Psi_2 & \Psi_2 = \frac{k_2}{k_1}\Psi_1 & \Psi_2 = \frac{m_2}{m_1}\Psi_1 \\ \hline \end{array} \right|$$

$$\Rightarrow \left\{ \begin{array}{|c|} \hline \frac{k_3}{k_1} = \frac{m_3}{m_1} \ , \ \frac{k_3}{k_2} = \frac{m_3}{m_2} \ , \ \Psi_3 = \frac{m_3}{m_2}\Psi_2 = \frac{m_3}{m_1}\Psi_1 \\ \hline \partial_3\Psi_2 = \frac{m_3}{m_2}\partial_2\Psi_2 \ , \ \partial_2\Psi_1 = \frac{m_2}{m_1}\partial_1\Psi_1 \ , \ \partial_3\Psi_1 = \frac{m_3}{m_1}\partial_1\Psi_1 \\ \hline \Rightarrow \Psi_i = \kappa_i\mathbf{m}\cdot\mathbf{r} + \psi_i(x^0) \\ \hline \Rightarrow A^i_{\overline{\sigma}} = [\kappa_i\mathbf{m}\cdot\mathbf{r} + \psi_i(x^0)]e^{-|\mathbf{m}\cdot\mathbf{r}|} \ , \ (i \in \{1,2,3\}) \\ \hline \end{array} \right|$$

If: $A^i_{\overline{\sigma}} = \Psi_i(x^0,x^1,x^2,x^3)e^{-|\mathbf{m}\cdot\mathbf{r}|}$ ($\mathbf{E}$ scenario)

$$\begin{cases} 0 = E^1_{\overline{\sigma}} \equiv -(\partial_0 + s(\overline{\overline{\sigma}})m_0)A^1_{\overline{\sigma}} - (\partial_1 + s(\overline{\sigma})m_1)A^0_{\overline{\sigma}} \Rightarrow (\partial_0 + s(\overline{\overline{\sigma}})m_0)A^1_{\overline{\sigma}} = -(\partial_1 + s(\overline{\sigma})m_1)A^0_{\overline{\sigma}} \\ 0 = E^2_{\overline{\sigma}} \equiv -(\partial_0 + s(\overline{\overline{\sigma}})m_0)A^2_{\overline{\sigma}} - (\partial_2 + s(\overline{\sigma})m_2)A^0_{\overline{\sigma}} \Rightarrow (\partial_0 + s(\overline{\overline{\sigma}})m_0)A^2_{\overline{\sigma}} = -(\partial_2 + s(\overline{\sigma})m_2)A^0_{\overline{\sigma}} \\ 0 = E^3_{\overline{\sigma}} \equiv -(\partial_0 + s(\overline{\overline{\sigma}})m_0)A^3_{\overline{\sigma}} - (\partial_3 + s(\overline{\sigma})m_3)A^0_{\overline{\sigma}} \Rightarrow (\partial_0 + s(\overline{\overline{\sigma}})m_0)A^3_{\overline{\sigma}} = -(\partial_3 + s(\overline{\sigma})m_3)A^0_{\overline{\sigma}} \end{cases}$$

$$\Rightarrow \begin{cases} \left[\partial_0\Psi_1 + \left(-\frac{\mathbf{k}\cdot\mathbf{r}}{|\mathbf{k}\cdot\mathbf{r}|}k_0 + s(\overline{\overline{\sigma}})m_0\right)\Psi_1\right]e^{-|\mathbf{k}\cdot\mathbf{r}|} = -\left[\partial_1\Psi_0 + \left(-\frac{\mathbf{k}\cdot\mathbf{r}}{|\mathbf{k}\cdot\mathbf{r}|}k_1 + s(\overline{\sigma})m_1\right)\Psi_0\right]e^{-|\mathbf{k}\cdot\mathbf{r}|} \\ \left[\partial_0\Psi_2 + \left(-\frac{\mathbf{k}\cdot\mathbf{r}}{|\mathbf{k}\cdot\mathbf{r}|}k_0 + s(\overline{\overline{\sigma}})m_0\right)\Psi_2\right]e^{-|\mathbf{k}\cdot\mathbf{r}|} = -\left[\partial_2\Psi_0 + \left(-\frac{\mathbf{k}\cdot\mathbf{r}}{|\mathbf{k}\cdot\mathbf{r}|}k_2 + s(\overline{\sigma})m_2\right)\Psi_0\right]e^{-|\mathbf{k}\cdot\mathbf{r}|} \\ \left[\partial_0\Psi_3 + \left(-\frac{\mathbf{k}\cdot\mathbf{r}}{|\mathbf{k}\cdot\mathbf{r}|}k_0 + s(\overline{\overline{\sigma}})m_0\right)\Psi_3\right]e^{-|\mathbf{k}\cdot\mathbf{r}|} = -\left[\partial_3\Psi_0 + \left(-\frac{\mathbf{k}\cdot\mathbf{r}}{|\mathbf{k}\cdot\mathbf{r}|}k_3 + s(\overline{\sigma})m_3\right)\Psi_0\right]e^{-|\mathbf{k}\cdot\mathbf{r}|} \end{cases}$$

$$\Rightarrow \left\{ \begin{array}{|c|c|c|} \hline \partial_0\Psi_1 = -\partial_1\Psi_0 & \Psi_1 = -\frac{k_1}{k_0}\Psi_0 & \Psi_1 = \frac{m_1}{m_0}\Psi_0 \\ \hline \partial_0\Psi_2 = -\partial_2\Psi_0 & \Psi_2 = -\frac{k_2}{k_0}\Psi_0 & \Psi_2 = \frac{m_2}{m_0}\Psi_0 \\ \hline \partial_0\Psi_3 = -\partial_3\Psi_0 & \Psi_3 = -\frac{k_3}{k_0}\Psi_0 & \Psi_3 = \frac{m_3}{m_0}\Psi_0 \\ \hline \end{array} \right|$$

$$\Rightarrow \begin{cases} \dfrac{k_1}{k_0} = -\dfrac{m_1}{m_0} \;,\; \dfrac{k_2}{k_0} = -\dfrac{m_2}{m_0} \;,\; \dfrac{k_3}{k_0} = -\dfrac{m_3}{m_0} \Rightarrow \dfrac{k_3}{k_1} = \dfrac{m_3}{m_1} \;,\; \dfrac{k_3}{k_2} = \dfrac{m_3}{m_2} \\ \Rightarrow \Psi_0 = k \Rightarrow \Psi_i = \dfrac{m_i}{m_0} k \\ \Rightarrow A^h_{\overline{\sigma}} = \dfrac{m_h}{m_0} k e^{-|\mathbf{m}\cdot\mathbf{r}|} \;,\; (h \in \{0,1,2,3\}) \end{cases}$$

Combining these two yields these potentials for the vacuum.

The arbitrary function of x^0 will be forced to a constant, and the arbitrary constants

κ_i will have to match vanish, yielding the following potentials.

$$A^h_{\overline{\sigma}} = k e^{-|\mathbf{m}\cdot\mathbf{r}|} \;,\; (h \in \{0,1,2,3\})$$

Note that:

$$\Box(k e^{-|\mathbf{m}\cdot\mathbf{r}|}) = k \sum_{h=0}^{3} \partial_i (\partial_i e^{-|\mathbf{m}\cdot\mathbf{r}|}) = k \sum_{h=0}^{3} \partial_i \left(-\frac{\mathbf{m}\cdot\mathbf{r}}{|\mathbf{m}\cdot\mathbf{r}|} m_i e^{-|\mathbf{m}\cdot\mathbf{r}|} \right)$$

$$= -k \sum_{h=0}^{3} m_i \partial_i \left(\frac{\mathbf{m}\cdot\mathbf{r}}{|\mathbf{m}\cdot\mathbf{r}|} e^{-|\mathbf{m}\cdot\mathbf{r}|} \right)$$

$$= -k \sum_{h=0}^{3} m_i \left[\left(\frac{1}{|\mathbf{m}\cdot\mathbf{r}|} m_i + (\mathbf{m}\cdot\mathbf{r})[-|\mathbf{m}\cdot\mathbf{r}|^{-2}] \frac{\mathbf{m}\cdot\mathbf{r}}{|\mathbf{m}\cdot\mathbf{r}|} m_i \right) e^{-|\mathbf{m}\cdot\mathbf{r}|} + \frac{\mathbf{m}\cdot\mathbf{r}}{|\mathbf{m}\cdot\mathbf{r}|} \left(-\frac{\mathbf{m}\cdot\mathbf{r}}{|\mathbf{m}\cdot\mathbf{r}|} m_i \right) e^{-|\mathbf{m}\cdot\mathbf{r}|} \right.$$

$$= -k \sum_{h=0}^{3} m_i \left[\frac{m_i}{|\mathbf{m}\cdot\mathbf{r}|} \left(1 - \frac{(\mathbf{m}\cdot\mathbf{r})^2}{|\mathbf{m}\cdot\mathbf{r}|^2} \right) e^{-|\mathbf{m}\cdot\mathbf{r}|} - \frac{(\mathbf{m}\cdot\mathbf{r})^2}{|\mathbf{m}\cdot\mathbf{r}|^2} m_i e^{-|\mathbf{m}\cdot\mathbf{r}|} \right]$$

But: $(\mathbf{m}\cdot\mathbf{r})^2 = |\mathbf{m}\cdot\mathbf{r}|^2 \Rightarrow \frac{(\mathbf{m}\cdot\mathbf{r})^2}{|\mathbf{m}\cdot\mathbf{r}|^2} = 1$

(naturally, since $\frac{\mathbf{m}\cdot\mathbf{r}}{|\mathbf{m}\cdot\mathbf{r}|}$ is a unit vector

$\Rightarrow \frac{\mathbf{m}\cdot\mathbf{r}}{|\mathbf{m}\cdot\mathbf{r}|} = \pm 1 \Rightarrow \partial_i \left(\frac{\mathbf{m}\cdot\mathbf{r}}{|\mathbf{m}\cdot\mathbf{r}|} \right) = 0$)

$$\Rightarrow \Box(k e^{-|\mathbf{m}\cdot\mathbf{r}|}) = k \sum_{h=0}^{3} m_i^2 e^{-|\mathbf{m}\cdot\mathbf{r}|} = \left(\sum_{h=0}^{3} m_i^2 \right) (k e^{-|\mathbf{m}\cdot\mathbf{r}|}) = |\mathbf{m}|^2 (k e^{-|\mathbf{m}\cdot\mathbf{r}|})$$

$$\Rightarrow (\Box - |\mathbf{m}|^2)(k e^{-|\mathbf{m}\cdot\mathbf{r}|}) = 0$$

where, as usual: $\mathbf{m}\cdot\mathbf{r} \equiv \sum_{i=0}^{3} m_i x^i \;,\; |\mathbf{m}|^2 \equiv \sum_{i=0}^{3} m_i^2$

So: $k e^{-|\mathbf{m}\cdot\mathbf{r}|}$ satisfies the Klein-Gordon equation.

[mass is not generally considered as a four-vector, but considering it
[as such is consistent with the mass-generalized Maxwell's equations and
[this solution.

Since $x^0 = ct$, $\mathbf{m}\cdot\mathbf{r}$ is of the form $\mathbf{k}\cdot\mathbf{r} - \omega t$ (for 3-space $\mathbf{r}$).
Thus, such a potential is represented as a traveling wave soliton solution of the
Klein-Gordon equation.
(Note that zero-densities dual potentials (σ & $\overline{\sigma}$) may exist as such in the vacuum.)

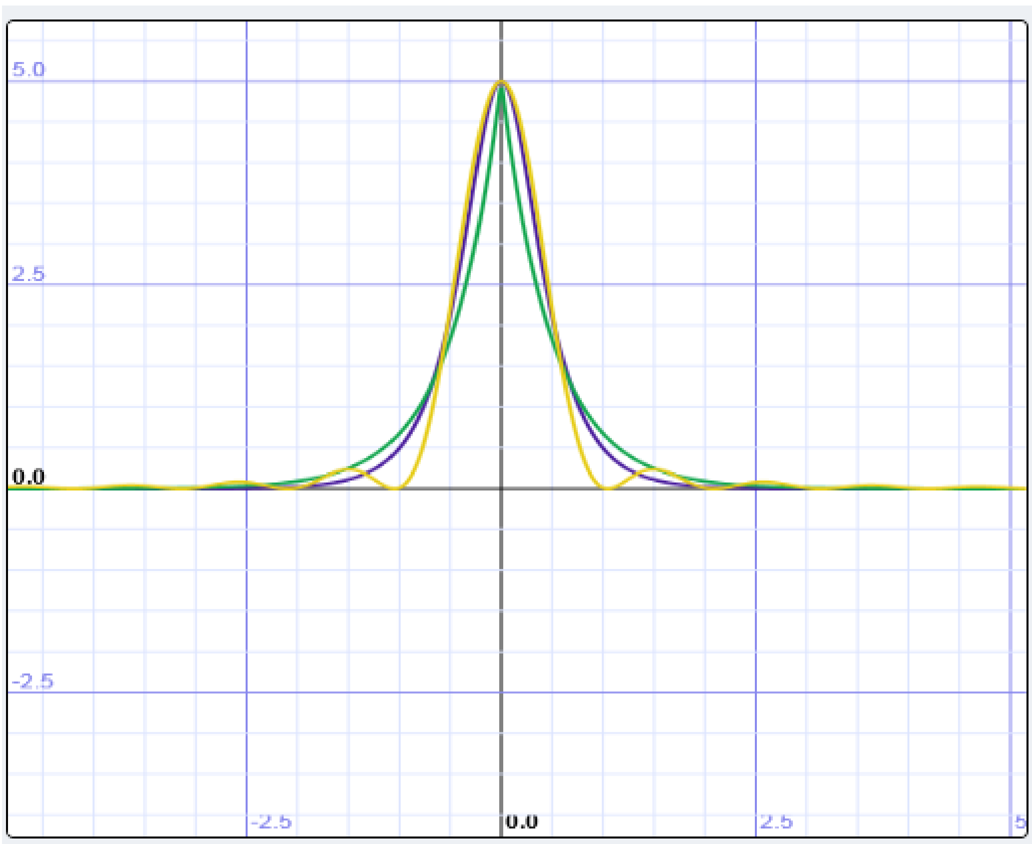
Thus:

$ke^{-|\mathbf{m}\cdot\mathbf{r}|}$ **is a soliton solution of the Klein-Gordon equation**,

where, as usual: $\mathbf{m}\cdot\mathbf{r} \equiv \sum_{i=0}^{3} m_i x^i$, $|\mathbf{m}|^2 \equiv \sum_{i=0}^{3} m_i^2$

Note the similarity of the graphs of some functions used to describe such phenomena by well known and respected quantum physics expert authors.

$\frac{5}{\cosh(3x)}$ (blue)[9] , $5e^{-|2x|}$ (green) , $5\frac{\sin^2(3x)}{(3x)^2}$ (gold)[10]



Note that $\frac{5}{\cosh(3x)}$ (blue) & $5\frac{\sin^2(3x)}{(3x)^2}$ (gold) are both differentiable functions, so there is no need to do analysis of it in $\mathbf{L}^2$. $5e^{-|2x|}$ (green) , on the other hand, is not differentiable at $x = 0$. It is Lebesgue integrable, so is in $\mathbf{L}^1$ (the equivalence classes of Lebesgue integrable functions). It is also easy to demonstrate that it is in $\mathbf{L}^2$ the traditional quantum mechanics Hilbert space. So, though analysis as a differentiable function may not always be appropriate on this function, analysis as an $\mathbf{L}^2$ function always is. There must be a reason why differential analysis is insufficient to describe quantum mechanical effects, while Hilbert space analysis gives good results. $ke^{-|\mathbf{m}\cdot\mathbf{r}|}$ field strength potentials gives such a reason.

Further, $\psi\psi^*$ is a mathematical construct, so that it is a soliton is interesting, but not necessarily physically relevent. $ke^{-|\mathbf{m}\cdot\mathbf{r}|}$, on the other hand, is a physical manifestation of the field strengths of the object in question; so representing a soliton in 3-space gives an accurate represention of the physical phenomenon.

And, further still, neither $\frac{5}{\cosh(3x)}$ (blue) nor $5\frac{\sin^2(3x)}{(3x)^2}$ (gold) are solutions of the Klein-Gordon equation, which the chargeless and vacuum states must be. $ke^{-|\mathbf{m}\cdot\mathbf{r}|}$, on the other hand, is a solution of the Klein-Gordon equation, so gives a meaningful represention of the chargeless and vacuum conditions; and some variation leading to this with zero charge may give an accurate represention of the field strengths.

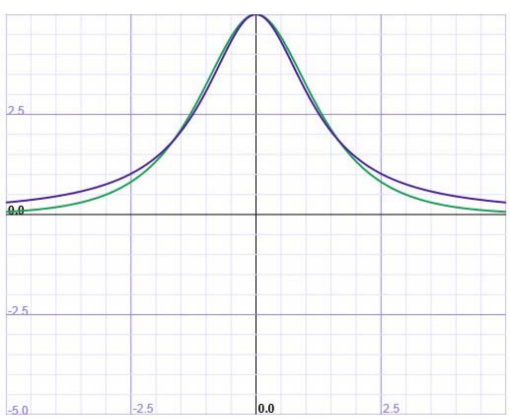
$e^{-\lambda|x|}$ is noted in exercise 2.2 of [9] , but the above establishes a proof of the generalzation as a soliton solution of the Klein-Gordon equation.

The correlation appears to be:

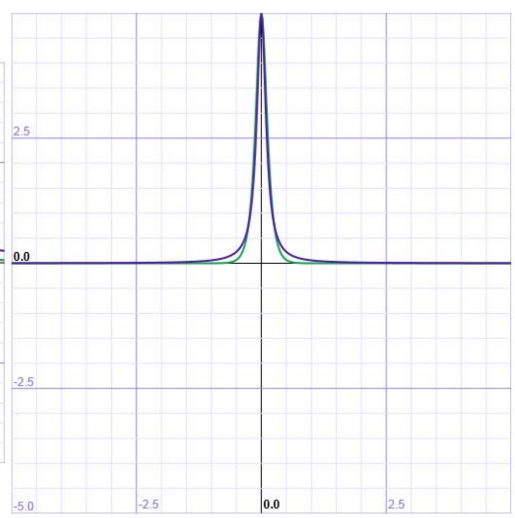
$$\frac{k}{\cosh(\lambda x)} : k\frac{\sin^2(\lambda x)}{(\lambda x)^2} : ke^{-|ax|}, [a \approx \tfrac{3}{4}\lambda].$$

On a related note, a similar correlation may be made to a Cauchy/Breit–Wigner distribution.

$$\frac{k}{\cosh(\lambda x)} : \frac{ka^2}{(kx)^2 + a^2}, \left[\lambda \approx \frac{c}{(a^2)^m}; \left\{ \begin{array}{l} m \approx \frac{\ln 2}{\ln 5} \\ c \approx 10(.3)^m \end{array} \right\} \right].$$



a^2=40



a^2=0.3

Now that a soliton solution of the Klein-Gordon equation has been established, indicative of a stable object which the preceeding analysis demonstrates that certain of these are fundamental; the use of this soliton solution in modeling the probability density function of quantum mechanics would be a natural following step.

After decades of experimental confirmations, there is little doubt that the probability density distribution model of QED accurately represents facts of physical phenomena within it's domain. However, all other probability distributions prescribe information concerning an underlying real range from a real domain. Some of QED, like statistical mechanics is statistical in nature, requiring probability theory analysis (like describing behavior of systems of indistinguishible identical particles - akin to the various problems of picking colored balls from a hat). Other parts of QED may be better understood if the underlying physical reality was well modeled. That is the object of this, earlier, and future efforts.

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