Are the Kelvin and Clausius statements of second law of thermodynamics mutually consistent?

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Abstract: The most standard and fundamental statements of the second law of thermodynamics are those of Kelvin and Clausius. They form the foundation for the structure of thermodynamics. Essentially they are statements of impossibility of certain processes in nature. Every standard book on thermodynamics ritualistically demonstrates the consistency between these two statements by showing violation of one leads to violation of the other. We show in this note, that the two statements are mutually inconsistent. That is, we show violation of one does not lead to violation of the other. We adopt a procedure similar to the one used by Fermi for our demonstration.

Keywords: Thermodynamics, Second law, Kelvin's proposition, Clausius proposition, Inconsistent second law statements.

1. Introduction

Thermodynamics occupies a very important place in the arena of modern science, engineering, and technology. Its importance stems mainly from the second of the four laws of thermodynamics. It has one and a half centuries of history and remains most controversial among the laws of physics even to this day. It plays crucial role in philosophical discussions on evolution, origin of universe, Black holes and so on. The importance attached to thermodynamics can be understood from the fact that, unlike any other subject (with the exception of mathematics perhaps) it is taught to the students of Physics, chemistry, biology, astronomy, geology, chemical, mechanical, metallurgical, automobile, architecture engineering & technology. There is, however, another side to thermodynamics (see next paragraph) that must be kept in mind while dealing with thermodynamics:

No other part of science has contributed as much as to the liberation of the human spirit as the second law of thermodynamics. Yet at the same time, few other parts of science are held to be so recondite. Mention of the second law raises visions of lumbering steam engines, intricate mathematics, and infinitely incomprehensible entropy. Not many would pass C. P. Snow's test of general literacy, in which not knowing the second law is equivalent to not having read a work of Shakespeare [1]. In the introduction of his book, "The Tragicomedy of Classical thermodynamics", Truesdell [2] says: I shall discuss one (field) accursed by misunderstanding, irrelevance, retreat, and failure: Thermodynamics in the 19th century."

The most important second law of thermodynamics is stated in many ways, two of which are treated as standard – one due to Kelvin and the other due to Clausius. The two statements are given in a precise form by Fermi [3]. We quote them below.

"*A transformation whose only final result is to transform into work heat extracted from a source which is at the same temperature throughout is impossible*" (Postulate of Lord Kelvin).

"*A transformation whose only final result is to transfer heat from a body at a given temperature to a body at a higher temperature is impossible*" (Postulate of Clausius).

Fermi [3] demonstrates the consistency between the above two postulates by showing violation of one leads to violation of the other. Most of the books on thermodynamics [3-5] do the same. Following a similar approach adopted by Fermi, we demonstrate the inconsistency between the two statements. That is, we show violation of one does not lead to violation of the other.

We first give the two postulates in terms of the processes they assert to be impossible in Fi.1 and Fig. 2. Then we give Fermi's demonstration of consistency between the two. We then follow it by our demonstration of the mutual inconsistency between the two postulates.

2. The impossible process underlying Kelvin's postulate of the second law

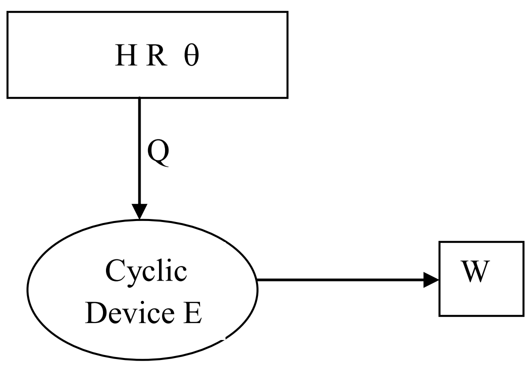


Fig. 1 depicts a cyclic process in which the device E absorbs Q units of heat from a heat reservoir (HR) at uniform temperature θ and transforms it completely into W units of work with no other change elsewhere. Kelvin asserts that it is an impossible process.

3. The impossible process underlying Clausius postulate of the second law

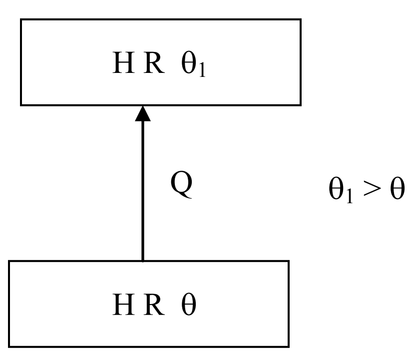


Fig. 2 depicts a process whose only final result is transfer of heat from a HR at a uniform temperature θ to a HR at a higher temperature θ_1 ($> \theta$) with no other change elsewhere. Clausius asserts that it is an impossible process.

4. Fermi's demonstration of consistency between Kelvin and Clausius postulates of the second law.

This demonstration consists of two parts. The first part (**4a**) shows violation of Kelvin's process leads to violation of Clausius process. The second part (**4b**) shows violation of Clausius process leads to violation of Kelvin's process.

4a. Violation of Kelvin's process leads to violation of Clausius process

Let us assume, in violation of Kelvin's statement that a process whose only final result is transformation into work heat absorbed from a body (say, a HR) at a uniform temperature θ throughout is possible. We can then transform this work completely into heat using Joule's heating process and supply the heat to a HR at a higher temperature θ_1 ($>\theta$). The only final result of the process then would be: transfer of heat from a lower temperature body (HR at θ) to a higher temperature body (HR at θ_1) with no other change elsewhere. Such a process violates Clausius statement of second law. Thus Fermi demonstrates that violation of Kelvin's statement leads to violation of Clausius statement. This whole process is depicted in Fig. 3 below.

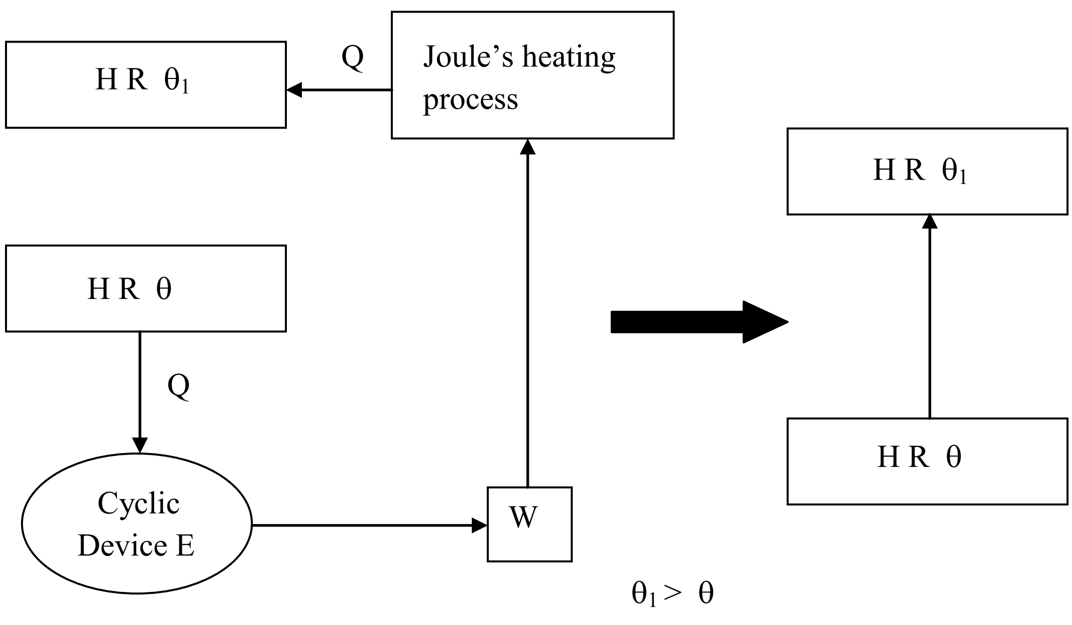


Fig. 3 depicts a compound cyclic process in which Kelvin's process shown in Fig. 1 is coupled to a Joule's heating process that transforms W units of work completely into Q units of heat. The only final result of the whole process is transfer of heat from a HR at temperature θ to a HR at temperature θ_1 ($>\theta$). This process violates Clausius assertion that it is an impossible process.

4a. Violation of Kelvin's process leads to violation of Clausius process

Let us assume in violation of Clausius statement that, a process whose only final result is transfer of heat from a body at a lower temperature to a body at a higher temperature is impossible, Q units of heat is transferred from a low temperature body (HR at temperatureθ) to a HR at temperature θ_1 ($>\theta$) with no other change elsewhere. We operate a Carnot heat engine (CHE) that interacts with HRs at temperatures θ_1 and θ. Let the CHE absorb Q units of heat from HR at θ_1 convert a portion of it into W units of work and reject the remaining portion of heat Q_2 (= Q-W) to HR at θ. The only final result of this whole process is a loss of ($Q - Q_2$) units of heat from HR at θ and its transformation completely into W units of work with no other change elsewhere. Such a process violates Kelvin's statement of the second law. Thus Fermi demonstrates that violation of Clausius process leads to violation of Kelvin's process. This whole process is depicted in Fig. 4 below.

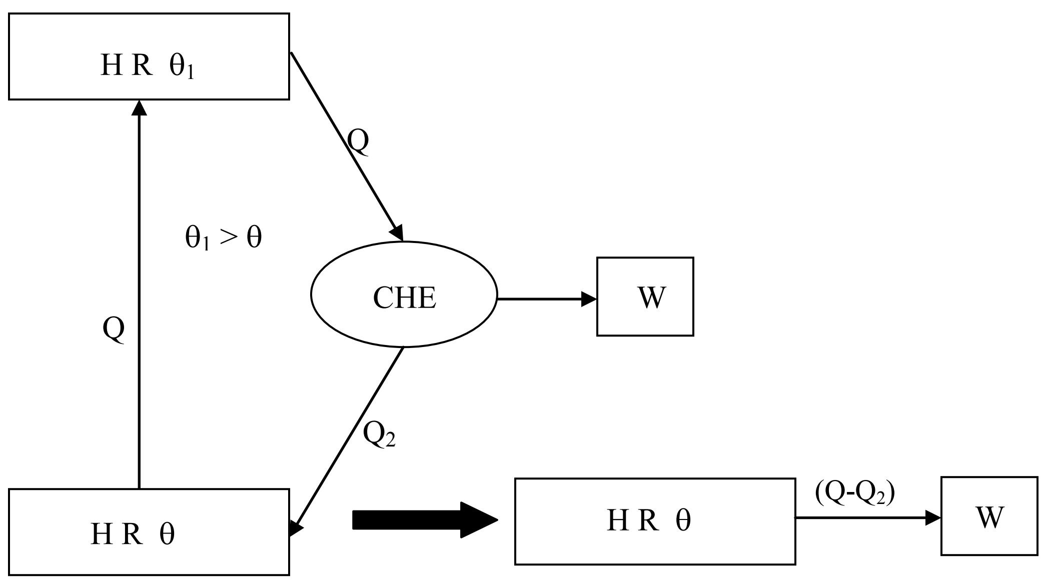


Fig. 4 depicts a compound cycle in which the process shown in Fig. 2 is coupled to a Carnot heat engine (CHE) interacting with HRs at temperatures θ and θ_1. It absorbs Q units of heat from HR at θ_1 converts a portion of it into W units of work and rejects the remaining heat Q_2 (= Q-W) to HR at θ. The only final result of this whole process is transformation into work heat absorbed from a HR at a uniform temperature θ. This process violates Kelvin's assertion that it is an impossible process.

5. Violation of Kevin's process does not violate Clausius process

We now demonstrate that violation of Kevin's process does not violate Clausius process. A simple modification of the process used by Fermi is employed for this purpose.

Let us assume in violation of Kelvin's assertion that Q units of heat absorbed from a HR at θ is completely transformed into W units of work with no other change elsewhere. We transform this W units of work into Q units of heat using Joule's heating process and transfer it to HR at temperature θ_2 ($<\theta$). The net result of the process is transfer of heat from a HR at θ to a HR at θ_2 ($<\theta$) with no other change elsewhere. This process does not violate Clausius process. Thus we demonstrate that violation of Kelvin's process does not necessarily lead to violation of Clausius process. This whole process is depicted in Fig.5.

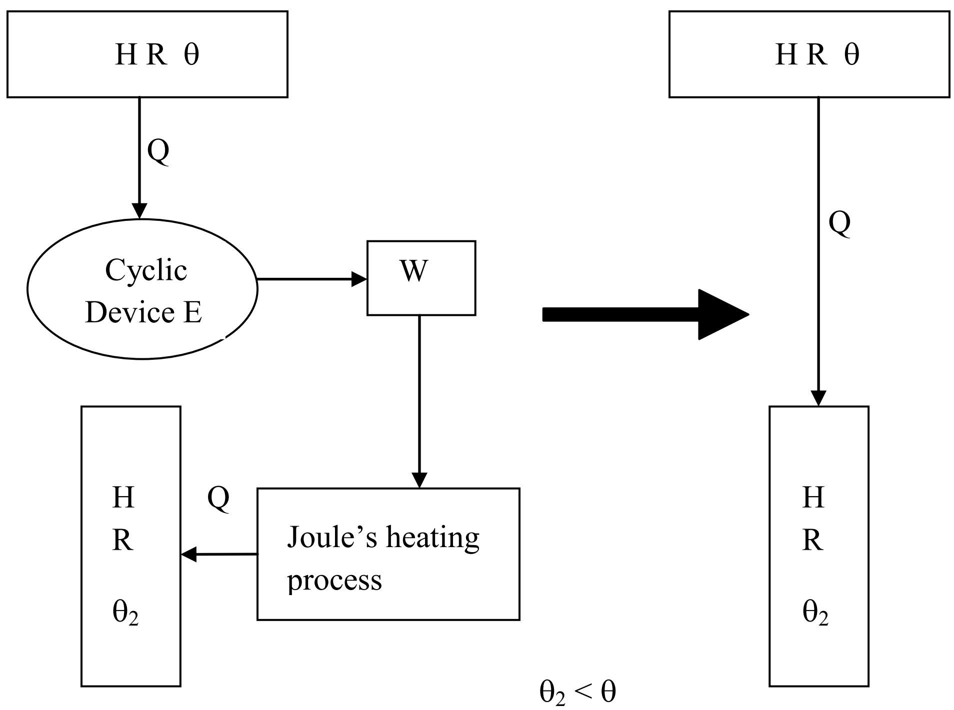
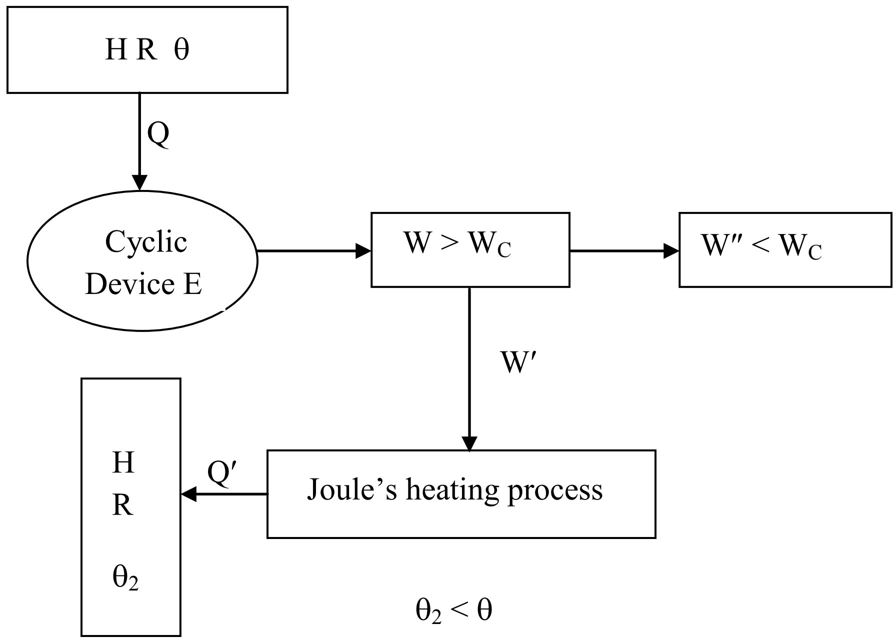


Fig. 5 depicts a compound cyclic process. A cyclic device, E absorbs Q units of heat from a HR at temperature θ and completely converts it into W units of work in violation of Kelvin's proposition. This work W is converted into Q units of heat by Joule's heating process and supplied to HR at temperature θ_2($<\theta$). The net result of the process is transfer of Q units of heat from a HR at temperatureθ to a HR at temperature θ_2 ($<\theta$). This process does not violate Clausius statement of second law. Therefore, violation of Kelvin's process does not necessarily lead to violation of Clausius process.

We can, in fact do even better. Instead of converting the all the W units of work into heat, we can convert an arbitrarily chosen fraction of W′ units into Q′ units of heat by Joule's heating process and transfer the same to a HR at a temperature θ_2. < θ_1. We get W'' (= W-W′) < W_C units of work for every Q units of heat absorbed at temperature θ. W_C is the work output given by a Carnot heat engine interacting with HRs at temperatures θ and θ2. Such a process is depicted in Fig. 6.



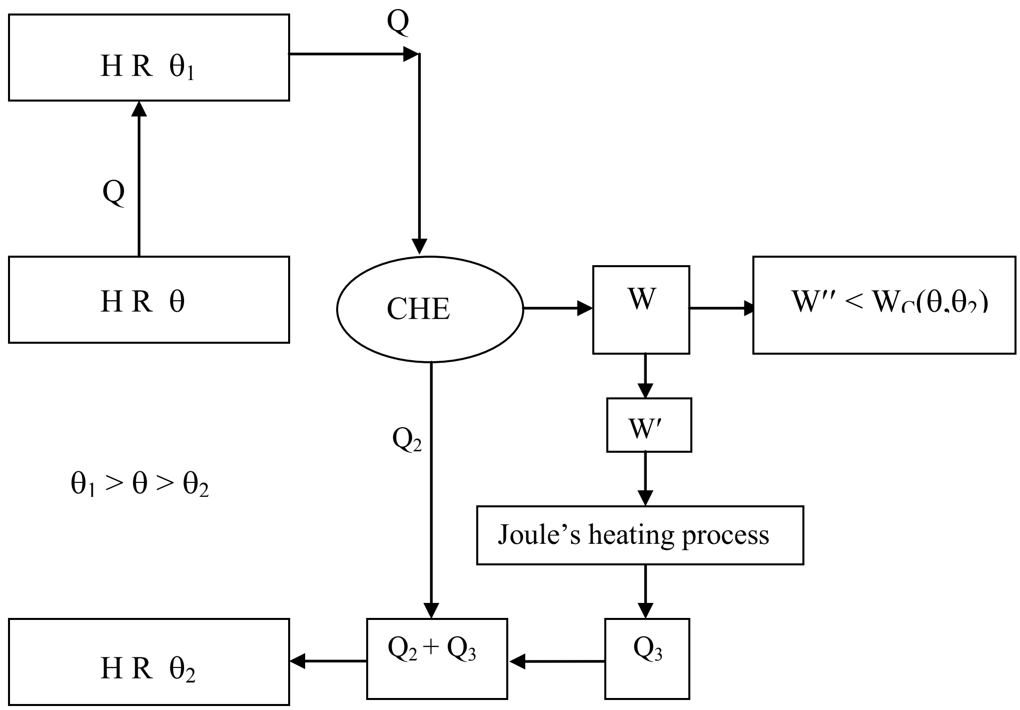
$$W = (W' + W''),\ W' = Q',\ Q = (Q' + W'')$$

Fig. 6 depicts a compound cyclic process. A cyclic device, E absorbs Q units of heat from a HR at temperature θ and completely converts it into W units of work. A fraction of W, say W' is converted into Q' units of heat by Joule's heating process and supplied to HR at temperature $\theta_2(< \theta)$. The net result of the process is loss of Q units of heat from a HR at temperature θ, production of $W'' < W_C$ units of work and rejection of the difference $Q' = (Q - W'')$ as heat to a HR at temperature $\theta_2(< \theta)$.

6. Violation of Clausius process does not violate Kevin's process

We now demonstrate that violation of Clausius process does not violate Kevin's process. Here again, a simple modification of the process used by Fermi is employed.

Let us assume in violation of Clausius assertion that Q units of heat flows spontaneously from a body say a HR, at temperature θ to a HR at higher temperature θ_1 with no other change elsewhere. We operate a Carnot heat engine between temperatures θ_1 and θ_2 (< θ). It absorbs the Q units of heat from HR at θ_1, transforms a fraction of it into W units of work and rejects the remaining Q_2 (= Q_1 - W) units of heat to HR at θ_2. A fraction W' units of work is converted into heat using Joules process and transferred to HR at temperature θ_2. This process does not violate Kelvin's statement of the second law. Thus we demonstrate that violation of Clausius process does not necessarily violate Kelvin's statement of second law. We depict this process in Fig.7.



$$\theta_1 > \theta > \theta_2,\ (Q - Q_2) = W > W_C(\theta,\theta_2),\ W'' < W_C(\theta,\theta_2)$$

Fig. 7 depicts a compound cyclic process. In this cycle the process shown in Fig. 2 is coupled to a Carnot heat engine (CHE) interacting with HRs at θ_1 and θ_2. A fraction W' of W is converted into heat using Joule's heating process and transferred to HR at θ_2, so that the net work obtained is less than that produced by a CHE interacting with HRs at θ and θ_2. The overall result is: HR at θ losses Q units of heat and HR at θ_2 gains heat and, W'' units of work is obtained.

Conclusion

Processes have been devised to demonstrate that violation of Kelvin's statement of the second law leads to violation of Clausius statement of the second law and vice versa, to prove the consistency between the two standard forms of the second law. We demonstrated, employing a procedure similar to the one adopted by Fermi that violation of Kelvin's statement of the second law does not necessarily lead to violation of Clausius statement of the second law and vice versa, to prove the inconsistency between the two standard forms of the second law.

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