PLANET SUPPORTING LIFE

Authored by

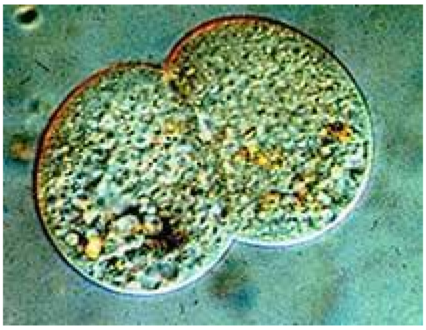
Nehul Yadav
nehul12@gmail.com
9013766717

ABSTRACT

Maths linked with Ecology

It is through mathematical descriptions of ecological systems that we abstract out the basic principles of these systems and determine the implications of these. Ecological systems are enormously complex. A major advantage of mathematical ecology is the capability to selectively ignore much of this complexity and determine whether by doing so we can still explain the major patterns of life on the planet.

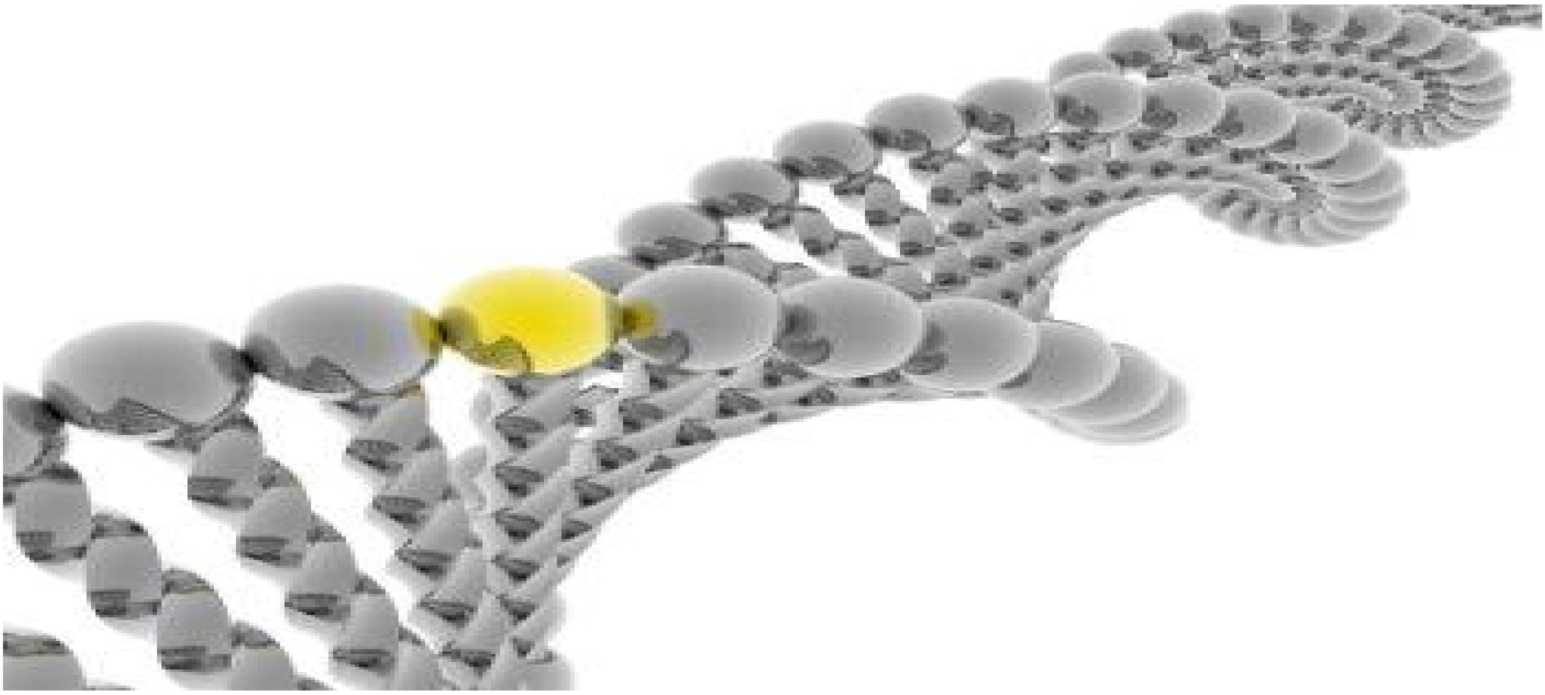
Thus simple population models group together all individuals of the same species and follow only the total number in the population.
Mathematical models in physiological ecology are often compartmental in form, in which the organism is assumed to be composed of several different components. For example, many plant growth models consider leaves, stem and roots as different compartments. The models then make assumptions about how different environmental factors affect the rate of change of biomass or nutrients in different compartments.
These models are typically framed as systems of differential equations with one equation for each compartment.



Maths linked with evolution

Humans are animals that specialize in thinking and knowing, and our extraordinary cognitive abilities have transformed every aspect of our lives. Evolution, as we all comprehend is development of individuals, thereby enhancing the thinking capability of the newer generation. Patterns in human thinking become the mathematics of logic.

There is Aristotelian logic and a deductive system (such as is encountered in Euclidean geometry which captures some of our perceived patterns of space). Such logical systems do reflect human thinking, but not all human thinking. Aristotelian logic is a useful approximation -- simplification -- of reality. It has served us very well, and will continue to do so. However, other patterns of thinking have been captured in the mathematics of fuzzy logic, and it is interesting to note that billions of dollars of business are built on it.



Here is a small example indicating the usefulness of maths in our environment -

Suppose you go into the jungles of Ecuador and start collecting orchids. You count the number of orchids of each different species that you find. You get a list of numbers, something like this:

14,10,18,6,2,1,1

What is the chance that the next orchid you find will belong to a new species?
Good gives a rule of thumb for solving problems of this type:

`n1/N`

Here `N` is the total number of orchid you collected, and `n1` is the number of species for which you found exactly `i` orchids of that species. In our example,

`n1 = 3`

since we found just one orchid of three different species: those are the three 1’s at the end of our list. Furthermore,

`N = 14+10+8+6+2+1+1`

So here is Good’s estimate the chance that the next orchid you collect will be of a new species:

`n1/N = 3/42`

Good’s argument is nontrivial—and of course it depends on some assumptions on the nature of the distribution of populations of different species!

METHODS

The Fibonacci Series

Our natural surroundings are dynamic, constantly changing, without pattern or cause; or so it seems. Consider the following sequence of numbers:

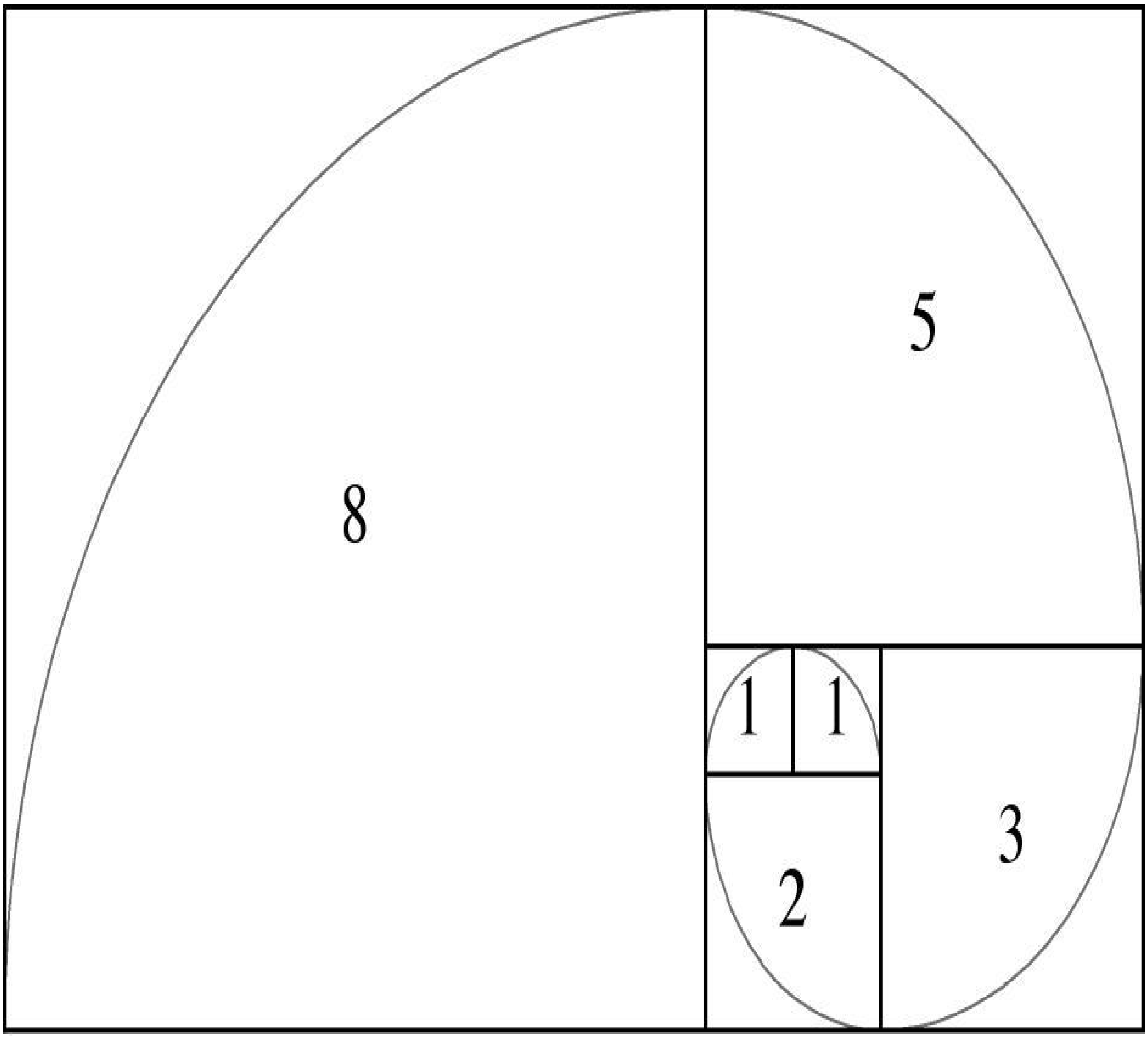
1 1 2 3 5 8 13 21 34 55 89 144 . . . .

The above series is based on a simple rule: each number is the sum of the two numbers preceding it. This seemingly useless sequence was first published in 1202 by a mathematician named Leonardo Fibonacci in his book *Liber Abaci.* In addition to modelling the position of leaves on a stem, the growth of florets on a flower and determining the number of ancestors a bee had a few generations ago, this sequence unmasks the underlying order in nature.

Consider a population which started out with two members of a species (say rabbits). After one breeding season, it was seen that the population had increased from **2** to **3**. The next breeding season it increased from **3** to **5** and so on. The population of rabbits after each breeding season follows the Fibonacci series, assuming that available resources such as land and food are unlimited.(We will get back to population growth later on.) Even the number of branches in a tree at successive levels follows the same sequence.

The Fibonacci Spiral

Consider a series of concentric quarter-arcs (outlines of a quadrant of a circle) with the radius given by successive terms of the Fibonacci sequence. The resulting spiral looks like this:



Now compare this with the growth of a pinecone and a nautilus shell:

As you can see, it is a combination of many fibonacci spirals.

The shape of the nautilus shell closely follows the Fibonacci spiral.

The Bee Ancestry Code:

Sex determination in bees is quite strange:
If an egg is laid by an unmated female, it hatches into a male.
If the egg is laid by a fertilised female, it hatches into a female.

Thus any one male bee **(1 bee)** has one parent **(1 bee),** two grandparents **(2 bees)**, three great-grandparents **(3 bees)**, five great-great-grandparents **(5 bees)** and so on. The number of ancestors **n** generations back is the **n**th term of the Fibonacci sequence.

If we divide any term **[n^{th} term]** in the Fibonacci sequence by the term preceding it **[$(n - 1)^{th}$ term]**, it can be shown that for large values of n, this quotient approaches 1.618. This number is also called the Golden Ratio or the Divine Proportion.

$$\lim_{\mathbf{n}\to\infty} \frac{\mathbf{Fn}}{\mathbf{Fn-1}} = \mathbf{1.618}$$

It is hidden in the basic anatomy of the human body.The ratio of arm length to distance between fingertips and shoulder, the ratio of palm length to arm length, the ratio of height of navel to overall height, the ratio of femur length to length of legs, all equal **1:1.618**.

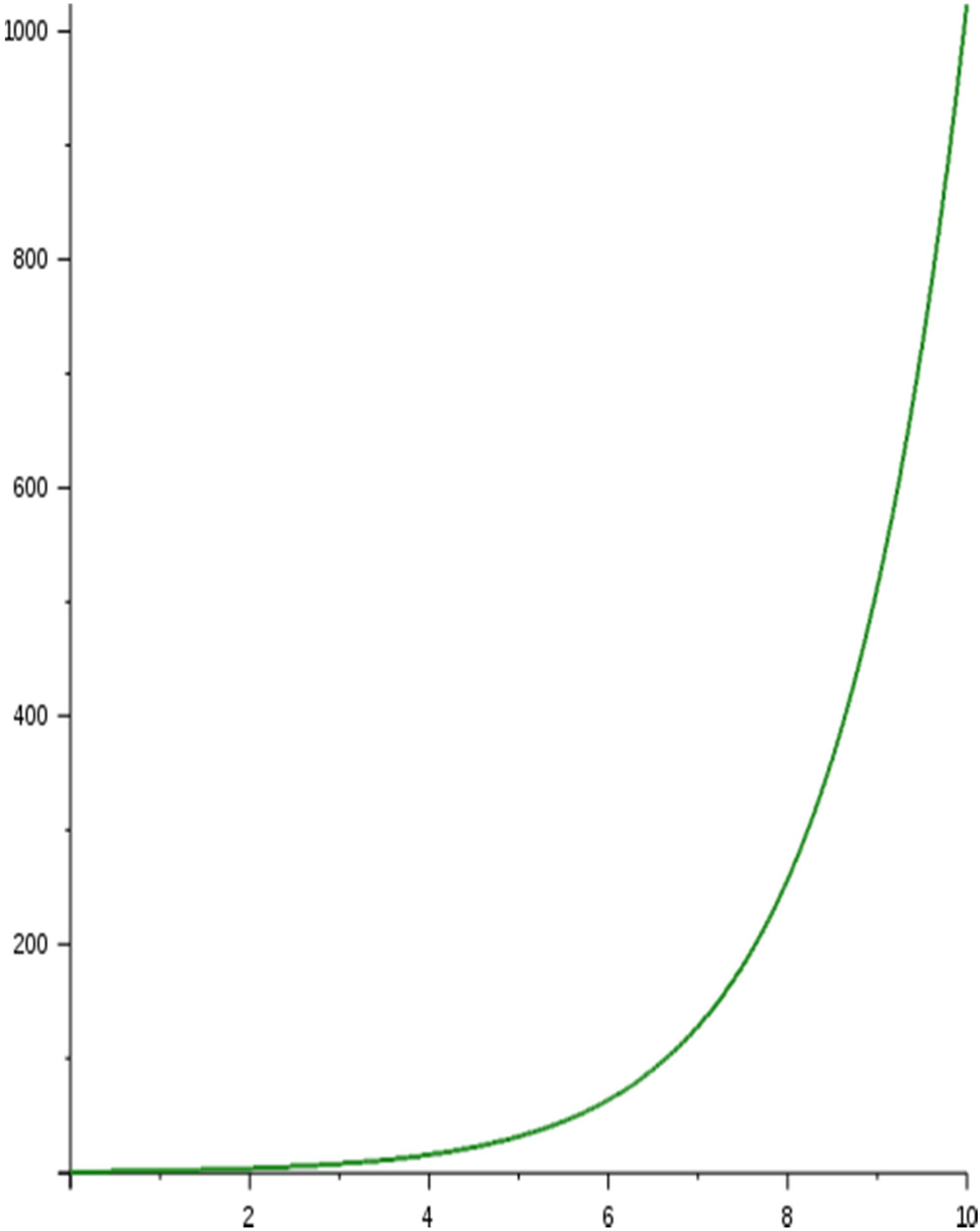
Consider the sunflower. Its seeds grow in opposing spirals and the ratio of each spiral's diameter to the next is also **1:1.618**. In a natural beehive, the females outnumber the males by a ratio of **1:1.618** also.

Now, while all of that was quite interesting, it doesn't have much of a practical application. The next topic, however, has far reaching implications and allows us to predict population of different species.

Population growth models:

Consider a bacterium which divides into two every minute. If, initially, one is introduced into a container and given appropriate conditions, it will start dividing. Its numbers will increase in the following manner after every minute: 1 2 4 8 16 32 64 128 . . . . .

We can simply state that the number of bacteria **y** in the container present at a time **x** minutes after the introduction of the first one is given by the equation: $y = 2^x$. The population growths of different are modeled by a class of functions known as exponential functions which have the general equation: $y = Ce^{kt}$ where *C, e, k* are constants and *t* is the time.



MODEL DEPICTING GROWTH

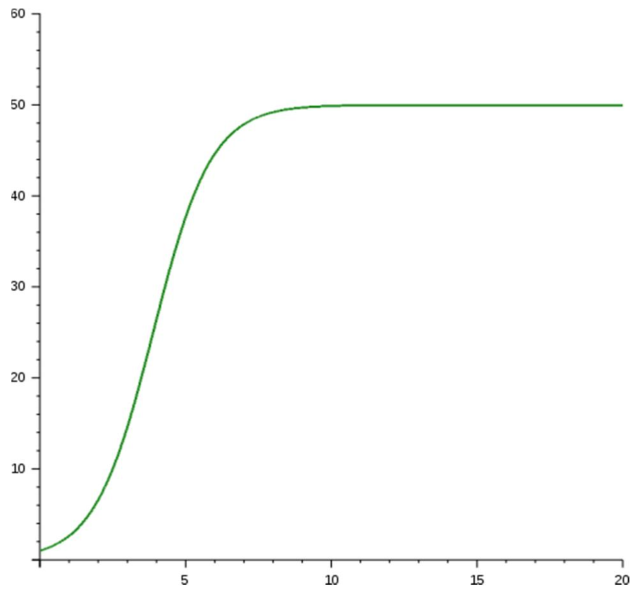
$$y = 2^x$$

It is evident from the graph that the population increases relatively slowly at first and then there is a sudden increase. Such is the case for any population growth, human or animal, provided resources such as land and food are adequate.

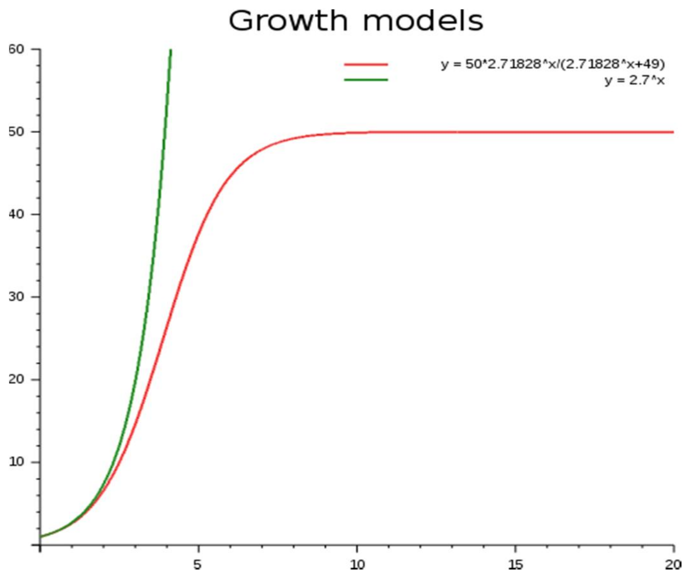
In actuality the population increases till a certain extent and then the graph levels off. So the growth is better modelled by logistic rather than exponential functions.

Logistic functions are of the form: $P(t) = \frac{KP_0e^{rt}}{K + P_0\left(e^{rt} - 1\right)}$

where $P(t)$ is the population at time t, e is Euler's number, P_0 is the initial population and K, r are constants that depend upon the species and its environment. As an example, consider the graph with K= 50 and P_0=r =1:



The graph shows that the population increases rapidly at first and then the growth rate gradually declines as resources like land and food become scarce and finally, due to intense competition the population reaches an upper limit. This model is much more realistic for long time periods than the exponential model as shown by the graph below:



All of this is of utmost importance in ecology. A conservationist will need to know what the population of a particular species will be at a particular time and what factors need to be controlled in order to maximise or minimise the propagation of a particular species; this also has importance in industries which are based upon the culture of animals. As an example, take fish. Most of the fish we eat is not caught but grown in artificial ponds with limited space and food which leads to intense competition for resources among the fishes. A logistic function will help because it indicates the right time to remove the fish from the pond (when their population is maximum).

NOTE : PLEASE SEE THE SUPPORTING ARTICLE ‘THEORETICAL ECOLOGY’ FOR RESULT AND CONCLUSION OF THIS SUPERFLOUS RESEARCH.

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Relation between mathematics and environment/blog.

PLEASE VIEW THE SUPPORTING ARTICLE ‘THEORETICAL ECOLOGY’ FOR FURTHER INFORMATION.