The Concept of the Effective Mass Tensor in GR

Clocks and Rods

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Abstract: *In the paper [1] we presented the concept of the effective mass tensor (EMT) in the General Relativity (GR). According to this concept under the influence of the gravitational field the bare mass tensor becomes the EMT. The concept of the EMT is a new physical interpretation of GR, where the curvature of space-time has been replaced by the EMT. In this paper we consider the concept of the EMT in GR but in the aspect of the clocks and rods.*

keywords: *the effective mass tensor, clocks and rods*

I. Introduction

In the paper [1] we presented the concept of *the effective mass tensor* (EMT) in the General Relativity (GR). According to this concept under the influence of the gravitational field *the bare mass tensor* becomes the EMT. In this paper we again consider the concept of the EMT in the GR but in the aspect of the clocks and rods.

Clocks measure the time and the rods measure the length. In the GR clocks and rods are massless. The main question is sounds: **whether the clocks and rods with the effective mass *m** will measure the same times and the same lengths as clocks and rods with the bare mass *m*?** Our mathematical considerations we will realize in the Schwarzschild metric.

In the paper [1] we postulated that in the particular case the EMT $m_{\mu\nu}$ can **mimics** the metric tensor $g_{\mu\nu}$ and

$$\frac{m_{\mu\nu}}{m} = g_{\mu\nu} \tag{1}$$

where components μ, ν = 0, 1, 2, 3. Therefore the metric

$$ds^2\left(g_{\mu\nu}\right) = ds^2\left(m_{\mu\nu}\right) \tag{2}$$

where: $ds^2\left(g_{\mu\nu}\right) = g_{\mu\nu}dx^\mu dx^\nu$ and $ds^2\left(m_{\mu\nu}\right) = \frac{m_{\mu\nu}}{m}dx^\mu dx^\nu$.

II. Clocks and rods in the Schwarzschild metric

The line element in the Schwarzschild metric has the form

$$c^2 d\tau^2 = -\left(1-\frac{2GM}{c^2 r}\right)c^2 dt^2 + \left(1-\frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \qquad (3)$$

where: τ is a proper time, c is the speed of light, t is the time coordinate, M is the mass of star, r – distance from the star , G is the gravitational constant, θ is the colatitude (angle from North, in units of radians), ϕ is the longitude (also in radians).

The same line element with the effective mass m^* has the form [1]

$$c^2 d\tau^2 = -\left(1-\frac{m^*}{m}\right)c^2 dt^2 + \left(1-\frac{m^*}{m}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \qquad (4)$$

If we take a slice given by t = const we obtain a tree-dimensional manifold with line element

$$ds^2 = \left(1-\frac{m^*}{m}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \qquad (5)$$

Equation (5) we obtained putting $dt = 0$ in equation (4). Thus, the linear element

$$ds^2 = \frac{m_{ij}}{m} dx^i dx^j \qquad (6)$$

where components i, j = 1, 2, 3, (x^1 = r, x^2 = θ, x^3 = ϕ).

We see that the $\frac{m_{ij}}{m}$ is positive-define EMT on this 3-manifold, so the slice is a space rather than a space-time and

$$m_{ij} = m\begin{pmatrix} \left(1-\frac{m^*}{m}\right)^{-1} & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \sin^2\theta \end{pmatrix} \qquad (7)$$

Components of the m_{ij} does not depends on the time t. If we assume that $m^* \to 0$ then the EMT has the form

$$m_{ij} = m\begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \sin^2\theta \end{pmatrix} \qquad (8)$$

Note that the 3-dimmensional EMT has also an interesting form. Component of the $m_{rr} = m$, component of the $m_{\theta\theta} = mr^2$, where mr^2 is *the moment of inertia*. Component of the $m_{\phi\phi} = mr^2 \cdot \sin^2\theta$. The $m_{rr} = m$ and does not depends on direction in the space, while the other components yes.

Let's go back to the equation (5). If we assume that $\theta = const$ and $\phi = const$ then we have the infinitesimal radial distance *dR* in the form

$$dR = dr\sqrt{\frac{m_{rr}}{m}} = \frac{dr}{\sqrt{1-\frac{m^*}{m}}} \tag{9}$$

If we assume that $m^* \uparrow$ then rods with the effective mass will measure different length than the rods with the bare mass and $dR > dr$. But if we assume that $m^* \to 0$ then rods with the effective mass and the rods with the bare mass will measure the same length $dR = dr$.

The measurement results are consistent with GR but their physical meaning was changed. Under the influence of the gravitational field only physical properties of the rods was changed, but not the space properties.

Let us now turn our attention to the time. According to the eq. (4) we have

$$d\tau = dt\sqrt{\frac{\tilde{m}_{tt}}{m}} = dt\sqrt{1-\frac{m^*}{m}} \tag{10}$$

where we assumed that $dr = d\theta = d\phi = 0$ and

$$\frac{\tilde{m}_{tt}}{m} = -\frac{m_{tt}}{m} = \left(1-\frac{m^*}{m}\right) \tag{11}$$

If we assume that $m^* \uparrow$ then clocks with the effective mass will measure different time than the clocks with the bare mass and $d\tau < dt$. But if we assume that $m^* \to 0$ then clocks with the effective mass and the clocks with the bare mass will measure the same time $d\tau = dt$.

The measurement results are consistent with GR but their physical meaning was changed. Under the influence of the gravitational field only physical properties of the clocks was changed, but not the time properties.

Simple two examples (see below) will illustrates our discussion for the properties of the EMT in the Schwarzschild metric.

Example 1. Let us consider a rod of one meter length (*dr* = 1 m) with the bare mass m. What is the length *dR* of one meter for the same rod if we will put it radially in a weak gravitational field for the $\frac{m^*}{m} = 2\cdot 10^{-2}$? The answer is: *dR* = 1.01 meter. In the gravitational field a rod with effective mass m^* will show a length greater than the rod with the bare mass.

Example 2. Let us consider a clock with the bare mass m, which measures the time with an accuracy of the one second (*dt* = 1 s). What is the length $d\tau$ of the one second for the same clock if we will put it in a weak gravitational field for the $\frac{m^*}{m} = 2\cdot 10^{-2}$? The answer is: $d\tau =$ 0.99 s. In the gravitational field a clock with effective mass m^* runs slower than the clock with the bare mass.

III. The EMT in the Schwarzschild metric

The EMT in the Schwarzschild metric has the form (see eq. (1))

$$m_{\mu\nu} = m\begin{bmatrix} -\left(1-\frac{m^*}{m}\right) & 0 & 0 & 0 \\ 0 & \left(1-\frac{m^*}{m}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \, sin^2 \, \theta \end{bmatrix} \tag{12}$$

If we assume that $m^* \to 0$ then the 4-dimmensional EMT has the form

$$m_{\mu\nu} = m\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \, sin^2 \, \theta \end{bmatrix} \tag{13}$$

IV. The EMT anisotropy in *the Solar System*

As we known from the paper [1]

$$\frac{m^*}{m} = \frac{2GM}{c^2 r} \tag{14}$$

Many physicists looks for the (inertial) mass anisotropy [2]. We will show where to look for the mass anisotropy *in the Solar System* if the concept of EMT is true.

The EMT for the Schwarzschild metric has form

$$m_{\mu\nu} = m\begin{bmatrix} -\left(1-\frac{2GM}{c^2 r}\right) & 0 & 0 & 0 \\ 0 & \left(1-\frac{2GM}{c^2 r}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \, sin^2 \, \theta \end{bmatrix} \tag{15}$$

Components of the m_{tt} and m_{rr} depends on the distance r between the planet and the star. As we known in *the Solar System* the distance r between the Earth and the Sun is changed (the orbit is an ellipse). The relative changes in the mass anisotropy for the components m_{tt} and m_{rr} we should observe during a year, measuring from perihelion (aphelion) to perihelion (aphelion).

Component of the m_{tt} is the smallest in the perihelion while the component m_{rr} is the biggest. Annual the biggest relative changes (for example from perihelion to perihelion) for the m_{tt} and for m_{rr} should be measured in *the Solar System* and the estimated value is equal to

$$\frac{|\delta m_{tt}|}{m} = \frac{|\delta m_{rr}|}{m} = \frac{1}{m}\left|(m_{tt})_{perih} - (m_{tt})_{aphel}\right| = \frac{1}{m}\left|(m_{rr})_{perih} - (m_{rr})_{aphel}\right| =$$
$$= \frac{2GM}{c^2}\left|\frac{1}{r_{perih}} - \frac{1}{r_{aphel}}\right| \approx 6{,}6 \cdot 10^{-10} \tag{16}$$

V. Conclusion

In this paper we considered the concept of the EMT in GR in the aspect of the clocks and rods. We have found that the gravitational field has an impact on the masses of the clocks and rods located in this field. In the gravitational field all clocks and rods have the effective mass m^*. When there is no gravitational field all clocks and rods have the bare masses m.

All clocks with the effective mass runs slower than the clocks with the bare mass while a rods with the effective mass will show a length greater than the rods with the bare mass.

The concept of the EMT is a new physical interpretation of the GR, where the curvature of space-time has been replaced by the EMT. We believe that this concept will help better understand the gravitational phenomena.

Appendix

For the similar consideration but for the Newton's metric in the Cartesian coordinates we have 3-dimensional EMT m_{ij}

$$m_{ij} = m\begin{pmatrix} 1+\frac{m^*}{m} & 0 & 0 \\ 0 & 1+\frac{m^*}{m} & 0 \\ 0 & 0 & 1+\frac{m^*}{m} \end{pmatrix} \tag{A1}$$

Components of the m_{ij} does not depends on the time *t*. If we assume in eq. (A1) that $m^* \to 0$ then 3-dimensional EMT in the Cartesian coordinates takes the form

$$m_{ij} = m\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{A2}$$

and does not depends on direction in the space. We see that in Cartesian coordinates the EMT m_{ij} is *the 3-dimensional bare mass tensor*. This tensor is *isotropic*[1] and physically represents *the mass isotropy* in the 3-dimensional space. This mass isotropy is generally interpreted as *a scalar*.

[1] The 3-dimensional bare mass tensor is isotropic if and only if it his properties do not depend on the direction in the space (all components have the same value in all rotated coordinate systems).

The 4-dimensional EMT $m_{\mu\nu}$ in the Newton's metric in the Cartesian coordinates has form

$$m_{\mu\nu} = m\begin{bmatrix} -\left(1-\frac{m^*}{m}\right) & 0 & 0 & 0 \\ 0 & \left(1+\frac{m^*}{m}\right) & 0 & 0 \\ 0 & 0 & \left(1+\frac{m^*}{m}\right) & 0 \\ 0 & 0 & 0 & \left(1+\frac{m^*}{m}\right) \end{bmatrix} \tag{A3}$$

If we assume that $m^* \to 0$ then EMT has the form

$$m_{\mu\nu} = m\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A4}$$

and does not depends on direction in the space-time. We see that in Cartesian coordinates the EMT $m_{\mu\nu}$ is *the 4-dimensional bare mass tensor*. This tensor is also isotropic[2] and physically represents the mass isotropy in the 4-dimensional space.

So far, the concept of the mass isotropy was associated only with the 3-dimensional space but not with the 4-dimensional space-time. The 4-dimensional bare mass tensor it is a new term in our considerations. The eq. (A4) we can rewrite in the form

$$m_{\mu\nu} = m \cdot \eta_{\mu\nu} \tag{A5}$$

where: $\eta_{\mu\nu}$ is *the Minkowski tensor*.

References

[1]. M. J. Kubiak, *The Concept of the Effective Mass Tensor in the General Relativity*, http://vixra.org/abs/1301.0060 .

[2]. Cross D. J., *Anisotropy of Inertia from the CMB Anisotropy*, http://www.haverford.edu/physics-astro/dcross/academics/papers/oral.pdf .

[2] The 4-dimensional bare mass tensor is isotropic if and only if it his properties do not depend on direction in the space-time.