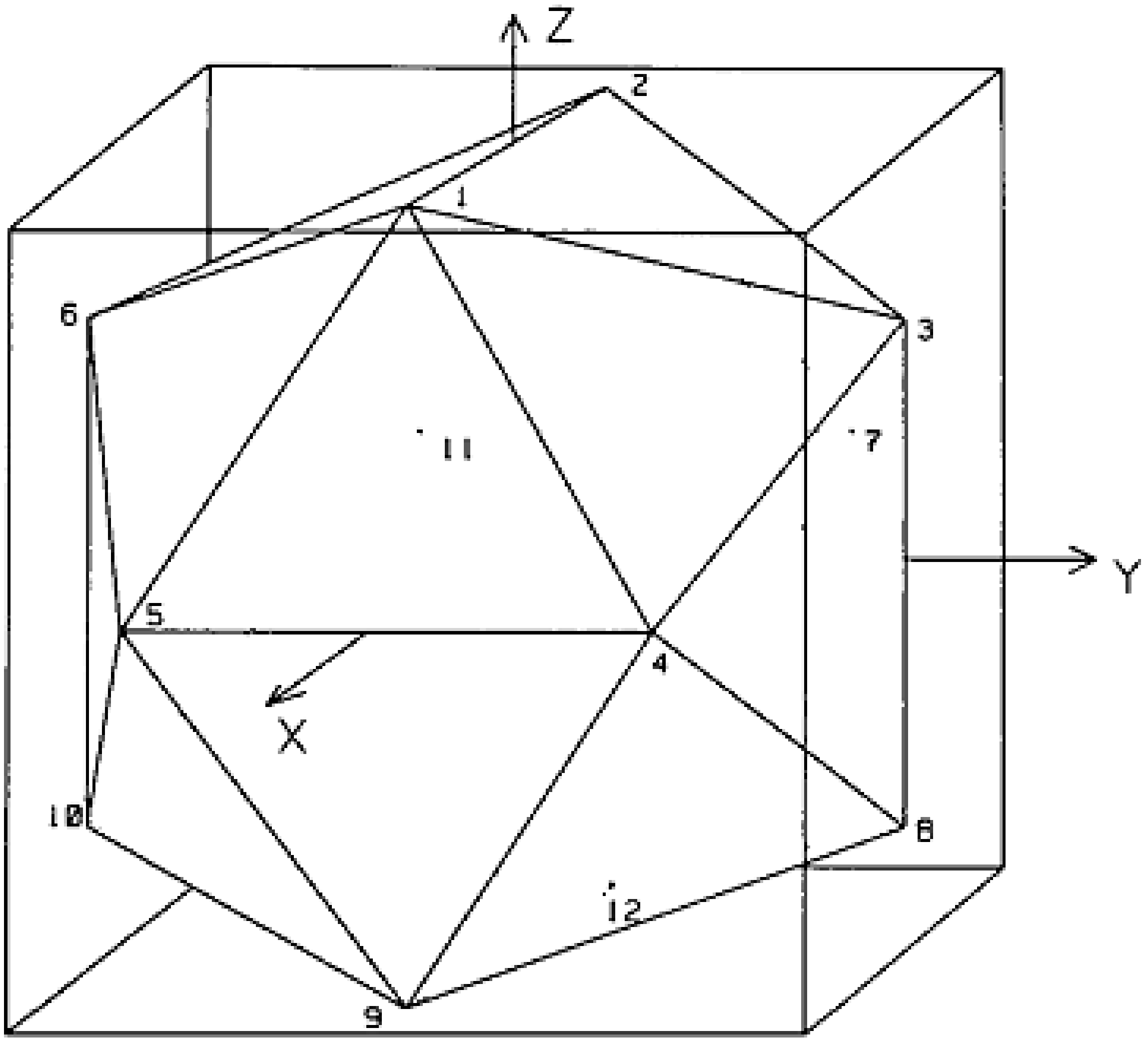
Sedenion Space



De – Constructing De Marrais Series

By John Frederick Sweeney

Abstract

The late Robert Marrais wrote of 3 distinct aspects of Sedenion Space: one belonging to the E8 x E8 Super – Symmetry of the embattled Super String Theory, one belonging to Icosahedral Rotation Groups or H3, and a third belonging to O. V. Lyashko. In addition, this paper discusses methods which De Marrais used to classify and to categorize the Sedenions.

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Introduction

One of the greatest surprises in writing this paper came from reading the Wikipedia entries for String Theory, Super String Theory and for M Theory. In fact, none of these are theories at all, and their Wikipedia descriptions sound more like obituaries than encyclopedia entries, and this from an "establishment" organization that is given special status on Google in order to promote the status quo view of the world.

One would imagine Wikipedia editors to defend these hackneyed pseudo – theories until death, but surprisingly even Wikipedia appears to have given up on these as not having any predictive or theoretical value. These ideas simply failed to measure up to expectations, after decades of waiting for evidence. Only establishment stalwarts such as Stephen Hawking support any of them, while Roger Penrose has entirely given up on them, as he has on Octonions.

After millions and billions spent, and decades of precious research time lost while chasing the implications of the failed "theoroids" - String Theory, Super String Theory and for M Theory – the Establishment has finally thrown up its hands and admits the powerlessness of the paradigm.

This string of defeats follows on the heels of the acknowledgement that the darling "genius" of 20th Century physics, Albert Einstein, not only stole "his" most famous equation, E=MC2, but tried seven times during the course of his lifetime to write a correct proof for the equation – and failed on each attempt. As de Marrais wrote, much of what was considered the basic architecture of modern physics turns out to have been nothing more than removable scaffolding.

The purpose of this paper is to separate the scaffolding from what remains of the foundation, in order to discard the useless scraps of out – dated theoroids onto the trash heap of math physics history, while retaining the useful concepts. The paper follows the intuitions and writings of de Marrais, since he is apparently the only human in recent times to comprehend the true value of Sedenions and Trigintaduonions, and to realize that these simply marked the foundation of the edifice that modern science is attempting to construct.

This paper proceeds by examining the specific statements made by de Marrais in 42 Assessors, then analyzing each statement in detail, by checking definitions with Wikipedia.

Robert de Marrais on Sedenion Space

In one or two pages of his “42 Assessors” paper, de Marrais indicates an area of potentially promising research:

“One of the greatest surprises in singularity theory after the “A-D-E Problem” was formulated was the incredible degree to which correspondences between fields could be traced: in particular, the icosahedral reflection group just alluded to was found to govern un - foldings in the 4-D boundary of a 16-D manifold (one per each plane of symmetry, plus the identity), as well as being connected with the generic problem of “bypassing an obstacle.” This latter, in turn, was shown to be antici pated –albeit not in modern language!

–in the “outdated” study of the evolvents of curves pioneered by Huyghens in wave-front theory,

In 1678, Huygens[1] proposed that every point to which a luminous disturbance reaches becomes a source of a spherical wave; the sum of these secondary waves determines the form of the wave at any subsequent time. He assumed that the secondary waves travelled only in the "forward" direction and it is not explained in the theory why this is the case. He was able to provide a qualitative explanation of linear and spherical wave propagation, and to derive the laws of reflection and refraction using this principle, but could not explain the deviations from rectilinear propagation that occur when light encounters edges, apertures and screens, commonly known as diffraction effects.[2]

In 1816, Fresnel[3] showed that Huygens' principle, together with his own principle of interference could explain both the rectilinear propagation of light and also diffraction effects. To obtain agreement with experimental results, he had to include additional arbitrary assumptions about the phase and amplitude of the secondary waves, and also an obliquity factor. These assumptions have no obvious physical foundation but led to predictions that agreed with many experimental observations, including the Arago spot.

Poisson was a member of the French Academy, which reviewed Fresnel's work.[4] He used Fresnel's theory to predict that a bright spot will appear in the center of the shadow of a small disc and deduced from this that the theory was incorrect. However, Arago, another member of the committee, performed the experiment and showed that the prediction was correct. (Lisle had actually observed this fifty years earlier.[2]) This was one of the investigations that led to the victory of the wave theory of light over the then predominant corpuscular theory.

specifically in diagrams Benniquen discovered of the evolvents of the semi -cubical parabola (itself the germ of the simplest, or Fold, Catastrophe Y = X3) in the very first textbook on analysis, written by L'Hôpital from Bernoulli's lectures[28].

(As Arnol'd understates the case, "the appearance of the regular polyhedra is often unexpected.")

The former result, though, due to Lyashko[29], suggests a frontier in "zero -divisor" study that remains unexplored: what connections, if any, can be elicited between the three 16-D structures we've seen are of interest?

The superstring theorist's E8 x E8 is clearly not the same as the 16-D realization of the icosahedral reflection group's symmetry whose (symplectic) boundary is Lyashko's focus: it is just the direct product of two icosahedral rotation groups."

The 16-D Lyashko singularity with boundary, especially given its close relationship to the "obstacle bypass" evolvent context, is quite suggestive in its own right: in particular, a novel opening to exploring some recent approaches to quantum non-localization would seem indicated. Readers are encouraged to
pursue the URL and/or text version of the source containing the quotes, while keeping the "Lissajous ping - pong" motif broached much earlier clearly in mind. First, by implication, the "obstacle bypass" problem:

The Obstacle Bypass Problem

"One way of explaining quantum non-locality is through a hand-shaking space-time interaction between an emitter and its potential absorbers. The transactional interpretation does just this by postulating an advanced wave travelling back in time from the [future] absorber to the emitter. This interferes with the retarded wave, travelling in the usual direction from emitter to absorber to form the exchanged particle.

Because both waves are zero -energy crossed phase waves, they interfere destructively outside the particle path but constructively between the emitter and absorber. The emitter sends out an offer wave and the absorber responds with a confirmation wave. Together they form a photon, just as an anti-electron (positron) travelling backwards in time is the same as an electron travelling forwards.[33]

De Marrais explains this handshake elsewhere in great detail. For now, this paper points out that the author has published a paper on this topic (Crop Circles Across the Universe) which indicates that the handshake takes place directly across the Universe.

Wikipedia on Singularity Theory

In mathematics, **singularity theory** is the study of the failure of manifold structure. A loop of string can serve as an example of a one-dimensional manifold, if one neglects its width. What is meant by a singularity can be seen by dropping it on the floor. Probably there will appear a number of double points, at which the string crosses itself in an approximate 'x' shape. These are the simplest kinds of *singularity*. Perhaps the string will also touch itself, coming into *contact* with itself without crossing, like an underlined 'U'. This is another kind of singularity. Unlike the double point, it is not *stable*, in the sense that a small push will lift the bottom of the 'U' away from the '_'.

How singularities may arise

In singularity theory the general phenomenon of points and sets of singularities is studied, as part of the concept that manifolds (spaces without singularities) may acquire special, singular points by a number of routes. Projection is one way, very obvious in visual terms when three-dimensional objects are projected into two dimensions (for example in one of our eyes); in looking at classical statuary the folds of drapery are amongst the most obvious features. Singularities of this kind include caustics, very familiar as the light patterns at the bottom of a swimming pool.

Other ways in which singularities occur is by degeneration of manifold structure. That implies the breakdown of parametrization of points; it is prominent in general relativity, where a gravitational singularity, at which the gravitational field is strong enough to change the very structure of space-time, is identified with a black hole. In a less dramatic fashion, the presence of symmetry can be good cause to consider orbifolds, which are manifolds that have acquired 'corners' in a process of folding up resembling the creasing of a table napkin.

Singularities in algebraic geometry

Algebraic curve singularities

Historically, singularities were first noticed in the study of algebraic curves. The *double point* at (0,0) of the curve

$$y^2 = x^2 - x^3$$

and the cusp there of

$$y^2 = x^3$$

are qualitatively different, as is seen just by sketching. Isaac Newton carried out a detailed study of all cubic curves, the general family to which these examples belong. It was noticed in the formulation of Bézout's theorem that such *singular points* must be counted with multiplicity (2 for a double point, 3 for a cusp), in accounting for intersections of curves.

It was then a short step to define the general notion of a singular point of an algebraic variety; that is, to allow higher dimensions.

The general position of singularities in algebraic geometry

Such singularities in algebraic geometry are the easiest in principle to study, since they are defined by polynomial equations and therefore in terms of a coordinate system. One can say that the *extrinsic* meaning of a singular point isn't in question; it is just that in *intrinsic* terms the coordinates in the ambient space don't straightforwardly translate the geometry of the algebraic variety at the point. Intensive studies of such singularities led in the end to Heisuke Hironaka's fundamental theorem on resolution of singularities (in birational geometry in characteristic 0).

This means that the simple process of 'lifting' a piece of string off itself, by the 'obvious' use of the cross-over at a double point, is not essentially misleading: all the singularities of algebraic geometry can be recovered as some sort of very general *collapse* (through multiple processes). This result is often implicitly used to extend affine geometry to projective geometry: it is entirely typical for an affine variety to acquire singular points on the hyperplane at infinity, when its closure in projective space is taken. Resolution says that such singularities can be handled rather as a (complicated) sort of compactification, ending up with a *compact* manifold (for the strong topology, rather than the Zariski topology, that is).

The smooth theory, and catastrophes

At about the same time as Hironaka's work, the catastrophe theory of René Thom was receiving a great deal of attention. This is another branch of singularity theory, based on earlier work of Hassler Whitney on critical points. Roughly speaking, a *critical point* of a smooth function is where the level set develops a singular point in the geometric sense.

This theory deals with differentiable functions in general, rather than just polynomials. To compensate, only the *stable* phenomena are considered. One can argue that in nature, anything destroyed by tiny changes is not going to be observed; the visible *is* the stable. Whitney had shown that in low numbers of variables the stable structure of critical points is very restricted, in local terms. Thom built on this, and his own earlier work, to create a *catastrophe theory* supposed to account for discontinuous change in nature.

Arnold's view

While Thom was an eminent mathematician, the subsequent fashionable nature of elementary catastrophe theory as propagated by Christopher Zeeman caused a reaction, in particular on the part of Vladimir Arnold.[1] He may have been largely responsible for applying the term ***singularity theory*** to the area including the input from algebraic geometry, as well as that flowing from the work of Whitney, Thom and other authors. He wrote in terms making clear his distaste for the too-publicised emphasis on a small part of the territory.

The foundational work on smooth singularities is formulated as the construction of equivalence relations on singular points, and germs. Technically this involves group actions of Lie groups on spaces of jets; in less abstract terms Taylor series are examined up to change of variable, pinning down singularities with enough derivatives. Applications, according to Arnold, are to be seen in symplectic geometry, as the geometric form of classical mechanics.

Duality

An important reason why singularities cause problems in mathematics is that, with a failure of manifold structure, the invocation of Poincaré duality is also disallowed. A major advance was the introduction of intersection cohomology, which arose initially from attempts to restore duality by use of strata. Numerous connections and applications stemmed from the original idea, for example the concept of perverse sheaf in homological algebra.

Other possible meanings

The theory mentioned above does not directly relate to the concept of mathematical singularity as a value at which a function isn't defined. For that, see for example isolated singularity, essential singularity, removable singularity. The monodromy theory of differential equations, in the complex domain, around singularities, does however come into relation with the geometric theory. Roughly speaking, *monodromy* studies the way a covering map can degenerate, while *singularity theory* studies the way a *manifold* can degenerate; and these fields are linked.

Elsewhere, de Marrais wrote of Whitney Umbrellas, therefore this concept may shed some light on his meaning concerning Singularity Theory. The animated diagram on Wikipedia under this entry reminds one of a square or cubic wave:

In mathematics, the **Whitney umbrella** (or **Whitney's umbrella** and sometimes called a **Cayley umbrella**) is a self-intersecting surface placed in three dimensions. It is the union of all straight lines that pass through points of a fixed parabola and are perpendicular to a fixed straight line, parallel to the axis of the parabola and lying on its perpendicular bisecting plane.

Whitney's umbrella is a ruled surface and a right conoid. It is important in the field of singularity theory, as a simple local model of a pinch point singularity. The pinch point and the fold singularity are the only stable local singularities of maps from $\mathbf{R}^2$ to $\mathbf{R}^3$.

It is named after the American mathematician Hassler Whitney.

In string theory, a Whitney brane is a D7-brane wrapping a variety whose singularities are locally modeled by the Whitney Umbrella. Whitney branes appear naturally when taking Sen's weak coupling limit of F-theory.

Lyashenko on 16-D Manifolds

"One of the greatest surprises in singularity theory after the "A-D-E Problem" was formulated was the incredible degree to which correspondences between fields could be traced: in particular, the icosahedral reflection group just alluded to was found to govern un - foldings in the 4-D boundary of a 16-D manifold (one per each plane of symmetry, plus the identity), as well as being connected with the generic problem of "bypassing an obstacle."

Wikipedia on F4

A search for a 16 – dimensional manifold led to the Exceptional Lie Algebra F4, which Wikipedia notes was formally known as E4, and thus belongs more properly to the E series.

In mathematics, $\mathbf{F}_4$ is the name of a Lie group and also its Lie algebra $\mathbf{f}_4$. It is one of the five exceptional simple Lie groups. F_4 has rank 4 and dimension 52. The compact form is simply connected and its outer automorphism group is the trivial group. Its fundamental representation is 26-dimensional.

The compact real form of F_4 is the isometry group of a 16-dimensional Riemannian manifold known as the octonionic projective plane $\mathbf{OP}^2$.

This can be seen systematically using a construction known as the *magic square*, due to Hans Freudenthal and Jacques Tits.

There are 3 real forms: a compact one, a split one, and a third one.

The F_4 Lie algebra may be constructed by adding 16 generators transforming as a spinor to the 36-dimensional Lie algebra **so**(9), in analogy with the construction of E_8.

In older books and papers, F_4 is sometimes denoted by E_4.

John Baez on F4 and OP2

The second smallest of the exceptional Lie groups is the 52-dimensional group $\mathbf{F}_4$. The geometric meaning of this group became clear in a number of nearly simultaneous papers by various mathematicians. In 1949, Jordan constructed the octonionic projective plane using projections in $\mathfrak{h}_3(\mathbb{O})$. One year later, Armand Borel [8] noted that $\mathbf{F}_4$ is the isometry group of a 16-dimensional projective plane. In fact, this plane is none other than than $\mathbb{OP}^2$. Also in 1950, Claude Chevalley and Richard Schafer [18] showed that $\mathbf{F}_4$ is the automorphism group of $\mathfrak{h}_3(\mathbb{O})$. In 1951, Freudenthal [35] embarked upon a long series of papers in which he described not only $\mathbf{F}_4$ but also the other exceptional Lie groups using octonionic projective geometry. To survey these developments, one still cannot do better than to read his classic 1964 paper on Lie groups and the foundations of geometry [38].

Let us take Chevalley and Schafer's result as the definition of $\mathbf{F}_4$:

$$\mathbf{F}_4 = \mathrm{Aut}(\mathfrak{h}_3(\mathbb{O})).$$

Its Lie algebra is thus

$$\mathfrak{f}_4 = \mathfrak{der}(\mathfrak{h}_3(\mathbb{O})).$$

As we saw in Section 3.4, points of $\mathbb{OP}^2$ correspond to trace-1 projections in the exceptional Jordan algebra. It follows that $\mathbf{F}_4$ acts as transformations of $\mathbb{OP}^2$. In fact, we can equip $\mathbb{OP}^2$ with a Riemannian metric for which $\mathbf{F}_4$ is the isometry group. To get a sense of how this works, let us describe $\mathbb{OP}^2$ as a quotient space of $\mathbf{F}_4$.

In Section 3.4 we saw that the exceptional Jordan algebra can be built using natural operations on the scalar, vector and spinor representations of $\mathrm{Spin}(9)$. This implies that $\mathrm{Spin}(9)$ is a subgroup of $\mathbf{F}_4$. Equation (3.4) makes it clear that $\mathrm{Spin}(9)$ is precisely the subgroup fixing the element

$$\left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right).$$

Since this element is a trace-one projection, it corresponds to a point of $\mathbb{OP}^2$. We have already seen that $\mathbf{F}_4$ acts transitively on $\mathbb{OP}^2$. It follows that

$$\mathbb{OP}^2 \cong \mathbf{F}_4/\mathrm{Spin}(9).$$

This fact has various nice spinoffs. First, it gives an easy way to compute the dimension of $\mathbf{F}_4$:

$$\dim(\mathrm{F}_4) = \dim(\mathrm{Spin}(9)) + \dim(\mathbb{O}\mathrm{P}^2) = 36 + 16 = 52.$$

Second, since $\mathbf{F}_4$ is compact, we can take any Riemannian metric on $\mathbb{O}\mathrm{P}^2$ and average it with respect to the action of this group. The isometry group of the resulting metric will automatically include $\mathbf{F}_4$ as a subgroup. With more work [5], one can show that actually

$$\mathrm{F}_4 = \mathrm{Isom}(\mathbb{O}\mathrm{P}^2)$$

and thus

$$\mathfrak{f}_4 = \mathfrak{isom}(\mathbb{O}\mathrm{P}^2).$$

Equation (4.2) also implies that the tangent space of our chosen point in $\mathbb{O}\mathrm{P}^2$ is isomorphic to $\mathfrak{f}_4/\mathfrak{so}(9)$. But we already know that this tangent space is just $\mathbb{O}^2$, or in other words, the spinor representation of $\mathfrak{so}(9)$. We thus have

$$\mathfrak{f}_4 \cong \mathfrak{so}(9) \oplus S_9$$

as vector spaces, where $\mathfrak{so}(9)$ is a Lie subalgebra. The bracket in $\mathfrak{f}_4$ is built from the bracket in $\mathfrak{so}(9)$, the action $\mathfrak{so}(9) \otimes S_9 \to S_9$, and the map $S_9 \otimes S_9 \to \mathfrak{so}(9)$ obtained by dualizing this action. We can also rewrite this description of $\mathfrak{f}_4$ in terms of the octonions, as follows:

$$\mathfrak{f}_4 \cong \mathfrak{so}(\mathbb{O} \oplus \mathbb{R}) \oplus \mathbb{O}^2$$

This last formula suggests that we decompose $\mathfrak{f}_4$ further using the splitting of $\mathbb{O} \oplus \mathbb{R}$ into $\mathbb{O}$ and $\mathbb{R}$. It is easily seen by looking at matrices that for all n, m we have

$$\mathrm{so(n+m)} \cong \mathfrak{so}(n) \oplus \mathfrak{so}(m) \oplus V_n \otimes V_m.$$

Moreover, when we restrict the representation S_9 to $\mathfrak{so}(8)$, it splits as a direct sum $S_8^+ \oplus S_8^-$. Using these facts and equation (4.2), we see

$$\mathfrak{f}_4 \cong \mathfrak{so}(8) \oplus V_8 \oplus S_8^+ \oplus S_8^-$$

This formula emphasizes the close relation between $\mathfrak{f}_4$ and triality: the Lie bracket in $\mathfrak{f}_4$ is completely built out of maps involving $\mathfrak{so}(8)$ and its three 8-dimensional irreducible representations! We can rewrite this in a way that brings out the role of the octonions:

$$\mathfrak{f}_4 \cong \mathfrak{so}(\mathbb{O}) \oplus \mathbb{O}^3$$

While elegant, none of these descriptions of $\mathfrak{f}_4$ gives a convenient picture of all the derivations of the exceptional Jordan algebra. In fact, there is a nice picture of this sort for $\mathfrak{h}_3(\mathbb{K})$ whenever $\mathbb{K}$ is a normed division algebra. One way to get a derivation of the Jordan algebra $\mathfrak{h}_3(\mathbb{K})$ is to take a derivation of $\mathbb{K}$ and let it act on each entry of the matrices in $\mathfrak{h}_3(\mathbb{K})$. Another way uses elements of

$$\mathfrak{sa}_3(\mathbb{K}) = \{x \in \mathbb{K}[3]\colon\ x^* = -x,\ \mathrm{tr}(x) = 0\}.$$

Given $x \in \mathfrak{sa}_3(\mathbb{K})$, there is a derivation ad_x of $\mathfrak{h}_3(\mathbb{K})$ given by

$$\mathrm{ad}_x(a) = [x, a].$$

In fact [4], every derivation of $\mathfrak{h}_3(\mathbb{K})$ can be uniquely expressed as a linear combination of derivations of these two sorts, so we have

$$\mathfrak{der}(\mathfrak{h}_3(\mathbb{K})) \cong \mathfrak{der}(\mathbb{K}) \oplus \mathfrak{sa}_3(\mathbb{K})$$

as vector spaces. In the case of the octonions, this decomposition says that

$$\mathfrak{f}_4 \cong \mathfrak{g}_2 \oplus \mathfrak{sa}_3(\mathbb{O}).$$

In equation (4.2), the subspace $\mathfrak{der}(\mathbb{K})$ is always a Lie subalgebra, but $\mathfrak{sa}_3(\mathbb{K})$ is not unless $\mathbb{K}$ is commutative and associative -- in which case $\mathfrak{der}(\mathbb{K})$ vanishes. Nonetheless, there is a formula for the brackets in $\mathfrak{der}(\mathfrak{h}_3(\mathbb{K}))$ which applies in every case [70]. Given $D, D' \in \mathfrak{der}(\mathbb{K})$ and $x, y \in \mathfrak{sa}_3(\mathbb{K})$, we have

$$\begin{array}{rcl} [D, D'] & = & DD' - D'D \\ {[D, \mathrm{ad}_x]} & = & \mathrm{ad}_{Dx} \\ {[\mathrm{ad}_x, \mathrm{ad}_y]} & = & \mathrm{ad}_{[x,y]_0} + \dfrac{1}{3} \displaystyle\sum_{i,j=1}^{3} D_{x_{ij}, y_{ij}} \end{array}$$

where D acts on x componentwise, $[x,y]_0$ is the trace-free part of the commutator $[x,y]$, and $D_{x_{ij},y_{ij}}$ is the derivation of $\mathbb{K}$ defined using equation (4.1).

Summarizing these different descriptions of $\mathfrak{f}_4$, we have:

Theorem 5. The compact real form of $\mathfrak{f}_4$ is given by

$$
\begin{aligned}
\mathfrak{f}_4 &\cong \mathfrak{isom}(\mathbb{OP}^2) \\
&\cong \mathfrak{der}(\mathfrak{h}_3(\mathbb{O})) \\
&\cong \mathfrak{der}(\mathbb{O}) \oplus \mathfrak{sa}_3(\mathbb{O}) \\
&\cong \mathfrak{so}(\mathbb{O} \oplus \mathbb{R}) \oplus \mathbb{O}^2 \\
&\cong \mathfrak{so}(\mathbb{O}) \oplus \mathbb{O}^3
\end{aligned}
$$

where in each case the Lie bracket is built from natural bilinear operations on the summands.

Wikipedia on the Exceptional Jordan Algebra / Albert Algebra

In mathematics, an **Albert algebra** is a 27-dimensional exceptional Jordan algebra. They are named after Abraham Adrian Albert, who pioneered the study of non-associative algebras, usually working over the real numbers. Over the real numbers, there are two such Jordan algebras up to isomorphism.[1] One of them, which was first mentioned by Jordan, Neumann & Wigner (1934) and studied by Albert (1934), is the set of 3×3 self-adjoint matrices over the octonions, equipped with the binary operation

$$x \circ y = \frac{1}{2}(x \cdot y + y \cdot x),$$

where . denotes matrix multiplication. The other is defined the same way, but using split octonions instead of octonions.

Over any algebraically closed field, there is just one Albert algebra. For example, the complexifications of the two Albert algebras over the real numbers are isomorphic Albert algebras over the complex numbers.

The Tits - Koecher construction applied to an Albert algebra gives a form of the E7 Lie algebra.

See also

- Euclidean Jordan algebra for the Jordan algebras considered by Jordan, von Neumann and Wigner
- Euclidean Hurwitz algebra for details of the construction of the Albert algebra for the octonions

Let A be a Euclidean Hurwitz algebra and let $M_n(A)$ be the algebra of n-by-n matrices over A. It is a unital nonassociative algebra with an involution given by

$$(x_{ij})^* = (x_{ji}^*).$$

The trace $\mathrm{Tr}(X)$ is defined as the sum of the diagonal elements of X and the real-valued trace by $\mathrm{Tr}_{\mathbf{R}}(X) = \mathrm{Re}\,\mathrm{Tr}(X)$. The real-valued trace satisfies:

$$\mathrm{Tr}_{\mathbf{R}}\, XY = \mathrm{Tr}_{\mathbf{R}}\, YX, \;\; \mathrm{Tr}_{\mathbf{R}}\,(XY)Z = \mathrm{Tr}_{\mathbf{R}}\, X(YZ).$$

These are immediate consequences of the known identities for $n = 1$.

In A define the *associator* by

$$[a, b, c] = a(bc) - (ab)c.$$

It is trilinear and vanishes identically if A is associative. Since A is an alternating algebra $[a, a, b] = 0$ and $[b, a, a] = 0$. Polarizing it follows that the associator is antisymmetric in its three entries. Note also that a, b or c lie in $\mathbf{R}$ then the $[a, b, c] = 0$. This implies that $M_3(A)$ has certain commutation properties. In fact if X is a matrix in $M_3(A)$ with real entries on the diagonal then

$$[X, X^2] = aI,$$

with a in A. In fact if $Y = [X, X^2]$, then

$$y_{ij} = \sum_{k,\ell} [x_{ik}, x_{k\ell}, x_{\ell j}].$$

Since the diagonal entries of X are real, the off diagonal entries of Y vanish. Each diagonal entry of Y is a sum of two associators involving only off diagonal terms of X. Since the associators are invariant under cyclic permutations, the diagonal entries of Y are all equal.

Let $H_n(A)$ be the space of self-adjoint elements in $M_n(A)$ with product $X \circ Y = 1/2(X\ Y + Y\ X)$ and inner product $(X, Y) = \mathrm{Tr}_{\mathbf{R}}(X\ Y)$.

Theorem. $H_n(A)$ is a Euclidean Jordan algebra if A is associative (the real numbers, complex numbers or quaternions) and $n \geq 3$ or if A is nonassociative (the octonions) and $n = 3$.

The exceptional Jordan algebra $H_3(\mathbf{O})$ is called the **Albert algebra** after A.A. Albert.

To check that $H_n(A)$ satisfies the axioms for a Euclidean Jordan algebra, note that the real trace defines a symmetric bilinear form with $(X, X) = \Sigma \| x_{ij} \|^2$. So it is an inner product. It satisfies the associativity property $(Z \circ X, Y) = (X, Z \circ Y)$ because of the properties of the real trace. The main axiom to check is the Jordan condition for the operators $L(X)$ defined by $L(X)Y = X \circ Y$:

$$[L(X), L(X^2)] = 0.$$

This is easy to check when A is associative, since $M_n(A)$ is an associative algebra so a Jordan algebra with $X \circ Y = 1/2(X\ Y + Y\ X)$. When $A = \mathbf{O}$ and $n = 3$ a special argument is required, one of the shortest being due to Freudenthal (1951).[7]

In fact if T is in $H_3(\mathbf{O})$ with $\mathrm{Tr}\ T = 0$, then

$$D(X) = TX - XT$$

defines a skew-adjoint derivation of $H_3(\mathbf{O})$. Indeed

$$\mathrm{Tr}(T(X(X^2)) - T(X^2(X))) = \mathrm{Tr}\, T(aI) = \mathrm{Tr}(T)a = 0,$$

so that

$$(D(X), X^2) = 0.$$

Polarizing yields:

$$(D(X), Y \circ Z) + (D(Y), Z \circ X) + (D(Z), X \circ Y) = 0.$$

Setting $Z = 1$, shows that D is skew-adjoint. The derivation property $D(X \circ Y) = D(X) \circ Y + X \circ D(Y)$ follows by this and the associativity property of the inner product in the identity above.

With A and n as in the statement of the theorem, let K be the group of automorphisms of $E = H_n(A)$ leaving invariant the inner product. It is a closed subgroup of O(E) so a compact Lie group. Its Lie algebra consists of skew-adjoint derivations.

Freudenthal (1951) showed that given X in E there is an automorphism k in K such that $k(X)$ is a diagonal matrix. (By self-adjointness the diagonal entries will be real.) Freudenthal's diagonalization theorem immediately implies the Jordan condition, since Jordan products by real diagonal matrices commute on $M_n(A)$ for any non-associative algebra A.

To prove the diagonalization theorem, take X in E. By compactness k can be chosen in K minimizing the sums of the squares of the norms of the off-diagonal terms of $k(X)$. Since K preserves the sums of all the squares, this is equivalent to maximizing the sums of the squares of the norms of the diagonal terms of $k(X)$.

Replacing X by $k X$, it can be assumed that the maximum is attained at X. Since the symmetric group S_n, acting by permuting the coordinates, lies in K, if X is not diagonal, it can be supposed that x_{12} and its adjoint x_{21} are non-zero. Let T be the skew-adjoint matrix with (2, 1) entry a, (1, 2) entry $-a^*$ and 0 elsewhere and let D be the derivation ad T of E. Let $k_t = \exp tD$ in K.

Then only the first two diagonal entries in $X(t) = k_tX$ differ from those of X. The diagonal entries are real. The derivative of $x_{11}(t)$ at $t = 0$ is the (1, 1) coordinate of $[T, X]$, i.e. $a^* x_{21} + x_{12} a = 2(x_{21}, a)$. This derivative is non-zero if $a = x_{21}$. On the other hand the group k_t preserves the real-valued trace. Since it can only change x_{11}

and x_{22}, it preserves their sum. However on the line $x + y$ =constant, $x^2 + y^2$ has no local maximum (only a global minimum), a contradiction. Hence X must be diagonal.

Unless the author is missing something important, all of this mathematics simply returns to H3.

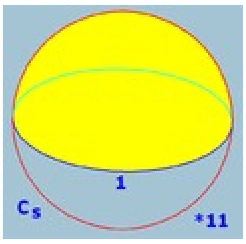
Theorem. $H_n(A)$ is a Euclidean Jordan algebra if A is associative (the real numbers, complex numbers or quaternions) and $n \geq 3$ or if A is nonassociative (the octonions) and $n = 3$.

Icosahedral Rotation Group - H3

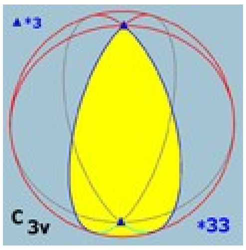
Wikipedia on Icosahedral Symmetry

A regular icosahedron has 60 rotational (or orientation-preserving)

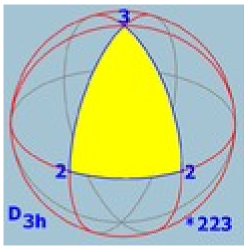
Point groups in three dimensions



Involutional symmetry
C_s, [1], (*)

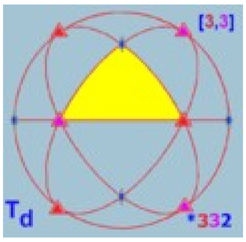


Cyclic symmetry
C_{nv}, [n], (*nn)

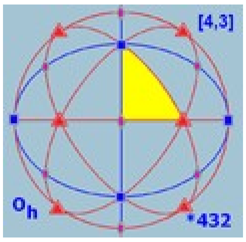


Dihedral symmetry
D_{nh}, [n,2], (*n22)

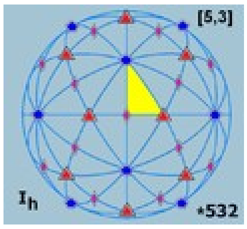
Polyhedral group, [n,3], (*n32)



Tetrahedral symmetry
T_d, [3,3], (*332)

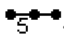


Octahedral symmetry
O_h, [4,3], (*432)



Icosahedral symmetry
I_h, [5,3], (*532)

symmetries, and a symmetry order of 120 including transformations that combine a reflection and a rotation. A regular dodecahedron has the same set of symmetries, since it is the dual of the icosahedron.

The set of orientation-preserving symmetries forms a group referred to as A_5 (the alternating group on 5 letters), and the full symmetry group (including reflections) is the product $A_5 \times Z_2$. The latter group is also known as the Coxeter group H_3, and is also represented by Coxeter notation, [5,3] and Coxeter diagram .

Apart from the two infinite series of prismatic and antiprismatic symmetry, **rotational icosahedral symmetry** or **chiral icosahedral symmetry** of chiral objects and **full icosahedral symmetry** or **achiral icosahedral symmetry** are the discrete point symmetries (or equivalently, symmetries on the sphere) with the largest symmetry groups.

Icosahedral symmetry is *not* compatible with translational symmetry, so there are no associated crystallographic point groups or space groups.

Schönflies crystallographic notation	Coxeter notation	Orbifold notation	Order
I	$[3,5]^+$	532	60
I_h	[3,5]	*532	120

Presentations corresponding to the above are:

$$I : \langle s,t \mid s^2, t^3, (st)^5 \rangle$$

$$I_h : \langle s,t \mid s^3(st)^{-2}, t^5(st)^{-2} \rangle.$$

These correspond to the icosahedral groups (rotational and full) being the (2, 3, 5) triangle groups.

The first presentation was given by William Rowan Hamilton in 1856, in his paper on icosian calculus.[1]

Note that other presentations are possible, for instance as an alternating group (for I).

Group structure

The **icosahedral rotation group** I is of order 60. The group I is isomorphic to A_5, the alternating group of even permutations of five objects. This isomorphism can be realized by I acting on various compounds, notably the compound of five cubes (which inscribe in the dodecahedron), the compound of five octahedra, or either of the two compounds of five tetrahedra (which are enantiomorphs, and inscribe in the dodecahedron).

The group contains 5 versions of T_h with 20 versions of D_3 (10 axes, 2 per axis), and 6 versions of D_5.

The **full icosahedral group** I_h has order 120. It has I as normal subgroup of index 2. The group I_h is isomorphic to $I \times Z_2$, or $A_5 \times Z_2$, with the inversion in the center corresponding to element (identity,-1), where Z_2 is written multiplicatively.

I_h acts on the compound of five cubes and the compound of five octahedra, but −1 acts as the identity (as cubes and octahedra are centrally symmetric). It acts on the compound of ten tetrahedra: I acts on the two chiral halves (compounds of five tetrahedra), and −1 interchanges the two halves. Notably, it does *not* act as S_5, and these groups are not isomorphic; see below for details.

The group contains 10 versions of D_{3d} and 6 versions of D_{5d} (symmetries like antiprisms).

I is also isomorphic to $PSL_2(5)$, but I_h is not isomorphic to $SL_2(5)$.

Wolfram on Icosahdral Groups

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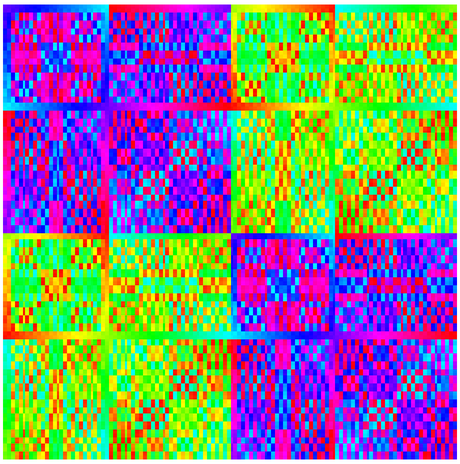
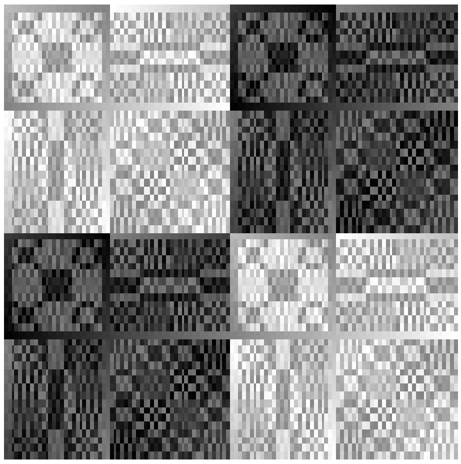
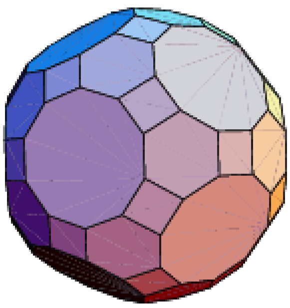
I_h



I_h

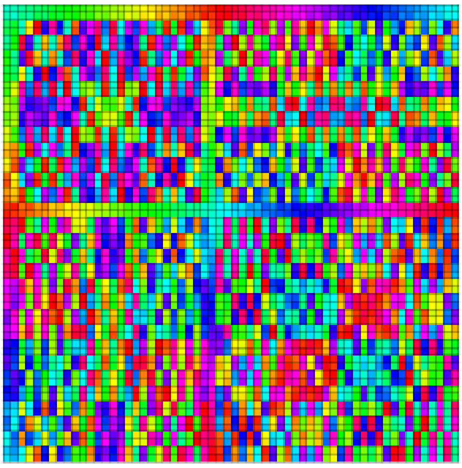


The icosahedral group I_h is the point group of symmetries of the icosahedron and dodecahedron having order 120, equivalent to the group direct product $A_5 \times \mathbb{Z}_2$ of the alternating group A_5 and cyclic group $\mathbb{Z}_2$. The icosahedral group consists of the conjugacy classes 1, $12\,C_5$, $12\,C_5^2$, $20\,C_3$, $15\,C_2$, i, $12\,S_{10}$, $12\,S_{10}^3$, $20\,S_6$, and $15\,\sigma$ (Cotton 1990, pp. 49 and 436). Its multiplication table is illustrated above. The icosahedral group is a subgroup of the special orthogonal group $SO(3)$.

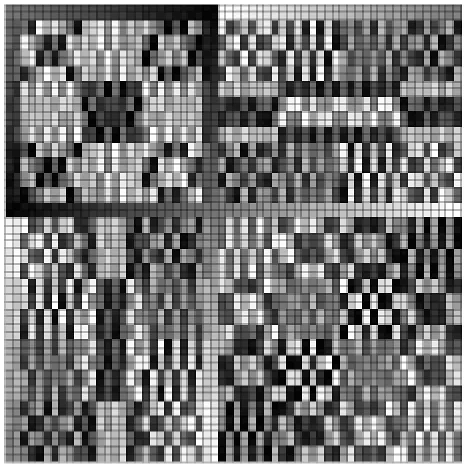


The great rhombicosidodecahedron can be generated using the matrix representation of I_h using the basis vector $(\phi, 3, 2\phi)$, where ϕ is the golden ratio.

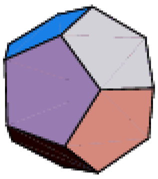
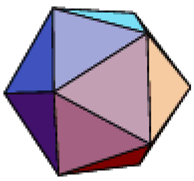
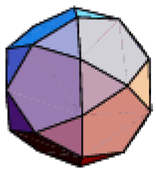
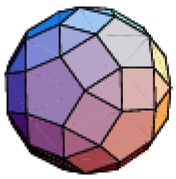
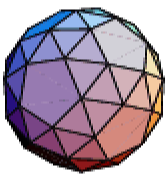
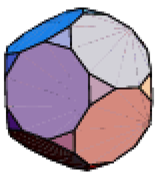
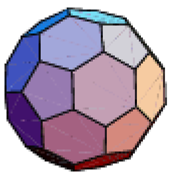
I



I



The icosahedral group I_h has a pure rotation subgroup denoted I that is isomorphic to the alternating group A_5. I is of order 60 and has conjugacy classes 1, $12\,C_5$, $12\,C_5^2$, $20\,C_3$, and $15\,C_2$ (Cotton 1990, pp. 50 and 436). Its multiplication table is illustrated above.

Platonic and Archimedean solids that can be generated by group I are illustrated above, with the corresponding basis vector summarized in the following table, where ϕ is the golden ratio and a and b are the largest positive roots of two sixth-order polynomials.

solid	basis vector
dodecahedron	$(1, 1+\phi, 0)$
icosahedron	$(1, 0, \phi)$
icosidodecahedron	$(0, 0, 1)$
small rhombicosidodecahedron	$(\phi, -1, 2)$
snub dodecahedron	$(1, a, b)$
truncated dodecahedron	$(\phi, 2, 1+\phi)$
truncated icosahedron	$(\phi, 1+2\phi, 2)$

Parsing Sedenions

De Marrais goes into great detail to figure out which Sedenion Triplet, Zero Divisor, Octonion and Twisted Octonion fit together, and his series of papers on Box Kites and Catamarans describes this in detail. Unfortunately, his highly idiosyncratic style perhaps prevents understanding, instead of serving as a heuristic device. Papers in this series will de – construct De Marrais' construction of Box Kites, Sails and Catamarans. For now, a good place to begin classifying Sedenions is this piece by Donald Chelsey.

2.1 Preliminary Classification of Sedenion Types

Testing each of the 2^{35} values of signmask in the XOR-based multiplication tables and analyzing the associators $(e_ie_j)e_k - e_i(e_je_k)$ shows that there are 9 broad classes of sedenions, classified by the nature of the heptads: of the 15 heptads, anywhere from 0 to 8 are true octonions, with the balance being twisted. Below, counts[N] shows how many signmask values give N true octonionic heptads in the corresponding multiplication table:

counts[0] = 4699455488
counts[1] = 9688596480
counts[2] = 10254827520
counts[3] = 6041190400
counts[4] = 2582200320
counts[5] = 817152000
counts[6] = 248299520
counts[7] = 25804800
counts[8] = 2211840
counts[9] = 0

Adding these up gives 2^{35}, establishing the fact that (at least for representations derived *via* permutation from the XOR-based multiplication tables) all sedenion types must include at least 7 twisted octonion subalgebras.

M – Theory / Wikipedia

In theoretical physics, **M-theory** is an extension of string theory in which 11 dimensions of spacetime are identified as seven *higher-dimensions* plus the four *common dimensions* (11D st = 7 hd + 4D). Proponents believe that the 11-dimensional theory unites all five 10 dimensional string theories and supersedes them. Though a full description of the theory is not known, the low-entropy dynamics are known to be supergravity interacting with 2- and 5-dimensional membranes.

This idea is the unique supersymmetric theory in 11 dimensions, with its low-entropy matter content and interactions fully determined, and can be obtained as the strong coupling limit of type IIA string theory because a new dimension of space emerges as the coupling constant increases.

Drawing on the work of a number of string theorists (including Ashoke Sen, Chris Hull, Paul Townsend, Michael Duff and John Schwarz), Edward Witten of the Institute for Advanced Study suggested its existence at a conference at USC in 1995, and used M-theory to explain a number of previously observed dualities, initiating a flurry of new research in string theory called the second superstring revolution.

In the early 1990s, it was shown that the various superstring theories were related by dualities which allow the description of an object in one super string theory to be related to the description of a different object in another super string theory. These relationships imply that each of the super string theories is a different aspect of a single underlying theory, proposed by Witten, and named "M-theory".

Originally the letter M in M-theory was taken from *membrane*, a construct designed to generalize the strings of string theory. However, as Witten was more skeptical about membranes than his colleagues, he opted for "M-theory" rather than "Membrane theory". Witten has since stated that the different interpretations of the M can be a matter of taste for the user, such as magic, mystery, and mother theory.[1]

M-theory (and string theory) has been criticized for lacking predictive power or being untestable. Further work continues to find mathematical constructs that join various surrounding theories. However, the tangible success of M-theory can be questioned, given its current incompleteness and limited predictive power.

String Theory / Wikipedia

In physics, **string theory** is a theoretical framework in which the point-like particles of particle physics are replaced by one-dimensional objects called strings. In string theory, the different types of observed elementary particles arise from the different quantum states of these strings. In addition to the types of particles postulated by the standard model of particle physics, string theory naturally incorporates gravity, and is therefore a candidate for a theory of everything, a self-contained mathematical model that describes all fundamental forces and forms of matter. Aside from this hypothesized role in particle physics, string theory is now widely used as a theoretical tool in physics, and it has shed light on many aspects of quantum field theory and quantum gravity.[1]

The earliest version of string theory, called bosonic string theory, incorporated only the class of particles known as bosons, although this theory developed into superstring theory, which posits that a connection (a "supersymmetry") exists between bosons and the class of particles called fermions. String theory requires the existence of extra spatial dimensions for its mathematical consistency. In realistic physical models constructed from string theory, these extra dimensions are typically compactified to extremely small scales.

String theory was first studied in the late 1960s as a theory of the strong nuclear force before being abandoned in favor of the theory of quantum chromodynamics. Subsequently, it was realized that the very properties that made string theory unsuitable as a theory of nuclear physics made it an outstanding candidate for a quantum theory of gravity. Five consistent versions of string theory were developed before it was realized in the mid-1990s that these theories could be obtained as different limits of a conjectured eleven-dimensional theory called M-theory.[2]

Many theoretical physicists (among them Stephen Hawking, Edward Witten, and Juan Maldacena) believe that string theory is a step towards the correct fundamental description of nature. This is because string theory allows for the consistent combination of quantum field theory and general relativity, agrees with general insights in quantum gravity such as the holographic principle and black hole thermodynamics, and because it has passed many non-trivial checks of its internal consistency. According to Hawking in particular, "M-theory is the *only* candidate for a complete theory of the universe."[3] Other physicists, such as Richard Feynman,[4][5] Roger Penrose,[6] and Sheldon Lee Glashow,[7] have criticized string theory for not providing novel experimental predictions at accessible energy scales and say that it is a failure as a theory of everything.

String theory details by type and number of spacetime dimensions

Type	Spacetime dimensions	Details
Bosonic	26	Only bosons, no fermions, meaning only forces, no matter, with both open and closed strings; major flaw: a particle with imaginary mass, called the tachyon, representing an instability in the theory.
I	10	Supersymmetry between forces and matter, with both open and closed strings; no tachyon; group symmetry is SO(32)
IIA	10	Supersymmetry between forces and matter, with only closed strings bound to D-branes; no tachyon; massless fermions are non-chiral
IIB	10	Supersymmetry between forces and matter, with only closed strings bound to D-branes; no tachyon; massless fermions are chiral
HO	10	Supersymmetry between forces and matter, with closed strings only; no tachyon; heterotic, meaning right moving and left moving strings differ; group symmetry is SO(32)
HE	10	Supersymmetry between forces and matter, with closed strings only; no tachyon; heterotic; group symmetry is $E_8 \times E_8$

Super Symmetry / Wikipedia

In particle physics, **supersymmetry**, **SUSY**, is a proposed extension of spacetime symmetry that relates two basic classes of elementary particles: bosons, which have an integer-valued spin, and fermions, which have a half-integer spin. Each particle from one group is associated with a particle from the other, called its **superpartner**, whose spin differs by a half-integer.

In a theory with unbroken supersymmetry each pair of superpartners shares the same mass and internal quantum numbers besides spin, but since no superpartners have been observed yet, supersymmetry must be a spontaneously broken symmetry[citation needed]. The failure of the Large Hadron Collider to find evidence for supersymmetry has led some physicists to suggest that the theory should be abandoned.[1] Experiments with the Large Hadron Collider also yielded an extremely rare particle decay event which casts doubt on supersymmetry.[2]

Supersymmetry differs notably from currently known symmetries in that its corresponding conserved charge (via Noether's theorem) is a fermion called a **supercharge** and carrying spin-1/2, as opposed to a scalar (spin-0) or vector (spin-1). A supersymmetry may also be interpreted as new fermionic (anticommuting) dimensions of spacetime, superpartners of the usual bosonic spacetime coordinates, and in this formulation the theory is said to live in superspace.

There is only indirect evidence for the existence of supersymmetry, primarily in the form of evidence for gauge coupling unification.[3] Supersymmetry is also motivated by solutions to several theoretical problems, for generally providing many desirable mathematical properties, and for ensuring sensible behavior at high energies.

Supersymmetric quantum field theory is often much easier to analyze, as many more problems become exactly solvable. When supersymmetry is imposed as a *local* symmetry, Einstein's theory of general relativity is included automatically, and the result is said to be a theory of supergravity. It is also a feature of a candidate of a theory of everything, superstring theory.

A central motivation for supersymmetry close to the TeV energy scale is the resolution of the hierarchy problem of the Standard Model. Without the extra supersymmetric particles, the Higgs boson mass is subject to quantum corrections which are so large as to naturally drive it close to the Planck mass barring its fine tuning to an extraordinarily tiny value. In the supersymmetric theory, on the other hand, these quantum corrections are canceled by those from the corresponding superpartners above the supersymmetry breaking scale, which becomes the new characteristic natural scale for the Higgs mass.

Other attractive features of TeV-scale supersymmetry are the fact that it often provides a candidate dark matter particle at a mass scale consistent with thermal relic abundance calculations,[4][5] provides a natural mechanism for electroweak symmetry breaking and allows for the precise high-energy unification of the weak, the strong and electromagnetic interactions. Therefore, scenarios where supersymmetric partners appear with masses not much greater than 1 TeV are considered the most well-motivated by theorists.[6]

These scenarios would imply that experimental traces of the superpartners should begin to emerge in high-energy collisions at the LHC relatively soon. As of September 2011, no meaningful signs of the superpartners have been observed,[7][8] which is beginning to significantly constrain the most popular incarnations of supersymmetry. However, the total parameter space of consistent supersymmetric extensions of the Standard Model is extremely diverse and can not be definitively ruled out at the LHC.

Another theoretically appealing property of supersymmetry is that it offers the only "loophole" to the Coleman–Mandula theorem, which prohibits spacetime and internal symmetries from being combined in any nontrivial way, for quantum field theories like the Standard Model under very general assumptions. The Haag-Lopuszanski-Sohnius theorem demonstrates that supersymmetry is the only way spacetime and internal symmetries can be consistently combined.[9]

The Minimal Supersymmetric Standard Model is one of the best studied candidates for physics beyond the Standard Model.

Conclusion

A book on Vedic Physics explains the relationship of String Theory, Super String Theory, Super Symmetry, etc. to Vedic Physics in the following passages:

“The basic principle of acquiring a mass is by synchronous super -positioning of oscillatory interactions on components into a coherent and super symmetric state that is relatively static, by triggering the spin angular momentum to perfectly synchronise, thereby acquiring a coherent potential (dense state) by super - positioning of interactive states by a self - similar proportional law.

The mathematical super symmetry existing in the substratum could never have been exposed by an accidental mistranslation of the word du:kha. Considering the published findings in current science, the numerical constants of super symmetry are not yet known, yet Vedic Physics principles accurately derive these numerical values.

For right now, all hopes are pinned on a theory based on 'super-symmetry' of 'super-strings'. What scientists remain unaware of is that the source for all such theories lies hidden in a strange corner and defined in a stranger language.

“Super-symmetry”, “GUT” & etc., are all covered in Vedic Physics as an integral part of its normal evolution, through its self-similar and scale-invariant axiomatic mathematical logic.

For this reason, no effort has been made to compare these newer theories explicitly. In any case, the essence of these theories does not differ from acknowledged relativistic concepts, except for the difference in mathematical procedures and experimental conformity.

In summary, the author of the book on Vedic Physics states that the contemporary, outdated conceptions of the west do not conflict with ancient Vedic science, they have simply failed in thoroughness, and by ignoring combinatorial methods. While Super Symmetry does in fact exist in the Substratum, western scientists continue to chase comic book versions of invisible donuts in the sky, which they assign the improbable name of “black hole.”

Vedic Physics contains the answers that western science has been struggling towards, too often like a drunken husband stumbling home in the early morning hours. The descriptions of String Theory, Super String Theory and M – Theory from Wikipedia speak for themselves.

Contemporary urban wisdom states that the expectation, if one repeats the same activity over and over, while expecting different outcomes each time, indicates mental illness. At last the Wikipedia entries suggest that the husband has awoken on the following morning with a horrible hangover.

Even Roger Penrose has gotten into the act, the man who criticizes Octonions as a “lost cause” in physics. Well, since Sir Penrose is perfectly capable of identifying “lost causes” in science, we ought to take his word about Super String Theory, despite its backing by Stephen Hawking.

When all is said and done, the thrust and momentum of mathematical physics at this point must focus on H3 and H4. As de Marrais wrote:

“But we know that H4 (the only higher -level analogue of H3) has a group structure which is the direct product of two icosahedral reflection groups, and plays a key role in the overarching theory of such five-fold singularities: Shcherbak’s classic investigation of these in fact shows five-fold -symmetric forms associated with H4, H3, and D6 [30] –the last item being shown by Arnol’d to underwrite quasicrystals and their non-algorithmizable construction via the 3-D analog of Penrose tiles[31].

De Marrais essentially comes to the position that the most important structures with regard to the Sedenions are in fact Coxeter Groups H3 and H4. As we have seen, the E8 x E8 Heteroitic String Theory has become a lost cause, and even de Marrais casually dismisses this in the same breath as he raises it as a point of interest. Singularity Theory holds some promise but the author lacks access to the mainly Russian language papers on this subject.

In previous papers published on Vixra during 2013, this author independently arrived at a similar conclusion, that the objects most worthy of research attention, in the process of the formation of visible matter, are H3 and H4. As indicated by the paper presented in Appendix I, these bear relationships to A5 and Pisano Periodicity, as well as to the 60 Stellated Icosahedrons. The 24 Hurwitz Quarternions are involved here as well, and these play a critical role in the formation of matter. A good starting point for exploration of H3 and H4 is found in Appendix I, where key pieces of the paper have been reproduced.

Appendix I

Branching of the W(H4) Polytopes and Their Dual Polytopes under the Coxeter Groups W(A4) and W(H3) Represented by Quaternions

Mehmet Koca, Nazife Ozdes Koca, Mudhahir Al-Ajmi

4-dimensional H4 polytopes and their dual polytopes have been constructed as the orbits of the Coxeter-Weyl group W(H4) where the group elements and the vertices of the polytopes are represented by quaternions. Projection of an arbitrary W(H4) orbit into three dimensions is made preserving the icosahedral subgroup W(H3) and the tetrahedral subgroup W(A3), the latter follows a branching under the Coxeter group W(A4) . The dual polytopes of the semi-regular and quasi-regular H4 polytopes have been constructed.

The rank-4 Coxeter group $W(H_4)$ is unique in the sense that it has no correspondence in higher dimensions and describes the quasi crystallography in 4-dimensions. It is the extension of the icosahedral group $W(H_3) \approx A_5 \times C_2 \approx I_h$ to 4-dimensions. The group $W(H_4)$ has four 4-dimensional irreducible representations. One of its 4-dimensional irreducible representations can be described by the transformations on the quaternions by performing a left-right multiplication of the binary icosahedral group elements I which constitute one of the finite subgroups of quaternions. The Coxeter group $W(H_4)$ has five maximal subgroups [6] described by the groups

$$Aut(A_4) \approx W(A_4):C_2,\ W(H_3 \oplus A_1),\ Aut(H_2 \oplus H_2),\ Aut(A_2 \oplus A_2),\ (\frac{W(D_4)}{C_2}):S_3. \qquad (2)$$

the Coxeter-Weyl group seems to show itself as an affine Toda field theory in the zero temperature (the coldest regime) and perhaps as a Lie group in the form of describing the heterotic superstring theory at very hot regime (Planck scale).

The Coxeter-Weyl group includes the crystallographic (tetrahedral and octahedral symmetries in 3Dand 4D) as well as the quasi crystallographic symmetries (icosahedral symmetry in 3D and its generalization to 4D).

The icosahedral group of rank-3 describes fully the structures of the fullerenes such as the molecule which is represented by a truncated icosahedron, the icosahedral quasicrystals and the viral structures displaying the icosahedral symmetry.

The Platonic solids, tetrahedron, cube, octahedron, icosahedron and dodecahedron have been discovered by the people lived in Scotland nearly

1000 years earlier than the ancient Greeks.

Appendix II Smith – de Marrais Discussion

Re – publishing of comments by Frank “Tony” Smith and Robert De Marrais from an online discussion group, for those who understand the metaphoric language of box kites, sails and catamarans:

Robert de Marrais says in his paper math.GM/0011260: "... We know that surprises will keep on coming at least up to the 2^8-ions, due to the 8-cyclical structure of all Clifford Algebras. And we know at least a little about what such surprises will entail: as can already be seen in low-dimensioned Clifford Algebras including non-real units which are square roots of positive one, numbers whose squares and even higher-order powers are 0 will appear.

The 8-cycle implies an iterable, hence ever-compoundable pattern, implying in its turn cranking out of numbers which are 2^Nth roots of 0, approaching as a limit-case an analog of the "Argand diagram" whose infinitude of roots form a "loop" of some sort. If we can work with this it all, it could only be by having as backdrop some sort of geometrical environment with an infinite number of symmetries . . . suggesting the "loop" resides on some sort of negative-curvature surface.

For the incomparably stable soliton waves which are deployed within such negatively curved arenas also are just about the only concrete wave-forms which meet the "infinite symmetries" (usually interpreted as "infinite number of conservation laws") requirement. ...".

I (Tony Smith) agree with Robert de Marrais's view of periodicity of the 2^N-ions, and it seems likely to me that the 2^8-ions might be regarded as a basic building block of number theory and group theory, just as I see the 2^8-dim Clifford algebra Cl(8) as a basic building block of physics in the D4-D5-E6-E7-E8 VoDou Physics model.

I (Tony Smith) conjecture further that these links might be usable to establish a relationship between the Riemann zeta function and quantum theory.

...since for 16-ions and larger you have interesting zero-divisor "sleeper-cell" substructures, could they be useful with respect to computational systems, perhaps doing things like forming loops that might let the computational system "adjust itself" and/or "teach itself"?

Robert de Marrais commented: "what is truly interesting is this: zero-divisor systems are, ironically, PRESERVERS of associative order! Specifically, each of the four "sails" on a box-kite can be represented (on an isomorphic box-kite diagram, in fact!) as a system of four interconnected Quaternion copies:

write each vertex as a pairing of one uppercase and one lowercase letter (with the 'generator' of the given 2^n-ions being the divider of the two: e.g., with the Sedenions, g = the index-8 imaginary, and the pure Sedenions of index > 8 are "uppercase," with the Octonions thereby being written with "lowercase" letters)... each sail can be seen as an ensemble of 5 Quaternion copies (the 4 associative triplets each are completed by the real unit, and the "sterile" zero-divisor-free triplet of generator, strut constant, and their XOR makes 5).

Viewing things in closest-packing-pattern style, we have 5 interacting "unit quaternion" algebras -- with the interactions entailing (1, u), where 'u' is the shared non-real unit.

Interestingly, this gives a nice way to think about the Tibetan Book of the Dead's "58 angry demons and 42 happy Buddhas," 100 in all = 5 * 16 + 2*10 = 100 distinct units in the interlinked 5-fold "unit quaternion" ensemble. So one first sees the "42 Assessors," then zooms in one one of the 7 isomorphic box-kites (which, as with all isomorphies, can be seen as identical at some higher level); then, one zooms in further on the "second box-kite" which has its struts defined by upper vs. lower case letters, and the triple zigzag analog being the "all lowercase" sail..."

Contact

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Some men see things as they are and say *why?* I dream things that never were and say *why not?*

Let's dedicate ourselves to what the Greeks wrote so many years ago: to tame the savageness of man and make gentle the life of this world.

Robert Francis Kennedy