Introducing the Metric Laplace Equation: A Disturbing Proposed Solution to the Cosmological Constant Problems

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In spite of the widespread fanfare of the 1998 discovery of a positive accelerating expansion and the subsequent need for a "Dark Energy" placeholder in physics, the one geometric component that seems to share a relationship, the Cosmological Constant, has become shrouded in even more questions by it. After a century of concentrated efforts, the mounting lack of forthcoming answers has "driven" the NSF/ NASA/DOE Dark Energy Task Force to consider whether general relativity is "incorrect". In keeping with this reluctant but forced skepticism we subject an early competitor to general relativity, Gunnar Nordströms version of the Poisson equation, to a more stringent definition utilizing an asymmetric property of the Fundamental Theorem of Calculus. We derive from this property a new metric Laplacian definition for flatness that in perturbed spherical symmetry form greatly resembles the Schwarzschild solution. However, this metric version would seem capable of uniting gravity with QFT by utilizing the widely considered equivalence of the Cosmological Constant with a proposed large value vacuum energy density but at the expense of differential topology and our understanding of tensors in general. A much larger penalty though seems to be that it results in a geometrical counterpoint to the physical explanations for general relativity, QFT and energy density.

Keywords: Dark Energy, Cosmological Constant, Quantum Gravity, Nordström

I. INTRODUCTION

Even prior to the discovery of a positive accelerating expansion [1–3], there existed fundamental questions as to how a Cosmological Constant (CC,λ, Λ) could be incorporated into general relativity (GR) through a seamless geometrical argument [4–7]. It could be argued that as one comes to understand the geometric basis of GR, the possibility of any CC becomes even more objectionable. The seemingly pure perturbations of basis unit vectors is jarringly interrupted by a term often considered as a constant of integration. The inability to unite GR with QFT has led to a conundrum. How can a geometrical theory be so exhaustively researched, provide a seemingly unassailable structure for gravitational phenomena but yet continue to refuse alignment with other theories and become even more paradoxical as time marches on? The strength of its geometrical basis would seem to also be a shield against unity.

While to the general science community, any detractions of general relativity may sound like ignorant hyperbole, quantum gravity theorists and cosmologists are quietly sounding an alarm bell. In 2006, the National Science Foundation, along with NASA and DOE, commissioned a Dark Energy Task Force [8] to lay out arguments that the existence of a Dark Energy is profoundly at odds with our current understanding of physics, thus requiring funding of special programs to determine its nature. It would appear that the energy density we are familiar with, defined through the Poisson equation, is fairly insignificant in the universe.

It is an intuitive geometric question that has led us backwards through the history of field theory to a subtle but different modification of a starting assumption, that of Laplace's equation. We begin our argument here, but caution against jumping to early conclusions based on any previous knowledge of what a "scalar" or "tensor" means (although previous knowledge of gauge invariance and metrics will help). This warning also applies to any equation which contains energy density ρ or mass, as we will be required to align our geometric equations using the same fluid analogies that lead to the energy-momentum tensor.

We will first give a simple graphical understanding of our interpretation, followed by a more formulaic explanation using the same techniques as Riemann sums. We avoid a drawn out proof here since a vast swath of material must necessarily be leaped over in order to arrive at what we propose is the significance of our argument. Formalization will be forthcoming but as this is a new theory that changes some basic equations, refuting peer review criticism will require more extensive knowledge of arguments presented tens, if not hundreds, of years ago. The phrase "state of the art" may be a bit of a misnomer in this research path.

In light of the fundamental questions concerning energy density, we are free to reconsider how scalar covariant theories would be affected by a geometrical reformulation of the Laplace and Poisson equations. To do this we require a new understanding of calculus, dubbed "Area Calculus" herein, that allows formulation of the metric Poisson and comparisons with other covariant forms.

II. GUNNAR NORDSTRÖM'S THEORIES

During the early days of relativity, various theories were studied to determine their predictive abilities. One of the most well known within the circle of geometric interest was that of extending the Newtonian scalar potential into four dimensions from the usual three. Although other theorists, including Einstein, have studied these covariant scalar theories, it is

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the Finnish physicist Nordström who is most widely associated with them due to a variable mass form he proposed [9] that was at one point was considered a serious contender. In more recent times, his theories are generally only utilized in introductory materials as an illustration of a geometric dead end. What we find interesting is that although there are examinations of the original assumptions that form covariant scalar and tensor theories and the predictions that should result, there seems to be a lack of research into the differences between fundamental equational mechanics of linearized tensor approximations and scalar theories from a first principals standpoint. Critics of these have no problem noting the superiority and relationship of $R_{\mu\nu} = 0$ to Poisson's $\nabla^2\Phi = f$, but seem to have cast a blind eye towards a rigorous conversion to Laplace's equation $\nabla^2\Phi = 0$.

Although it is often stated that the source reason for Einstein's Λ (sometimes misunderstood as "invented" instead of derived) was to create a static universe, a review of his 1917 paper *Cosmological Considerations On The General Theory Of Relativity* [10] also shows that his first geometric hunch was to use it as a modification to Poisson's equation so that it would become the limiting value of the Newtonian Φ at "spatial infinity". We present Figure 1 as a visual aid in understanding our research. It should be obvious that although all three figures result in similar outcomes at "spatial infinity", the Φ does not have the same value in each plot. However, keep Einstein's supposition on Λ in mind as we present Area Calculus.

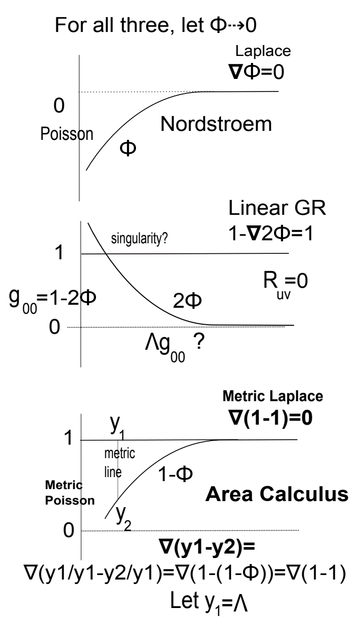

FIG. 1. A geometric illustration of similarities

III. A PROOF OF AREA

Therefore, we start by re-examining the concept of differentiation and integration from their most basic proof in graphical form. Let us draw a regular rectangle where the top and the bottom are line segments such that *each* is a function of x denoted as y_1^* and y_2^* (Fig. 2).

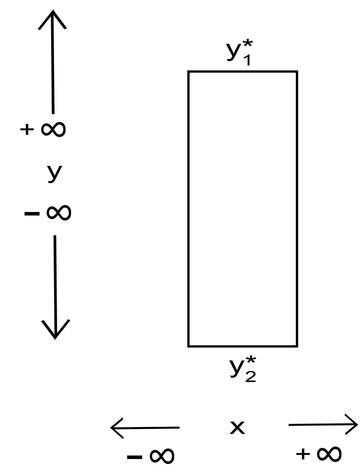

FIG. 2. Lines segments as two functions

We do not yet place these onto a coordinate plot as a solid line (Fig. 3) on the x axis can lead to an incorrect proof. We

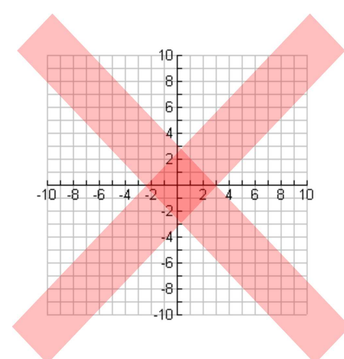

FIG. 3. Misleading solid axes

then incorporate multiple rectangles (A,B,C,D) (Fig. 4), of equal length ($\Delta x = x_2 - x_1$) and height ($\Delta y = y_1^* - y_2^*$) with an area of $((y_1^* - y_2^*) * (x_2 - x_1))$. Each horizontal line segment has a particular $|y_1^*|$,$|y_2^*|$ of $-\infty > y^* < \infty$ as measured from an unknown zero point. The vertical line segment between the y functions ($\Delta y = y_1^* - y_2^*$) that is perpendicular to x will become our metric.

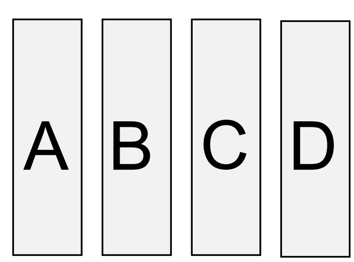

FIG. 4. Rectangles

Note that if we continue adding on rectangles with the same width and height (E,F,G,H) (Fig. 5), that the rate of the

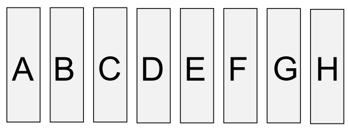

FIG. 5. Adding area via rectangles

addition of area is constant (**rate of change of addition of area=area next-area previous**). Should we instead add on blocks of decreasing area (E',F',G',H')(Fig. 6), then the rate of the addition of area is decreasing.

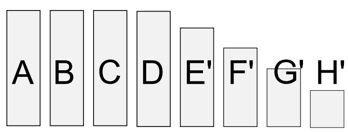

FIG. 6. Addition of area decreasing

From Fig. 7, we can see that a reflection of the rectangles about the lower line segment does not change the rate of the addition of area. It is still either constant, decreasing or similarly increasing (not shown) should we add on blocks of greater area (E",F",G",H"). We assume in this new interpretation of calculus that *area* is axiomatic such that the x and y line segments are simply the boundaries used to denote it. We designate the top line in all plots, even after reflection, as a variation of y_1 and the bottom line as a variation of y_2.

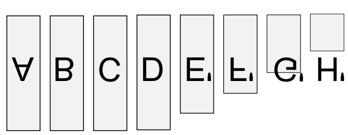

FIG. 7. Reflected but area still decreasing

Let us now use the standard proof technique of taking the number of individual rectangles to infinity by decreasing the width of each rectangle. It is important to always maintain in mind that whereas the width of Δx is decreasing, the distance between the line segments of Δy^* *is not* as line segments y_1^* and y_2^* are *two* separate functions of x. We must, however, denote the change in each individual line segment function as Δy_1^* and Δy_2^* as we take into account the decrease in Δx to zero (Fig. 8).

Thus we obtain an instantaneous rate of change of the addition of area (or the instantaneous change in area of rectangles of zero width) as $\frac{dy_1 - dy_2}{dx} \equiv \lim_{\Delta x \to 0} \frac{\Delta y_1^* - \Delta y_2^*}{\Delta x} = \lim_{h \to 0} \frac{(f_1(x+h) - f_1(x)) - (f_2(x+h) - f_2(x))}{(x+h) - x}$. To reiterate, although $x_2 - x_1 = \Delta x \to dx$ we must understand that $y_1^* - y_2^* = \Delta y_1^* - \Delta y_2^* \to dy_1 - dy_2$, ***not just* the single function form** dy! Thus our metric line segment is actually an instantaneous change in the addition of area and is the definition of our metric.

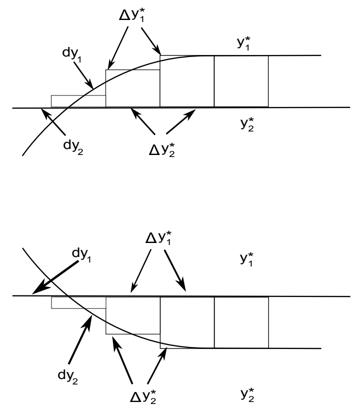

FIG. 8. Reflected line segments into continuous functions

Let us put some numerical values in to provide a better understanding. Let $y_1 = 10 - \frac{1}{x}$ and let $y_2 = 8$. The Area derivative is $\frac{dy_1 - dy_2}{dx} = \frac{d(10 - \frac{1}{x} - 8)}{dx} = \frac{1}{x^2}$. If we should reflect the area about y_2 we now have $y_1 = 8$ and $y_2 = 6 + \frac{1}{x}$. The Area derivative and the "direction" of a directional derivative is unchanged with $\frac{dy_1 - dy_2}{dx} = \frac{d(8 - (6 + \frac{1}{x}))}{dx} = \frac{1}{x^2}$ despite incorporating a function with a slope of opposite sign. This is a proof that differentiation happens with respect to the secondary constant function (or the change of the line segment metric), not whether the points that make up a line are changing positively or negatively.

IV. ANTI-DIFFERENTIATION

Assuming that y_2 or y_1 is a constant function (we do not go into it now, but if the functions only differ by a "scalar" amount, then this would seem to be a truer definition of an Einstein manifold with $R = nk = n(y_1 - y_2)$) and the area has been reflected, then $\frac{dy_1 - dy_2}{dx} = \frac{dy_1}{dx} = \frac{-dy_2}{dx}$. Graphically, these both reproduce the same function relative to the y=0 on the x axis ($\frac{1}{x^2}$ in our example). During integration, there is no difference between the answer numerically given through standard Single Function techniques, however Area Calculus views the "area under the curve" of y_1 as instead the "area between the functions" of y_1 and y_2, providing a major proof difference during the process of anti-differentiation. With area axiomatic, the x axis must be the function $y_2 = 0$ which has an indefinite integral form of $\int 0dx = k$, where k is an arbitrary constant. Thus $\int \frac{d}{dx}(y_1 - y_2)dx = \int (\frac{d}{dx}(y_1) - 0)dx = (\int \frac{d}{dx} y_1 dx) - (\int 0dx) = (y_1 + c) - (k)$ which demonstrates those dual functions within Area Calculus that are viable solutions, including $((10 - \frac{1}{x}) - 8)$, $(8 - (6 + \frac{1}{x}))$ and $((0 - \frac{1}{x}) - 0)$.

V. THE METRIC LAPLACIAN

While we have only briefly gone over some of the properties of our new metric definition, we move on to one required to advance our argument. As can be understood from the previous sections, the asymmetry in single function calculus is that differentiation and integration are viewed as a cyclical process. Generally, a function that can be differentiated can also be anti-differentiated. There is of course, the anti-symmetric nature of a constant of integration. Simply put, although there are an infinite number of functions that equal zero ($f' = 0$) when differentiated, there are NO functions where the anti-derivative equals zero. We view this as an important distinction between the functional forms $\nabla^2\Phi = 0$ and $\nabla^2 0 = 0$. The second equation can only be geometrically true, without assigning a function a scalar value, via a metric line segment. This would give us $\nabla^2 0 = \nabla^2(y_1 - y_2) = 0$. Due to the differential nature of the equation, we could normalize one of the y functions with respect to the other. By simply multiplying with the inverse of one of the functions, we normalize the length of our metric line segment such that $\frac{1}{y_1}\nabla^2(y_1 - y_2) = \nabla^2(1 - \frac{y_2}{y_1}) = \nabla^2(1 - 1) = 0$.

VI. THE METRIC POISSON EQUATION

In Figure 9 we have the graphical basis of the Poisson equation where the first derivative of the field potential is force and the second derivative is the definition of energy density.

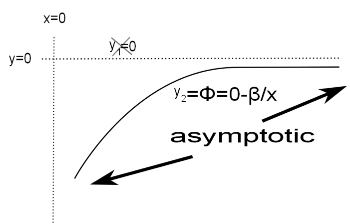

FIG. 9. Poisson field potential

Our metric Laplace equation would require that a normalized Poisson equation have a term with the form of $\frac{y_2}{y_1} \approx 1 - \zeta$ giving us $\nabla^2(1 - \frac{y_2}{y_1}) \approx \nabla^2(1 - (1 - \zeta)) = \mu$ (we used Φ instead of ζ earlier in our comparison). We must keep in mind though, that although the vectorial approximation would be just of $\nabla\zeta$, the normalization process falls away during anti-differentiation since the function becomes $\int(0 - (0 - \zeta')) \approx (y_1 - (y_2 - \zeta))$.

VII. CONCLUSION

Attempts at incorporating Λ from the linearized field equation of $1 - 2\Phi$ into Newton's Law of Gravity using the Poisson equation leads to the formula [11]

$$\vec{g} = -\nabla\Phi = -\frac{GM}{r^2}\hat{\vec{r}} + \frac{\Lambda c^2 r}{3}\hat{\vec{r}}.$$

There is no known way to reconcile a theoretical relationship between the observed magnitudes of M and Λ in this equation, even to the point of being referred to as the "worst" prediction in physics.

Comparing the Area Calculus version of the Metric Poisson equation against the linearized gravity portion demonstrates some short comings to incorporating the Cosmological Constant into GR (Fig. 1). In the standard Poisson we also see hidden assumptions that y_1 has been ignored, $y2 = 0 - \frac{\beta}{x}$ and that the area has no finite boundaries (perhaps not quantized 4-volume) since $|(y_2)| \to 0$ as $x \to \infty$ and $|(y_2)| \to -\infty$ as $x \to 0$. We are not aware that either of these boundary conditions match any known empirical evidence, nor how quantized energy levels can effect out to spatial infinity. From the most basic of linear solutions, it makes no sense where to place a multiple of the metric into a plot that already contains the metric and its perturbations. Where does Λg_{00} go in relation to the $g_{00} = 1$ line and how can it possibly be permissible for them both to occupy the same plot?

In light of the deep paradoxes and relationships of the Cosmological Constant and Dark Energy, we conclude that the simplest geometric solution is the one that should receive first consideration, even if it is disturbing to our current perspective.

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