Extended Electron in Constant Magnetic Field

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In this article we determine the net magnetic forces **Fm** developed on the extended electron which is moving in a constant magnetic field **B** with the velocity **V** . The electron has two particular motions : *parallel* and *normal* to the external magnetic field .
Part I : When the electron moves parallel to **B** (**V** ⁄⁄ **B**) , the net magnetic force **Fm** produced on the electron is zero : **Fm** = 0 .
Part II : When the electron moves normally to **B** (**V** ⊥ **B**) , two opposite forces **F** and **F'** are produced on the electron : **F** is the resultant of all magnetic forces **fm** produced on surface dipoles of the electron : **F** = Σ **fm** ; and **F'** is the magnetic force produced on the core ($-q_0$) of the electron . The net magnetic force produced on the electron is thus **Fm = F + F' .**
Part III : These magnetic forces (**fm , F , F', Fm)** will help illustrating the mechanism of radiation of the extended electron in the magnetic field .
Key words : extended electron , electric dipole (-q ,+q) , surface dipoles , interior dipoles , the core ($-q_0$) .

Introduction

The readers are recommended to read the previous article [1(a)] : "**A new extended model for the electron "** to have a view on the *extended model* of the electron and the *assumptions for calculations* ; since all the calculations in this article will be based on this model and the assumptions on the electric and magnetic boundary conditions .

In a nutshell , this extended model of the electron is a version of the image of the *screened electron by vacuum polarization* [1(a)] ; it is a spherical composite structure consisting of the point-charged core ($-q_0$) which is surrounded by countless electric dipoles (-q ,+q) as schematically shown by Figs.1 & 2 .
And hence , in the determination of the net force **Fm** , we will go through four sections :

1. Calculation of forces **fm** which are produced on *surface dipoles* of the extended electron and the resultant **F** = Σ **fm** .
2. Calculation of forces which are produced on *interior dipoles* of the extended electron. As we will argue in the text , the resultant of these forces cancels out .
3. Calculation of the force **F'** which is produced *on the core* $-q_0$ of the electron .
4. Finally , the net force **Fm = F + F'** .

To reduce the length of the main text , all long calculations are put into the **appendices** (A , B and C) ; only the results of these calculations are showed in the main text . The readers who are interested in these calculations can read them in the appendices .

Part I : Extended electron moving parallel to the magnetic field B : (V ⁄⁄ B)

To calculate magnetic forces produced on the extended electron we make use of boundary conditions [1(a)] to determine the magnetic field **B'** inside of the spherical surface of the electron and Lorentz's magnetic force equation $\mathbf{F_L} = q\,(\mathbf{V} \times \mathbf{B})$ to calculate magnetic forces produced on point charges like $-q$ and $+q$ (of a dipole) and on $-q_0$ (of the core) . As for the direction of the magnetic forces we will use the following rule of the observer (which is equivalent to *the rule of three fingers of the right hand*) :
*" An observer standing in the direction of the magnetic field **B** , looks at the charge q in the direction of its motion **V** . If q is a positive charge , the magnetic force **fm** directs in the direction of the right hand of the observer and is considered as a positive force . If q is a negative charge , the magnetic force **fm** directs in the direction of the left hand of the observer and is regarded as a negative force ".*

I. 1 Magnetic forces fm produced on surface dipoles of an extended electron moving parallel to B (V//B)

First , let us determine the *magnitude* and the *direction* of magnetic forces **fm** produced on a surface dipole M of the extended electron .
Let's consider four surface dipoles M , N , P , Q lying on the same great circle C of an electron moving parallel to the external constant magnetic field **B : V // B** (Figs.1 & 2).

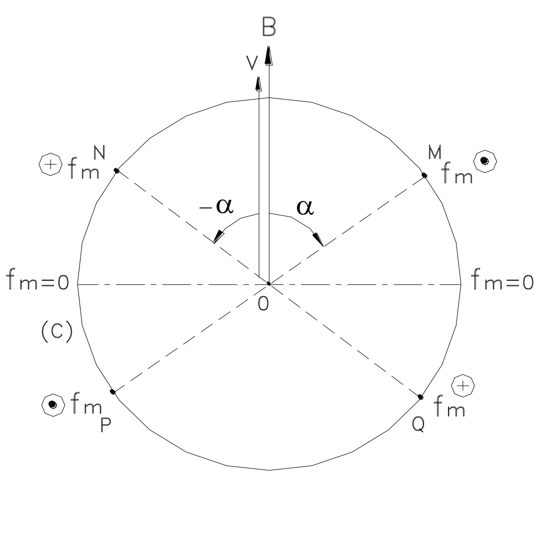


Fig.1 . **μ > 1** , **V ⁄⁄ B** : directions of magnetic forces **fm** acting on four surface dipoles M , N , P , Q on the great circle C .
(•) fm points up from the page ,
⊕ **fm** points down from the page .

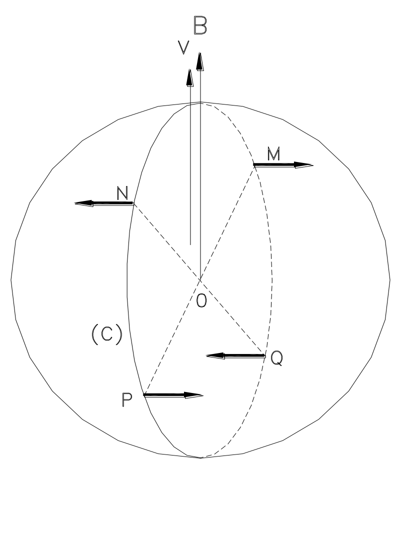


Fig.2 . Another view of the great circle C in Fig.1 : the directions of **fm** on four dipoles M , N , P , Q.

Calculations of magnitudes of **fm** produced on four surface dipoles M , N , P , Q are shown in the **Appendix A** . The results are :

When $\mu > 1$: $f't > f'n$, $fm = f't - f'n = (\mu - 1)\, q V B \sin\alpha \cos\alpha$: **fm** **(•)** (A.1)
where $0 \le \alpha \le \pi$ (Fig.5) .

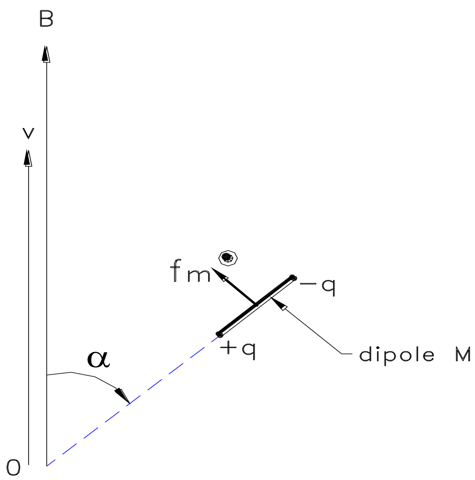


Fig.5 . **μ > 1 , V ∕∕ B** : the magnetic force **fm** acting on dipole M points up from the page (•)

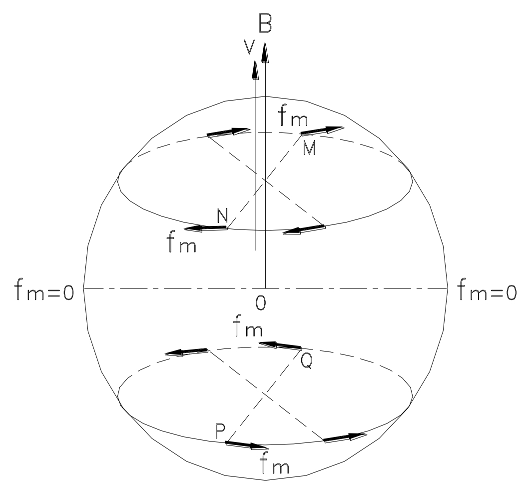


Fig.6 **. μ > 1 , V ∕∕ B :** on the upper hemisphere **fm** tend to rotate the electron clockwise ; on the lower hemisphere **fm** tend to rotate the electron counterclockwise (if the electron is viewed from above) .

The same calculations on the remaining dipoles N , P , Q give the results (Figs.1 & 2) :
- on dipole N : **fm** ⊕ ,
- on dipole P **:** **fm (•)** **,**
- on dipole Q : **fm** ⊕ ,

The magnitude of **fm** is the same on four dipoles M , N , P , Q :

$$fm = (\mu - 1)\, q V B \sin\alpha \cos\alpha \qquad (1) \text{ or } (A\text{-}1)$$

where $\mu > 1$ and $0 \le \alpha \le \pi$ is the angular position of the surface dipole .
On the equator of the electron : $\alpha = \pi/2$, $\cos\alpha = 0$, so $fm = 0$.
At two poles of the electron : $\alpha = 0$ and $\alpha = \pi$, $\sin\alpha = 0$, so $fm = 0$.

Fig.6 shows the case **$\mu > 1$** : four forces **fm** acting on four dipoles M , N , P , Q the they are tangent to the spherical surface of the electron .
On the upper hemisphere two forces **fm** at M and N form a couple of forces tending to rotate the electron *clockwise* . On the contrary, two forces **fm** at P and Q on the lower hemisphere form a couple of forces tending to rotate the electron *counterclockwise* . These two couples of forces cancel out : they have no effect on the motion of the electron in **B** because the net force **Fm** = Σ **fm** = 0 and the net torque **T** = $\Sigma\boldsymbol{\tau}$ = 0 .

When $\mu < 1$: f'n > f't , **fm** reverse their directions on all four dipoles M , N , P , Q , and their magnitudes are

$$fm = f'n - f't = (1 - \mu) q V B \sin\alpha \cos\alpha \qquad (2) \text{ or } (A\text{-}2)$$

Two expressions (1) and (2) or (A-1) and (A-2) are calculated in the **Appendix A** .

This result can be generalized as follows : magnetic forces produced on all surface dipoles on the upper hemisphere counteract magnetic forces on the lower hemisphere, and hence they have no effect on the motion of the electron in **B** .

In short , when an extended electron moves parallel (or anti-parallel) to **B ,** magnetic forces **fm** are developed on *all surface dipoles* but their net force Σ **fm** and net torque $\Sigma\boldsymbol{\tau}$ cancel out , and thus they have no effect on the motion of the electron (**V∕∕B**).

I. 2 The resultant of magnetic forces produced on all interior dipoles of the extended electron is cancelled out .

All *interior dipoles* have the same velocity **V** and are subject to the same magnetic field **B'** inside the electron . Two magnetic forces produced on two ends of an interior dipole (-q , +q) are thus equal and opposite ; they cause a slight "*rotation on-the-spot* " (re-orientation) of all electric dipoles , but the resultant force is zero . So , the resultant of all magnetic forces produced on all interior dipoles is zero .

I. 3 Magnetic force produced on the core $-q_0$ of the extended electron is zero .

The core ($-q_0$) is a point charge at the center of the electron where the magnetic field **B'** is created by the external field **B .** Because of the spherical symmetry of the structure of the electron , **B'** must be parallel to **B** . So when the electron moves parallel to **B** , the core $-q_0$ moves parallel to **B'** and hence the magnetic force that **B'** produces on the core is zero (according to the Lorentz 's force equation when **V∕∕B**) . The calculation of the magnitude of **B'** is presented in the appendix C **.**

I. 4 The net magnetic force Fm produced on the extended electron is equal to zero when V∕∕B

We have determined magnetic forces produced on different components of the extended electron when it moves parallel to the external magnetic field **B** (**V**∥**B**) :
- the resultant of forces produced on all surface dipoles is zero (section I.1),
- the resultant of forces produced on all interior dipoles is zero (section I.2) ,
- the magnetic force produced on the core ($-q_0$) is zero (section I.3) .

So , the net magnetic force **Fm** which is the sum of these forces is equal to zero .

Conclusion : This result appears to be the same as the Lorentz magnetic force $\mathbf{F_L}$ produced on a point electron of electric charge e that moves parallel to the external magnetic field **B** :

$$\mathbf{F_L} = e\,(\,\mathbf{V} \times \mathbf{B}\,) = 0$$

This expression means that the magnetic force $\mathbf{F_L}$ is simply not produced on the point electron , or in other words , $\mathbf{F_L}$ does not exist . And hence it has no effect at all on the electron .
But for the extended electron , the elementary magnetic forces **fm** $\neq$ 0 , although their resultant **Fm** = Σ **fm** = 0 as they are shown in Fig.6 . As noted from two expressions (1) and (2) , fm = 0 only on the equator and at two poles (north and south) and on the core of the extended electron, but **fm** $\neq$ 0 elsewhere .
These non-zero magnetic forces **fm** cause the electric dipoles of the electron to be slightly re-oriented , leading to a change in its permeability μ **;** and this change affects the effective electric charge of the electron .

The readers who are interested in this topic are invited to read the *thought experiment* described in section 4 of the article [1(b)] : **" A Foundational Problem in Physics : Mass versus Electric Charge "** which tried to prove the variability of the effective electric charge of the extended electron in the magnetic field.

Part II : Extended electron moving normally to B : V ⊥ B

When an extended electron moves normally to the external constant magnetic field **B ,** two opposite forces **F** and **F'** are produced on the electron : **F** is the resultant of all magnetic forces **fm** produced on surface dipoles of the electron (i.e., **F** = Σ **fm)** , and **F'** is the magnetic force produced on the core ($-q_0$) of the electron . The net magnetic force is thus **Fm** = **F + F'** .
As before , to determine the net force **Fm** , we have to go through four sections :
1. Calculation of forces **fm** which are produced on *surface dipoles* of the extended electron and the resultant **F** = Σ **fm** .
2. Calculation of forces which are produced on *interior dipoles* of the extended electron. As we will argue in the text , these forces cancel out .
3. Calculation of the force **F'** which is produced *on the core* $-q_0$ of the electron .
4. Finally , the net force **Fm = F + F'** .

II . 1 : Determination of fm on surface dipoles and the resultant F = Σ fm

First we determine the direction and magnitude of **fm** produced on surface dipoles lying on *three particular great circles* C_1 , C_2 and C_3 (Figs.7 , 8 & 9) . The results of these three steps will help determine **fm** on an *arbitrary surface dipole* in step 4 .

In the following figures , $\mathbf{V} \perp \mathbf{B}$ and the magnetic force-axis is normal to the plane (**V , B**) .
As before , we use the rule of the observer (stated at the beginning of Part I) to determine the direction of the magnetic force .

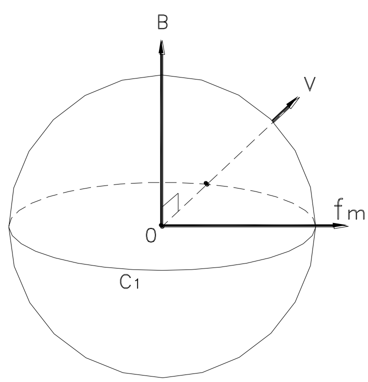


Fig.7 . The great circle C_1 lies in the horizontal plane (**V , fm**) .

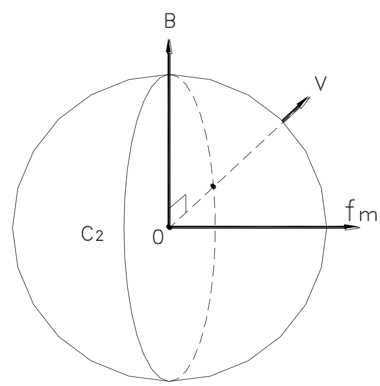


Fig.8 . The great circle C_2 lies in the plane (**B , V**) .

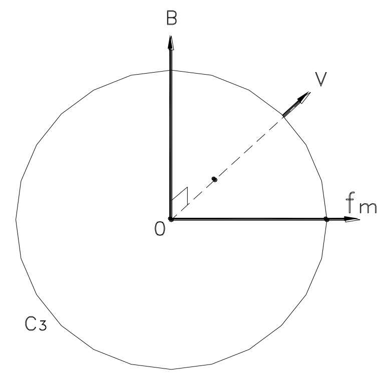


Fig.9 . The great circle C_3 lies in the plane (**B , fm**) .

Step 1 : Magnetic forces fm produced on surface dipoles lying on the great circle C_1

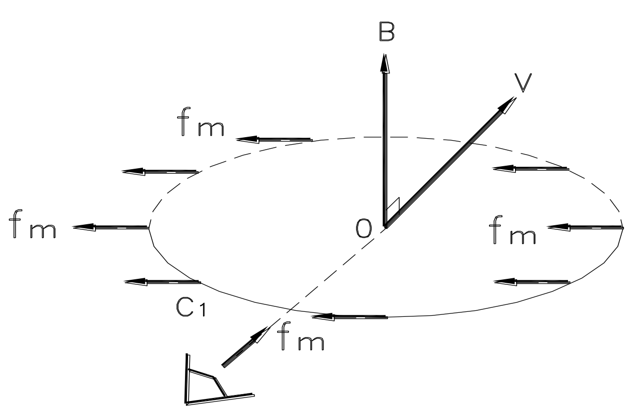
C_1 is the equatorial circle of the electron ; it lies in the plane (**V , fm**) (Fig.7) .
Calculations in Appendix B (for step 1) give the magnitudes and directions of the forces **fm** :
Fig.14 : for $\mu < 1$: all forces **fm** point to the left of the observer and have magnitudes

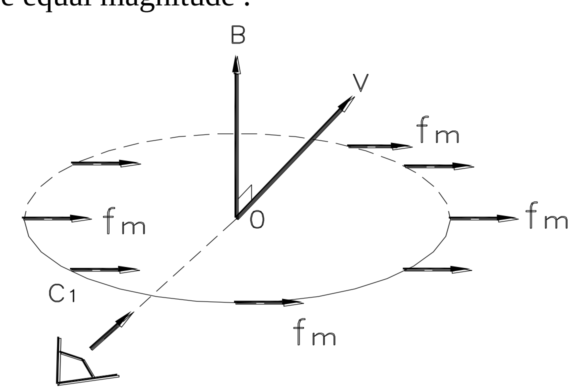
$$fm = ft - f't = (1 - \mu) q V B$$

Fig.15 : for $\mu > 1$: all forces **fm** point to the right of the observer and have magnitudes

$$fm = f't - ft = (\mu - 1) q V B \qquad (3) \text{ or } (B.1)$$

All forces **fm** produced on surface dipoles lying on C_1 are parallel to each other because they are perpendicular to the plane (**V , B**) and have equal magnitude .





Observer

Observer

Fig.14 . For $\mu < 1$: all magnetic forces **fm** produced on the great circle C_1 point to the left of the observer .

Fig.15 . For $\mu > 1$: all magnetic forces **fm** produced on the great circle C_1 point to the right of the observer .

Step 2 : Magnetic forces fm produced on surface dipoles lying on the great circle C_2

The great circle C_2 lies in the plane (**B , V**) (Fig.8) . Calculations in Appendix B (for step 2) give the magnitudes and the directions of the forces **fm** :

Fig.20 : For $\mu < 1$: all magnetic forces **fm** produced on the great circle C_2 point to the left of the observer and have magnitude

$$fm = ft - f't = (1 - \mu) q V B \sin^2 \alpha$$

Fig.21 : For $\mu > 1$: all magnetic forces **fm** produced on the great circle C_2 point to the right of the observer and have magnitude

$$fm = f't - ft = (\mu - 1) q V B \sin^2 \alpha \qquad (4) \text{ or } (B.2)$$

All magnetic forces **fm** produced on C_2 are parallel to each other because they are perpendicular to the plane (**V , B**) , but their magnitudes depend on the angle α ($0 \leq \alpha \leq \pi$) , the angular position of the dipole .

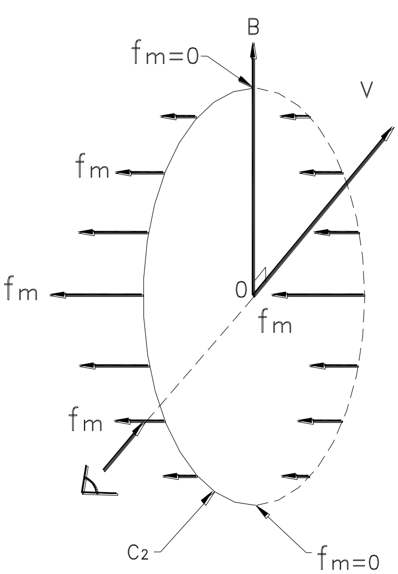


Fig.20 . For $\mu < 1$: all magnetic forces **fm** produced on the great circle C_2 point to the left of the observer .

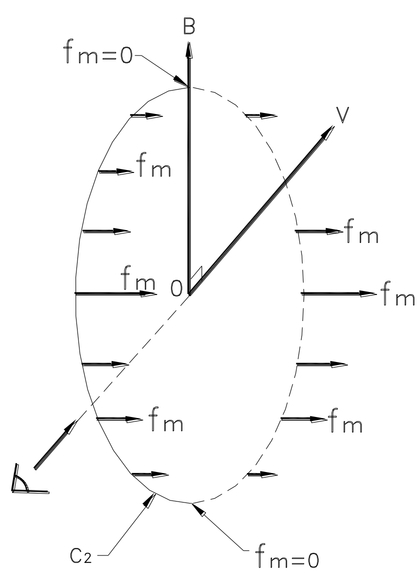


Fig.21 . For $\mu > 1$: all magnetic forces **fm** produced on the great circle C_2 point to the right of the observer .

Step 3 : Magnetic forces fm produced on surface dipoles lying on the great circle C_3

The great circle C_3 lies in the plane (**B , fm**) : Fig.9
Calculations in Appendix B (for step 3) give the following results :

Fig.26 : For $\mu < 1$: all magnetic forces **fm** produced on C_3 point to the left of the observer . Magnitudes of **fm** are

$$fm = ft - f't = (1 - \mu)\, q\, V\, B \sin\alpha$$

Fig.27 : For $\mu > 1$: all magnetic forces **fm** produced on C_3 point to the right of the observer . Magnitudes of **fm** are

$$fm = f't - ft = (\mu - 1)\, q\, V\, B \sin\alpha \qquad (5) \text{ or } (B.3)$$

Magnetic forces **fm** produced on C3 are not parallel to each other ; their magnitudes depend on the angle α .

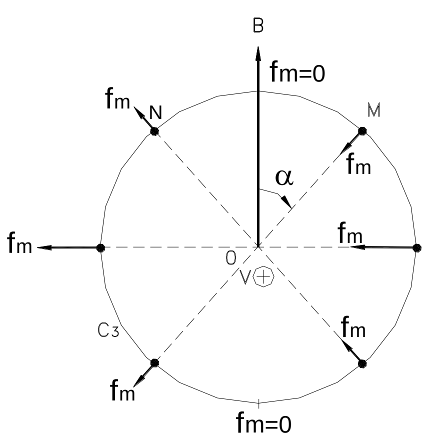


Fig.26 . For $\mu < 1$: all magnetic forces **fm** produced on C_3 point to the left of the observer .

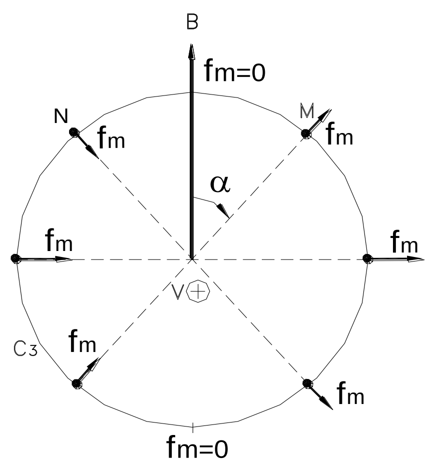


Fig.27 . For $\mu > 1$: all magnetic forces **fm** produced on C_3 point to the right of the observer .

We have determined the directions and magnitudes of magnetic forces **fm** produced on three great circles C_1 , C_2 and C_3 . By comparing the magnitudes of **fm** on these great circles we find that **fm** on C_1 (which is the equator of the electron) are strongest .

Figs.28 & 29 show an *overall view* of these forces **fm** on three great circles C_1 , C_2 and C_3 in two cases $\mu < 1$ and $\mu > 1$ respectively .

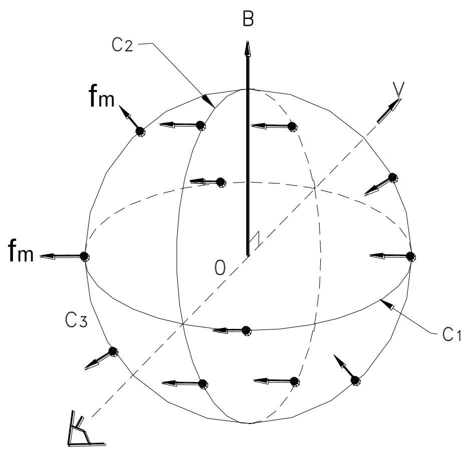


Fig.28 . For $\mu < 1$: all magnetic forces **fm** produced on surface dipoles lying on three great circles C_1 , C_2 , C_3 point to the left of the observer .

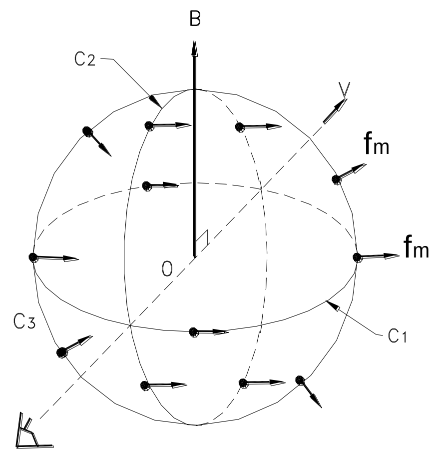


Fig.29 . For $\mu > 1$: all magnetic forces **fm** produced on surface dipoles lying on three great circles C_1 , C_2 , C_3 point to the right of the observer .

These two overall views of these forces **fm** allow us to confirm the direction of the magnetic force **fm** produced on an *arbitrary surface dipole* A :

- **fm** points to the left of the observer when $\mu < 1$ as shown in Fig.30
- **fm** points to the right of the observer when $\mu > 1$ as shown in Fig.31

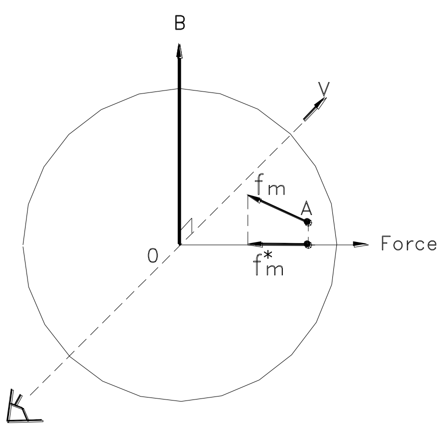


Fig.30 . For $\mu < 1$: magnetic force **fm** produced on an arbitrary surface dipole A points to the left of the observer ; and hence its projection **fm*** on the force-axis also points to the left : **fm*** is thus regarded as a negative force .

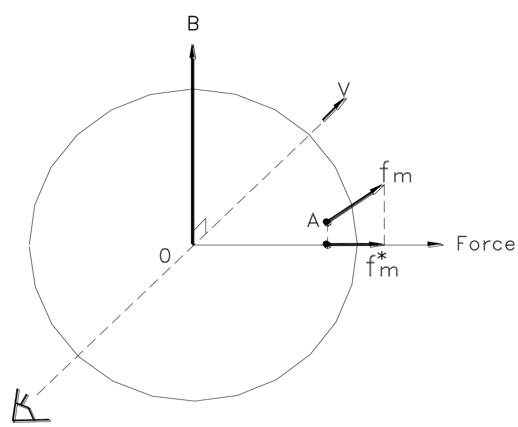


Fig.31. For $\mu > 1$: magnetic force **fm** produced on an arbitrary surface dipole A points to the right of the observer ; and hence its projection **fm*** on the force-axis also points to the right : **fm*** is thus regarded as a positive force .

The magnitude of **fm** and the resultant $\mathbf{F} = \sum \mathbf{fm}$ are calculated in the Appendix B (for step 4) . The projection **fm*** is used in the calculation of the resultant **F** .

Step 4 : Calculation of magnitude of fm produced on an arbitrary surface dipole and the resultant $\mathbf{F} = \sum \mathbf{fm}$

Let's consider an arbitrary surface dipole A ; **n** is the normal at A ; **t** is the tangent at A , lying in the plane (**OB , n**) ; $\alpha = (\mathbf{B}, \mathbf{n})$; $\beta = (\mathbf{V}, \mathbf{Bt}) = (\mathbf{V}, \mathbf{t})$; $\mathbf{V} \perp \mathbf{B}$ **.**

The appendix B (for step 4) gives the magnitude of **fm** produced on the dipole A :

- for $\mu < 1$: $fm = ft - f't = (1 - \mu)\, q\, V\, B \sin\alpha \sin\beta$ (6) or (B.4)
- for $\mu > 1$: $fm = f't - ft = (\mu - 1)\, q\, V\, B \sin\alpha \sin\beta$ (7) or (B.5)

Since **fm** are symmetric around the force-axis , their resultant $\mathbf{F} = \sum \mathbf{fm} = \sum \mathbf{fm}^*$.
When $\mu < 1$, **F** points to the left as shown in Fig.32 and is thus considered as a negative force .
When $\mu > 1$, **F** points to the right as shown in Fig.33 and is considered as a positive force .

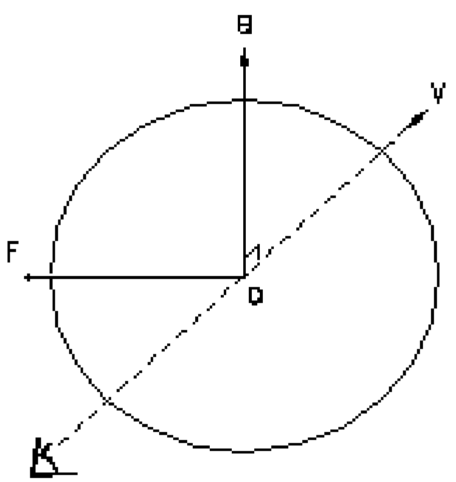


Fig.32 . For $\mu < 1$: the resultant force **F** of all magnetic forces **fm** produced on all surface dipoles of the electron points to the left of the observer : **F** is negative .

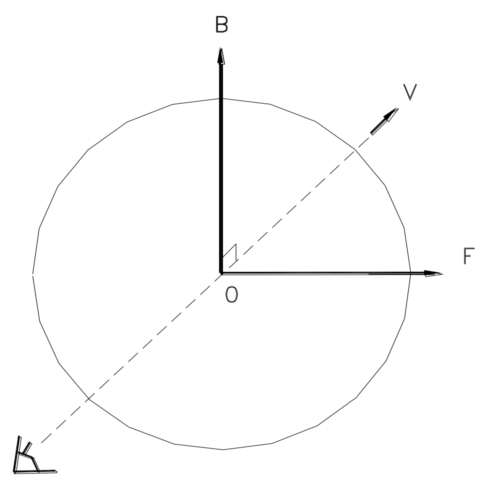


Fig.33 . For $\mu > 1$: the resultant force **F** of all magnetic forces **fm** produced on all surface dipoles of the electron points to the right of the observer : **F** is positive .

Let γ be the angle between **fm** and the force-axis , we have $fm^* = fm \cos\gamma$;
and hence , in magnitude $F = \sum fm^* = \sum fm \cos\gamma$; where fm are given by (6) and (7)
Therefore , for $\mu < 1$: $F = \sum (1 - \mu)\, q\, V\, B \sin\alpha \sin\beta \cos\gamma$
for $\mu > 1$: $F = \sum (\mu - 1)\, q\, V\, B \sin\alpha \sin\beta \cos\gamma$

Using the index i to indicate the surface dipole i and n is the total number of surface dipoles , these two expressions can be written in the general form

For $\mu < 1$ (Fig.32) $F = (1 - \mu)\, q\, V\, B \sum_{i=1}^{n} \sin\alpha_i \sin\beta_i \cos\gamma_i$ (8)

Since F is negative , the sum $\sum_{i=1}^{n} \sin\alpha_i \sin\beta_i \cos\gamma_i < 0$

For $\mu > 1$ (Fig.33) $F = (\mu - 1)\, q\, V\, B \sum_{i=1}^{n} \sin\alpha_i \sin\beta_i \cos\gamma_i$ (9)

Since F is positive , the sum $\sum_{i=1}^{n} \sin\alpha_i \sin\beta_i \cos\gamma_i > 0$ (10)

Through four steps 1 , 2 , 3 , 4 we have finished determining the direction and magnitude of the resultant force **F** which is produced on surface dipoles of the extended electron .
Its direction is shown in two figures 32 & 33 for $\mu < 1$ and $\mu > 1$, respectively .
Its magnitude is given by two expressions (8) and (9) for two cases $\mu < 1$ and $\mu > 1$.

II. 2 The resultant magnetic force produced on interior dipoles is zero

Here we repeat the Section I.2 (on page 4) that the resultant magnetic force produced on all interior dipoles of the electron cancels out .
But since these magnetic forces exist and create couples of forces on all interior dipoles , they cause a re-orientation on-the-spot of these dipoles . The result is a change in the permeability μ of the electron which affects its effective electric charge in a variable magnetic field .

II. 3 Calculation of the magnetic force F' produced on the core ($-q_0$) of the electron

To calculate the magnetic force **F'** produced on the core ($-q_0$) we need to know the magnetic field **B'** created on the core O by the external field **B** . **B'** is determined in Appendix C : **B'** is parallel to **B** because of the spherical symmetry of the structure of the extended electron ; it has magnitude

$$\mathbf{B'} = \mu \mathbf{B} \qquad (11) \text{ or } (C.6)$$

This means that when the magnetic field **B** is applied on the electron , the magnetic field **B'** parallel and equal to μ **B** is produced at the core of the electron .
Therefore , the magnetic force **F'** produced on the core ($-q_0$) has magnitude

$$\boxed{F' = -\mu\, q_0\, V\, B} \qquad (\mathbf{V} \perp \mathbf{B}) \qquad (12)$$

F' is negative , i.e., **F'** points to the left of the observer as shown in Fig.35

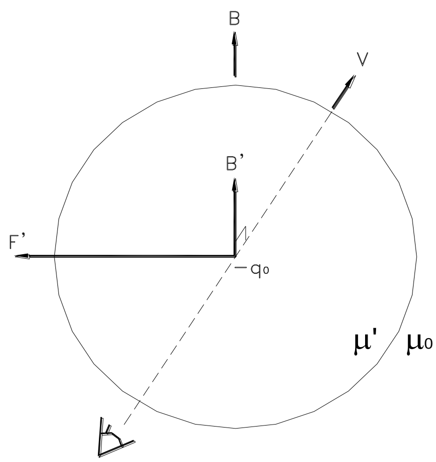


Fig.35 . **B** is the external magnetic field ;
B' is the magnetic field produced on the core .
F' is negative , i.e., **F'** points to the left of the observer .

II.4 Calculation of the net magnetic force Fm produced on the electron when V ⊥ B

So , the net magnetic force **Fm** produced on the extended electron when it moves normally to the magnetic field **B** is the sum **Fm = F + F'** .
While **F'** always points to the left of the observer for all values of μ (Fig.35) , **F** has two possible directions depending on μ :

- for $\mu < 1$ **F** points to the left of the observer : Fig.32
- for $\mu > 1$ **F** points to the right of the observer : Fig.33

Therefore , we come to two possible situations showed in two Figs. 36 & 37 below , of which we will choose one, based on two following experimental properties of the real electron :

1) Experiments of injecting electrons normally to the magnetic field have showed that the radius of curvature R of the trajectory of the electron increases with its velocity V and when $V \to c$, $R \to \infty$; that is , when the velocity of the electron approaches c , the electron traverses the magnetic field *without deflection* ; and this implies that **Fm** $\to 0$ as $V \to c$: this is *a required condition* for the magnetic force **Fm** .

Now let us examine Fig.36 for $\boldsymbol{\mu < 1}$: both **F** and **F'** are negative, they point to the left of the observer ; the net force **Fm** (= **F + F'**) is thus *always negative* ; i.e., in the interval $\mu < 1$ **Fm** cannot tend to zero ; the requirement **Fm** $\to 0$ is not satisfied ; so we reject the interval $\mu < 1$: i.e., the case shown in Fig.36 is rejected .
In Fig.37 for $\boldsymbol{\mu > 1}$: **F** > 0 and **F'** < 0 , they point in opposite directions ; so ,
Fm = **F + F'** = 0 when **F = - F'** ; the requirement **Fm** $\to 0$ can be satisfied , so we accept this case $\mu > 1$: i.e., the case shown in Fig.37 is acceptable .

Therefore $\boldsymbol{\mu > 1}$ so that Fm can tend to zero as $\boldsymbol{V \to c}$.

When $\mu > 1$, from Eqs.(9) and (12) we have

$$Fm \; = \; F + F' \; = \; (\mu - 1)\, q\, V\, B \sum_{i}^{n} \sin\alpha_i \; \sin\beta_i \; \cos\gamma_i \; - \; \mu\, q_0\, V\, B \qquad (13)$$

or
$$Fm = [\, (\mu - 1)\, (q / q_0) \sum_{i}^{n} \sin\alpha_i \; \sin\beta_i \; \cos\gamma_i \; - \; \mu \;]\; q_0\, V\, B \qquad (14)$$

In Eq.14 we set
$$\boxed{b \; \equiv \; (q / q_0) \sum_{i}^{n} \sin\alpha_i \; \sin\beta_i \; \cos\gamma_i} \qquad (15)$$

b is thus a dimensionless , positive number because the sum $\sum_{i}^{n} \sin\alpha_i \; \sin\beta_i \; \cos\gamma_i$ is positive according to Eq.(10) . Eq.(14) becomes

$$\boxed{Fm \; = \; [\, \mu\, (b-1) - b \,]\, q_0\, V\, B} \qquad (16)$$

So , the net magnetic force **Fm** developed on an extended electron when $\mathbf{V} \perp \mathbf{B}$ is modified by the factor $[\, \mu(b-1) - b \,]$ depending on two parameters μ and b which represent the *magnetic characteristic* and the *physical structure* of the extended electron respectively .The requirement $\mathbf{Fm} \to 0$ as $V \to c$ (for the electron to traverse the magnetic field **B** without deflection) occurs when the factor $[\, \mu(b-1) - b \,] \to 0$; i.e., when $\mu \to \dfrac{b}{b-1}$. Since $\mu > 1$, $\dfrac{b}{b-1} > 1$ gives **b > 1.**

2) <u>Experiments on the motion of the electron in **B**</u> (in the case $\mathbf{V} \perp \mathbf{B}$) showed that the electron deflects to the left of the observer (who stands in the direction of **B** , looking at the electron in the direction of velocity **V** as shown in Fig.38) . This means that the net magnetic force **Fm** is negative; and hence , from Eq.(16) we have

$$[\, \mu(b-1) - b \,] \; < \; 0 \qquad \text{or} \qquad \mu < \frac{b}{b-1}$$

So , in the Eq.(16) the relative permeability μ **is greater than 1 but less than** $\dfrac{b}{b-1}$.

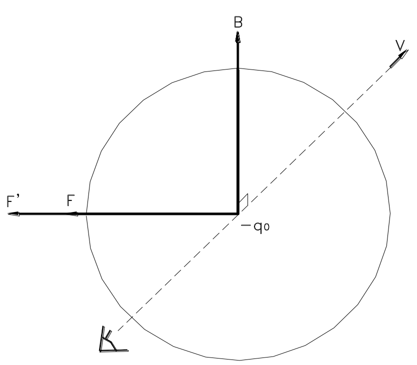


Fig.36 . $\mathbf{V} \perp \mathbf{B}$ **;** μ **< 1** : **F** and **F'** are both negative ; their resultant **Fm = F + F'** is thus always negative ; the condition $\mathbf{Fm} \to 0$ as $V \to c$ cannot be satisfied ; so , this case is rejected .

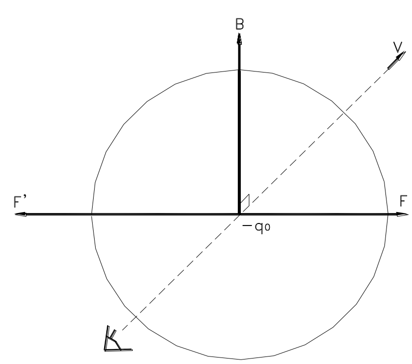


Fig.37 . $\mathbf{V} \perp \mathbf{B}$ **;** μ **> 1** : **F** is positive ; **F'** is negative; their resultant **Fm = F + F'** $\to$ 0 when $\mathbf{F} \to \mathbf{- F'}$ i.e., when $\mu = b / (b-1)$; this case is thus acceptable .

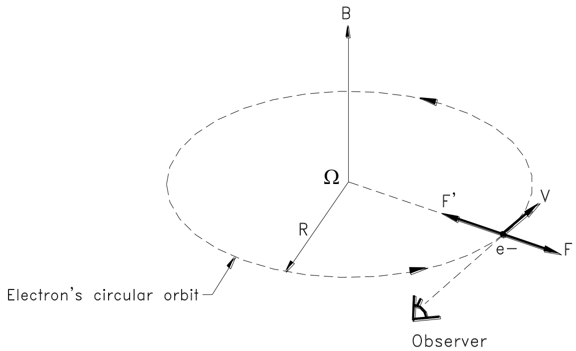


Fig.38 . Shows when $\mu > 1$ the circular orbit of the electron in constant **B** when $\mathbf{V} \perp \mathbf{B}$. **F** and **F'** point in opposite directions **:** **F** points to the right while **F'** to the left of the observer . The electron deflects to the left hand of the observer , this means that the net force **Fm (= F + F')** is negative .

In short , based on two experimental properties of the real electron which travels normally to the magnetic field **B** , namely :

- Fm → 0 as V → c : the electron traverses the magnetic field without deflection ,
- Fm < 0 when V < c : the electron deflects to the left hand side of the observer ,

we come to **the actual interval of variation of** $\boldsymbol{\mu}$ **:** $\mathbf{1 < \mu <} \frac{b}{b-1}$ where $b > 1$.

From (16) the effective electric charge Q of the electron can be deduced as

$$\boxed{Q = [\, \mu\,(b-1) - b\,]\, q_0} \qquad (17)$$

where $b > 1$, $1 < \mu < \frac{b}{b-1}$

For this interval of μ , Q varies in the interval $(-q_0 , 0)$.

If we insert b from the expression (15) into Eq.9 , we get a simpler expression for **F** :

$$\boxed{F = (\mu - 1)\, b q_0\, V B} \qquad (18)$$

If $\mu = 1$, $Q = -q_0$, $F = 0$ and $F' = -q_0 VB$ [from Eq.(12)] : this means that the electron has no surface dipoles , but only the core ($-q_0$) , i.e., it is a point electron .
So , when $\mu = 1$, the extended electron is equivalent (in term of charge and force) to the point electron .

We note that if b is a large number then $\frac{b}{b-1} \approx 1_+$, the interval of variation of μ of the extended electron becomes a narrow one : $1 < \mu < 1_+$; i.e., $\mu \approx 1_+$.
Macroscopic magnetic materials that have $\mu \approx 1_+$ are called *paramagnetic materials* [2] such as air (μ = 1. 00000037) , aluminum (1.000021) , tungsten (1.00008), platinum (1.0003) , manganese (1.001) ; these materials have nonzero permanent magnetic moment . The extended electron has its own intrinsic magnetic moment : its spin magnetic moment .

Conclusion

In this part II , we have come to the Eq.(16) : Fm = [μ (b-1) – b] q_0 V B which proves that the magnetic force **Fm** depends on **V** , **B** and also on two parameters μ and b . While b , representing the physical structure of the electron , may be considered as constant , the relative permeability μ changes with the strength of the elementary forces **fm** which eventually affects the effective electric charge of the electron .

From Eq.(16) we deduce the Eq.(17) :

$$Q = [\ \mu\ (b-1) - b\ ]\ q_0\ , \quad \text{where} \quad b > 1\ , \qquad 1 < \mu < \frac{b}{b-1}$$

which shows that the electric charge Q varies linearly with μ , the relative permeability of the extended electron . The variability of μ can be explained physically by the re-orientation of all electric dipoles inside the electron under the action of the couples of forces which are developed on two ends -q and +q of the dipole . The re-orientation of the dipoles (already mentioned in sections I.2 , page 4 and II.2 , page 11) causes a change in the *screening effect** and hence affects the electric charge Q of the electron :

- as $\mu \to 1$, $Q \to -q_0$: the screening effect is negligible (or zero): the electric charge of the extended electron tends to that of the core ; i.e., there is no screening effect at all ;
- as $\mu \to \frac{b}{b-1}$, $Q \to 0$: the screening effect becomes maximum : the electric charge of the extended electron tends to vanish .

We conclude that when the extended electron is subject to an external magnetic field , the field exerts couples of forces on all electric dipoles of the electron , causes them to be re-oriented and changes the screening effect on the core - q_0 , and thus affects the effective electric charge Q of the electron . The permeability μ is the measure of the strength of the screening effect .

***The screening of the electron** by the vacuum polarization is a concept of QED .
The image of the screened electron and its caption are showed on page 2 of the article [1(a)]
"A New Extended Model of the Electron " .
The extended model of the electron is a version of this image of the screened electron in which all virtual pairs (e^- , e^+) are replaced by the real electric dipoles (-q +q) which will be identified as photons when the electron radiates .

Part III. Radiation of the extended electron in constant magnetic field

III. 1 Introduction : radiation by forces

Radiation process is a tough topic to discuss because of the dual nature of light and the unknown structure of the electron that emits light . In this part III , the extended model of the electron[1(a)] will be used in the discussion of the radiation process . As for light , it is assumed that the electron emits light that may be **particles** (called photons , as conceived by **Feynman***) or it may be **a chunk of self-sustaining field**** which detaches from the electron and travels through space , as conceived by the classical theory of radiation .
In this article , we choose to consider **light as particles** which are identified as tiny dipoles carrying two opposite charges -q and +q on their two ends ; these **"electric dipoles"** form the outer part of the extended electron [1(a)] . When the electron emits its electric dipoles into the surrounding space under the action of an external field , we say it is radiating .
The following presentation will introduce the readers to *a novel way of explaining the radiation process* : *this is radiation by forces* .

*** Feynman** : " *I want to emphasize that light comes in this form – particles . It is very important to know that light behaves like particles , especially for those of you who have gone to school , where you were probably told something about light behaving like waves . I'm telling you the way it does behave – like particles* . " (Optics , E.Hecht , p.138)
****** " *If the point charge is subjected to a sudden acceleration caused by some external force , then pieces of the electric and magnetic fields break away from the point charge and propagate outward as a self-supporting electromagnetic wave pulse* . " (Classical Electrodynamics , 1988 , H. C. Ohanian , p. 411)

III. 2 Radiation of the extended electron moving normally to the magnetic field : cyclotron radiation

First let us recall the inherent forces **G** which attract all surfaces dipoles toward the core of the electron ; they are centripetal and produced by the self-field $\mathbf{E_0}$ of the electron ; Their magnitude is

$$G = [(1/\varepsilon) - 1]\, q\, E_0$$

which has been determined on page 8 (Fig.19) of the article [1(a)] **"A New Extended Model for the Electron ".**

The inherent forces G are crucial in the radiation process because they keep all surface dipoles attached to the electron until there exist external forces which counteract and overcome the forces G on certain dipoles ; these dipoles are then set free and emitted outwards ; i.e., the electron is radiating .

In the previous part (part II) , we have determined the magnitudes and directions of the magnetic forces **fm** which are produced on all surface dipoles of the electron as shown in Fig.29 in the case $\mu > 1$ (this is the case when the extended electron behaves like a real electron) .

Fig.29 shows that on *the right hemisphere* , **fm** point outwards , while G always point inwards : so , <u>the radiation can occur</u> if **fm > G** in magnitude .
On the contrary , on *the left hemisphere* , **fm** point inwards , i.e., in the same direction as the inherent forces **G** : <u>the radiation cannot occur</u> .

Therefore , the radiation of the electron moving normally to a constant **B** is characterized by *a beam of radiation which emits straight outwards in the direction of the right hand of the observer* . (The beam of radiation is in the same direction as the force **F** that points to the right hand of the observer as shown in Fig.38)
Moreover , the photons of the beam do not issue from the whole surface of the right hemisphere , but only from regions alongside the equator of the electron because **fm** are strongest there .
Fig.15 shows forces **fm** on the great circle C_1 (the equator) where **fm** are strongest ($\alpha = \pi/2$, $\sin\alpha = 1$) : $fm = (\mu - 1) q V B$ (Eq.(3) .
Since the strength of **fm** depends on the product V B and also on the angular position (α) of the surface dipole , if on the equator **fm < G** in magnitude , the electron cannot radiate .

This is the **cyclotron radiation** of the electron in constant magnetic field .
It is different from the **synchrotron radiation** which occurs in a time-varying magnetic field . *The beam (the cone) of synchrotron radiation does not emit straight outwards , (like the cyclotron radiation) but bends in the direction of spin of the electron* due to the spinning forces which are produced by the induced electric field (Synchrotron Radiation will be discussed in the next article) .

Summary & Conclusion

This article presents a theory on the extended model [1(a)] of the electron which shows that its electric charge is an effective one ; it is also intended to explain the radiation process of the extended electron by magnetic forces . This model is a version of the image of the screened electron in which the virtual pairs (e^- , e^+) are replaced by the real electric dipoles (-q +q) . This extended electron is a real particle consisting of a central core ($-q_0$) surrounded by countless electric dipoles . When subjected to an external field , the actions of the field on these point charges (-q ,+q , $-q_0$) create various properties of the electron such as the variability of its electric charge , its radiation and its spin .

In two parts I and II , we have determined the direction and strength of the magnetic forces **fm** produced on the dipoles of the electron when it moves parallel and normally to the constant magnetic field **B** . The determination of these forces led to the idea that

the electric charge of the electron is not constant but changes with the strength of the magnetic forces **fm** that exert on the dipoles of the electron .
In the conclusion of part I (page 5) a thought experiment was cited ; it is intended to prove the variability of the electric charge of the extended electron when it travels parallel to a magnetic field .

In part III , these forces **fm** help explain the radiation process of the electron in the magnetic field and in this way they help distinguish the cyclotron radiation from the synchrotron radiation when the electron moves normally to the magnetic field .

In the case when the <u>electron moves parallel to the magnetic field</u> , since the forces **fm** are <u>tangent to the spherical surface of the electron</u> (as shown in Figs.1, 2 & 6 , on pages 2 and 3 of part I), it is safe to think that <u>the radiation cannot occur</u> because as mentioned previously that the radiation occurs only when **fm** are in opposite direction and stronger than the cohesive forces G which are inherently centripetal .

As we will see in the coming articles , this extended model of the electron can provide a way to explore the mechanisms of radiation and spin of the electron in time-varying electric and magnetic fields .

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  (b) " **A Foundational Problem in Physics : Mass versus Electric Charge ",**
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2. **Sadiku M.N.O.** , Elements of Electromagnetics , 2nd Edition , p.361 , 784

Appendices

There are three appendices : A , B and C .
Two appendices A , B present the calculations of the forces **fm** and their resultant produced on the dipoles of the electron in two cases **V ∕∕ B** and **V ⊥ B** .
The appendix C is the determination of the magnetic field **B**' produced on the core ($-q_0$) of the electron by applying the boundary conditions .

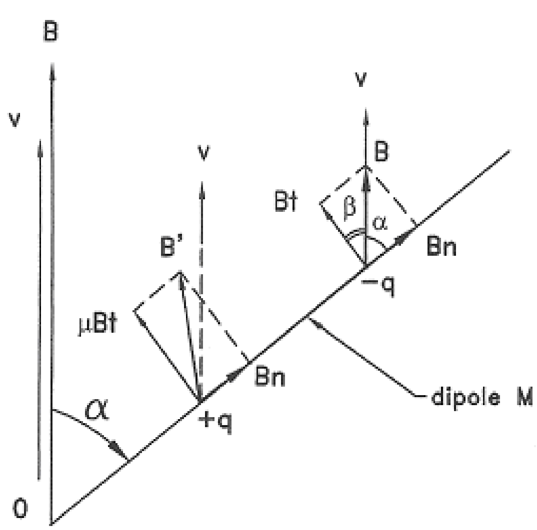
Appendix A : Calculation of fm when V ∕∕ B

Magnetic fields **B** and **B'** applied on two ends –q and +q of surface dipole M have components (Fig.3)

$$\mathbf{B} = \mathbf{Bn} + \mathbf{Bt}$$
$$\mathbf{B'} = \mathbf{B'n} + \mathbf{B't}$$

Boundary conditions give **B'n = Bn** and **B't = μBt**
where μ is the relative permeability of the electron to the free space ; i.e., $\mu = \mu'/\mu_0$.
Fig.3 shows normal and tangential components of **B** and **B'** at two ends of dipole M :
V //B , α = (**V, Bn**) , β = (**V , Bt**) , α + β = π/2 hence sinβ = cosα .
Two point charges -q and +q of dipole M have the same velocity **V** .
B'n = Bn = B cosα , Bt = B sinα , B't = μ Bt = μ B sinα .
Fig.4 shows components **fn , f'n , ft , f't** of magnetic forces acting on two ends –q and +q of dipole M ; their magnitudes and directions are

- fn = q V Bn sinα = q V B cosα sinα : **fn** (•) (i.e., **fn** points up from the page)
- f'n = fn = q V B cosα sinα : **f'n** ⊕ (i.e., **f'n** points down from the page)
- ft = q V Bt sinβ = q V B sinα cosα : **ft** ⊕ (**ft** points down)
- f't = μ ft = μ q V B sinα cosα : **f't** (•) (**f't** points up)



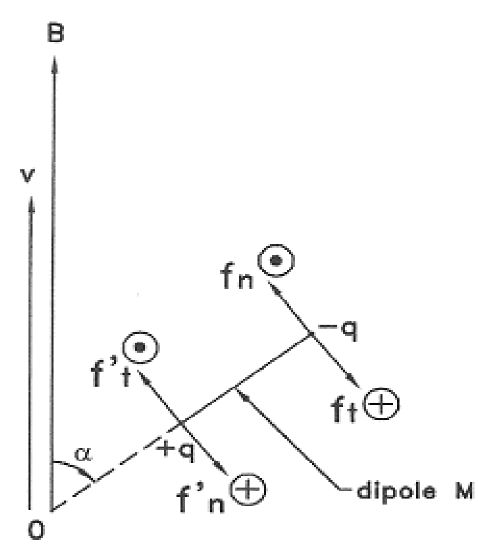


Fig.3 . Normal and tangential components of magnetic fields **B** and **B'** acting on two ends of dipole M .

Fig.4 . Components **fn , ft , f'n , f't** of magnetic forces acting on two ends of dipole M.
- **fn** and **f't** point up from the page (•)
- **ft** and **f'n** point down from the page ⊕

We notice that on the negative end (-q) of dipole M two components **fn (•)** and **ft ⊕** are equal and opposite **,** their resultant is thus equal to zero . This is because **V // B** at this end (Fig.3) . Meanwhile , at the positive end (+q) of dipole M , **V** and **B'** are not parallel, and hence **f'n ⊕** and **f't (•) ,** although in opposite direction , have different magnitude , their resultant is thus different from zero .
All four components **fn , f'n , ft , f't** are perpendicular to the plane (OB , M) ; i.e., perpendicular to the plane of the great circle C . Their resultant **fm** acting on dipole M is **fm = fn + f'n + ft + f't = f'n + f't** (because **fn + ft = 0**) .
(Since the dipole is extremely small the resultant **fm** of these four component forces can be regarded as applied at one point on the dipole M) .
When $\mu > 1$: f't > f'n , fm = f't - f'n = $(\mu - 1)\, q\, V\, B \sin\alpha \cos\alpha$: **fm (•)** (A.1)
where $0 \leq \alpha \leq \pi$ (Fig.5 , p. 3)

The same calculations on the remaining dipoles N , P , Q give the results :
- on dipole N : **fm ⊕** ,
- on dipole P **: fm (•) ,**
- on dipole Q : **fm ⊕** , (Figs.1 & 2 , p.2)

The magnitude of **fm** is the same on four dipoles M , N , P , Q :

$$fm = (\mu - 1)\, q\, V\, B \sin\alpha \cos\alpha \qquad \text{(A-1)}$$

where $\mu > 1$ and $0 \leq \alpha \leq \pi$ is the angular position of the surface dipole .
On the equator of the electron : $\alpha = \pi/2$, $\cos\alpha = 0$, so fm = 0 .
At two poles of the electron : $\alpha = 0$ and $\alpha = \pi$, $\sin\alpha = 0$, so fm = 0 .

When $\mu < 1$: f'n > f't , **fm** reverses its direction on all four dipoles M , N , P , Q , and its magnitude becomes

$$fm = f'n - f't = (1 - \mu)\, q\, V\, B \sin\alpha \cos\alpha \qquad \text{(A-2)}$$

Fig.6 (p. 3) shows four forces **fm** acting on four dipoles M , N , P , Q in the case **$\mu > 1$** ; they are tangent to the spherical surface of the electron .
On the upper hemisphere two forces **fm** at M and N form a couple of forces tending to rotate the electron *clockwise* . On the contrary, two forces **fm** at P and Q on the lower hemisphere form a couple of forces tending to rotate the electron *counterclockwise* .
These two couples of forces cancel out : they have no effect on the motion of the electron in **B** because the net force **Fm** = Σ **fm** = 0 and the net torque **T** = Στ = 0 .

This result can be generalized as follows : magnetic forces produced on all surface dipoles on the upper hemisphere counteract all magnetic forces on the lower hemisphere, and hence they have no effect on the motion of the electron in **B** .

In short , when an extended electron moves parallel (or anti-parallel) to **B ,** magnetic forces **fm** are developed on all surface dipoles but their net force **Fm** (= Σ **fm)** and net torque **T (**= Σ**τ)** cancel out, and thus they have no effect on the motion of the electron .

Appendix B : Determination of fm and the resultant F = ∑fm when V ⊥ B

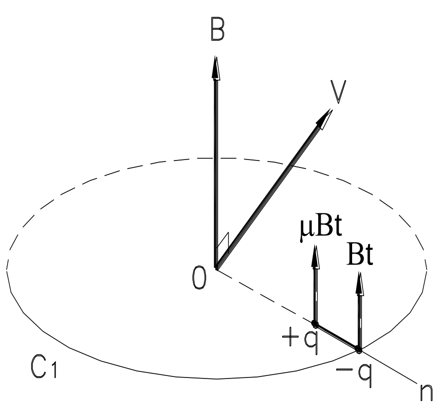
Step 1 : Magnetic forces fm produced on surface dipoles lying on the great circle C_1

C_1 is the equatorial circle of the electron ; it lies in the plane **(V , fm**) (Fig.7) (p.6)
Fig.10 shows an arbitrary surface dipole M lying on C_1 .
Boundary conditions for the magnetic field **B** on two ends of dipole M are

$$\mathbf{B} = \mathbf{Bn} + \mathbf{Bt} \quad \text{(on outside end } -q \text{ of the dipole)}$$
$$\mathbf{B'} = \mathbf{Bn} + \mu\,\mathbf{Bt} \quad \text{(on inside end } +q \text{ of the dipole)}$$

Since **B** is tangent to the spherical surface of the electron , **Bn** = 0 and **B = Bt**
B’ = μ **Bt** = μ **B ;** hence fn = f’n = 0
Fig.11 : on the end -q : since **V ⊥ B** , ft = q V Bt = q V B : **ft** points to the left of the observer ;
on the end +q : f’t = q V B’t = μ q VB : **f’t** points to the right of the observer .
Since **ft** and **f’t** are perpendicular to (**V** , **B**) , they are parallel . Their resultant is
- for μ < 1 : fm = ft - f’t = (1 - μ) q V B : **fm** points to the left (Fig.12) ,
- for μ > 1 : fm = f’t - ft = (μ - 1) q V B : **fm** points to the right (Fig.13) (B.1)

The same method of calculation gives the magnitude and direction of **fm** produced on all other dipoles lying on C_1 : they are parallel to one another because they are perpendicular to the plane (**V , B**) and have equal magnitude (Figs. 14 & 15 , p. 6) .



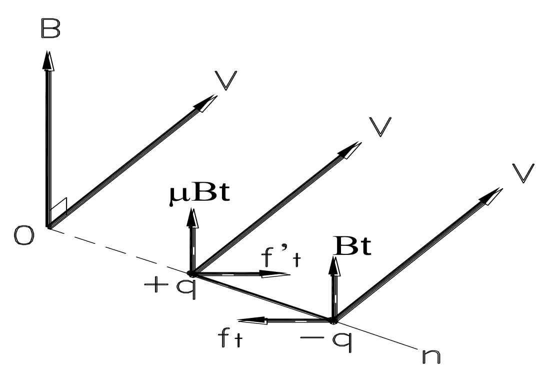


Fig.10 . Components **Bt** and μ**Bt** acting on two ends of an arbitrary dipole M on C_1 .

Fig.11 . Magnetic fields **Bt** and μ**Bt** produce magnetic forces **ft** and **f’t** on two ends of dipole M .

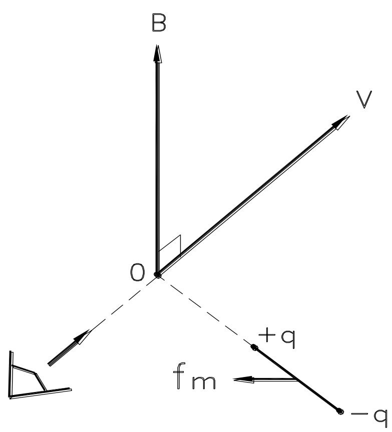


Fig.12 . For $\mu < 1$, $\mathbf{V} \perp \mathbf{B}$: the resultant force **fm** acting on dipole M points in the direction of the left hand of the observer .

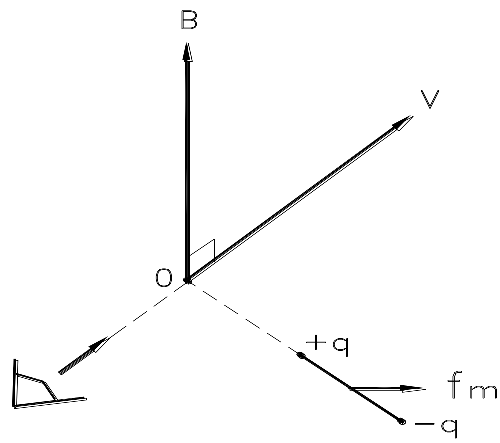


Fig.13 . For $\mu > 1$, $\mathbf{V} \perp \mathbf{B}$: the resultant force **fm** acting on dipole M points in the direction of the right hand of the observer .

Step 2 : Magnetic forces fm produced on surface dipoles lying on the great circle C_2

The great circle C_2 lies the plane (**V , B**) : Fig. 8 (p.6)
Using boundary conditions and the same method of calculation as above we come to the following results :
Fig.16: Components of **B** and **B’** act on two ends of dipole N on the great circle C_2 ,
Fig.17: Directions of component forces **fn , f’n , ft , f’t** , acting on two ends of dipole N :

- **fn** and **ft** point down from the page ⊕
- **f’n** and **f’t** point up from the page (•)

Fig.18 : For $\mu < 1$: the resultant force **fm** acting on dipole N points to the left of the observer ; i.e., **fm** ⊕
Fig.19 : For $\mu > 1$: the resultant force **fm** acting on dipole N points to the right of the observer ; i.e., **fm** (•)
Fig.20 (p. 7) : For $\mu < 1$: all magnetic forces **fm** produced on the great circle C_2 point to the left of the observer and have magnitude

$$fm = ft - f't = (1 - \mu)\, q V B \sin^2 \alpha$$

Fig.21 (p. 7): For $\mu > 1$: all magnetic forces **fm** produced on the great circle C_2 point to the right of the observer and have magnitude

$$fm = f't - ft = (\mu - 1)\, q\, V\, B \sin^2 \alpha \tag{B.2}$$

All magnetic forces **fm** produced on C_2 are parallel because they are perpendicular to the plane (**V , B**) , but their magnitudes depend on the angle α ($0 \le \alpha \le \pi$) .

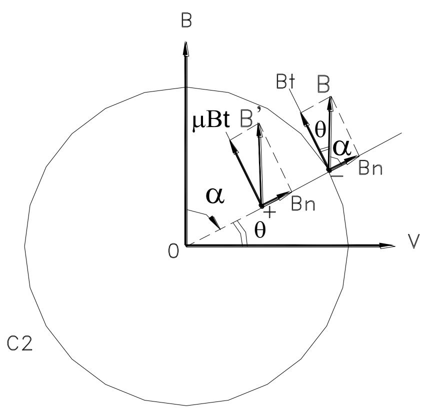


Fig.16 . Components of magnetic fields **B** and **B'** acting on two ends of dipole N on the great circle C_2

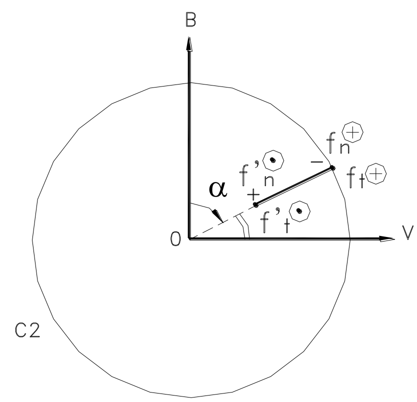


Fig.17 . Directions of component forces **fn , f'n , ft , f't** acting on two ends of dipole N :

- **fn** and **ft** point down from the page ⊕
- **f'n** and **f't** point up from the page (•)

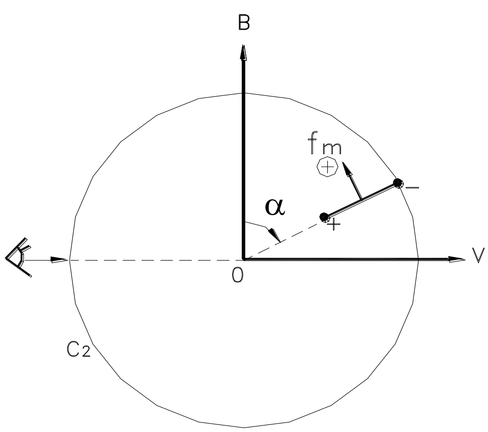


Fig.18 . For $\mu < 1$: the resultant force **fm** acting on dipole N on C_2 points to the left of the observer : **fm** ⊕

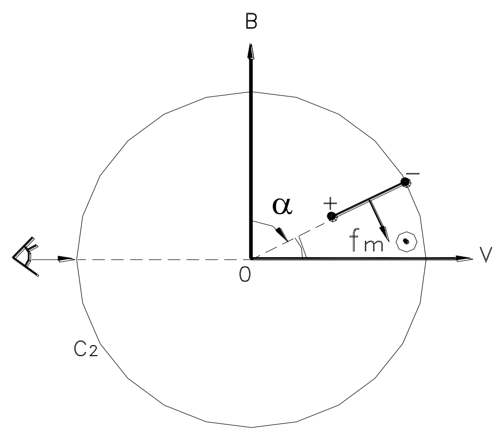


Fig.19 . For $\mu > 1$: the resultant force **fm** points to the right of the observer : **fm (•)**

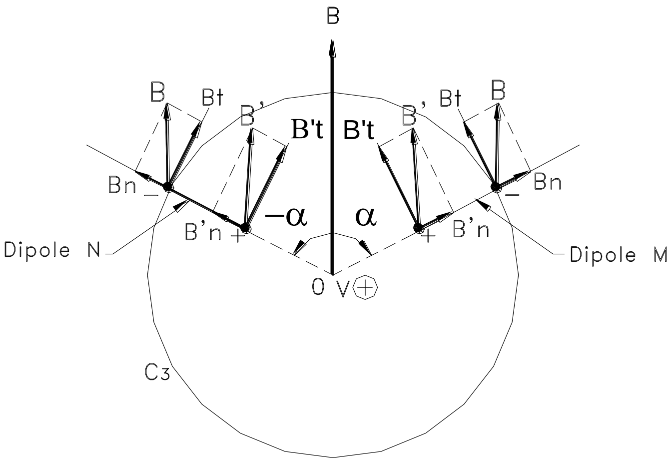


Fig.22 . **V** ⊕ ; **V** ⊥ **B** : components of magnetic fields **B** and **B'** acting on two ends of surface dipoles M and N on C_3 .

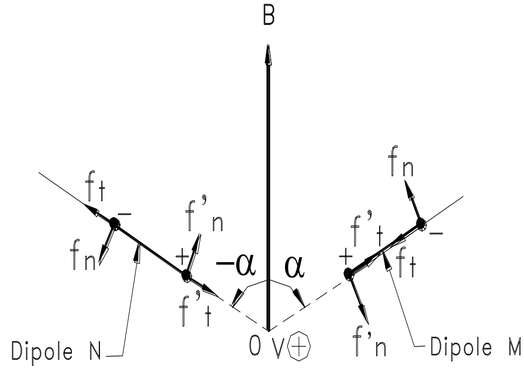


Fig.23 . Components of magnetic forces **fn , ft , f'n , f't** acting on two ends of surface dipoles M and N on C_3 .

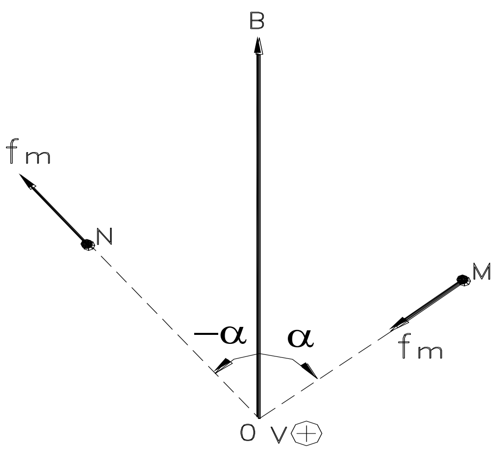


Fig.24 . For $\mu < 1$: the resultant force **fm** produced on dipole M is centripetal ; while **fm** acting on dipole N is centrifugal .

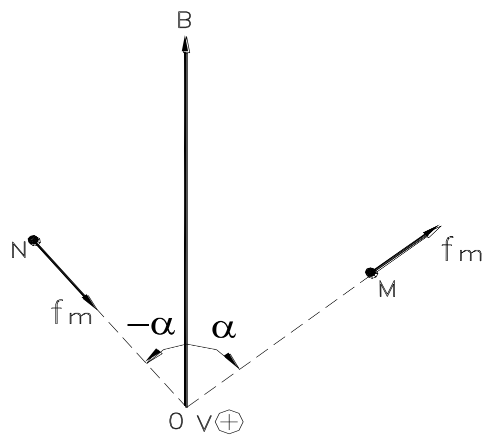


Fig.25 . For $\mu > 1$: the resultant force **fm** produced on dipole M is centrifugal ; while **fm** acting on dipole N is centripetal .

Step 3 : Magnetic forces fm produced on surface dipoles lying on the great circle C_3

The great circle C_3 lies in the plane (**B , fm**) : Fig.9 (p.6)

Using boundary conditions and the same method of calculation as above we come to the following results :

Fig.22 : Components of **B** and **B'** acting on two ends of two arbitrary dipoles M and N

on the great circle C_3 .

Fig.23 : Components forces **fn , f'n , ft , f't** acting on dipoles M and N on C_3 .

Fig.24 : For $\mu < 1$: the resultant **fm** acting on dipole M is centripetal ; meanwhile **fm** acting on dipole N is centrifugal .

Fig.25 : For $\mu > 1$: the resultant **fm** acting on dipole M is centrifugal ; meanwhile **fm** acting on dipole N is centripetal .

Fig.26 : For $\mu < 1$: all magnetic forces **fm** produced on C_3 point to the left of the observer . Magnitudes of **fm** are $fm = ft - f't = (1 - \mu)\, q\, V\, B \sin\alpha$

Fig.27 : For $\mu > 1$: all magnetic forces **fm** produced on C_3 point to the right of the observer . Magnitudes of **fm** are $fm = f't - ft = (\mu - 1)\, q\, V\, B \sin\alpha$ (B.3)

Magnetic forces **fm** produced on C3 are not parallel to each other ; their magnitudes depend on the angle α .

We have determined the directions and magnitudes of magnetic forces **fm** produced on three great circles C_1 , C_2 and C_3 .
Figs.28 & 29 (p.9) show an *overall view* of these forces **fm** on three great circles in two cases $\mu < 1$ and $\mu > 1$ respectively . These overall views allow us to confirm that the magnetic force **fm** produced on an *arbitrary surface dipole* A points to

- the left of the observer when $\mu < 1$ as shown in Fig.30 (p.9)
- the right of the observer when $\mu > 1$ as shown in Fig.31 (p.9)

Step 4 : Calculation of magnitude of fm produced on an arbitrary surface dipole and the resultant F = $\sum$fm

This result gives the direction of the projection **fm*** of **fm** onto the force-axis which is perpendicular to (**V , B) :**

- when $\mu < 1$: **fm*** points to the left of the observer (Fig.30) .
  Since **fm** are symmetric around the force-axis , their resultant
  F = $\sum$fm = $\sum$fm* points to the left as shown in Fig.32 (p.10)
  F is thus considered as a negative force .
- when $\mu > 1$: **fm*** points to the right of the observer (Fig.31).
  Since **fm** are symmetric around the force-axis , their resultant
  F = $\sum$fm = $\sum$fm* points to the right as shown in Fig.33 (p.10)
  F is thus considered as a positive force .

The magnitudes of **fm** , **fm* and F (= $\sum$fm = $\sum$fm*)** are calculated below

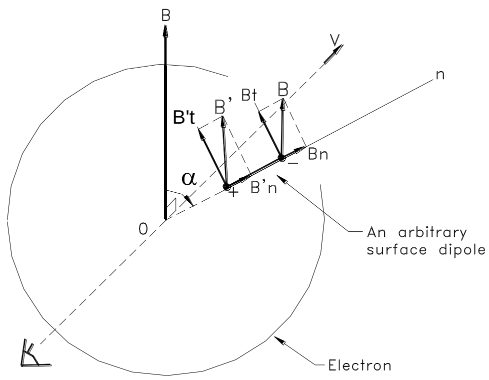


Fig.34 shows an arbitrary surface dipole with the external magnetic field **B** acting on the outside end -q , and **B'** acting on the inside end +q .

Let's consider an arbitrary surface dipole A (Fig.34) ; **n** is the normal at A ; **t** is the tangent at A lying in the plane (**OB , n**) ; $\alpha = (\mathbf{B}, \mathbf{n})$; $\beta = (\mathbf{V}, \mathbf{Bt}) = (\mathbf{V}, \mathbf{t})$; $\mathbf{V} \perp \mathbf{B}$.

Boundary conditions for the magnetic field **B** applied on dipole A are

B = Bn + Bt (on the outer end -q)

B' = Bn + μ **Bt** (on the inner end +q)

where $Bn = B \cos\alpha$ and $Bt = B \sin\alpha$.

Since **Bn = B'n** , **fn** and **f'n** are equal and opposite , hence **fn + f'n** = 0 ;

$ft = q\,V\,Bt \sin\beta = q\,V\,B \sin\alpha \sin\beta$; $f't = q\,V\,B't \sin\beta = \mu\,q\,V\,B \sin\alpha \sin\beta$.

The resultant **fm** produced on dipole A is **fm = fn + f'n + ft + f't = ft + f't**

The magnitude of **fm** is :

- for $\mu < 1$: $fm = ft - f't = (1 - \mu)\,q\,V\,B \sin\alpha \sin\beta$ (B.4)
- for $\mu > 1$: $fm = f't - ft = (\mu - 1)\,q\,V\,B \sin\alpha \sin\beta$ (B.5)

The direction of **fm** is perpendicular to the plane (**V , t**) .

Note : Two expressions (B.4) and (B.5) give magnitude of **fm** produced on *an arbitrary surface dipole* ; so we can use them to check the magnitudes of **fm** on three great circles C_1 , C_2 and C_3 that we have calculated in three sections II .1 , II .2 and II .3 .

For dipoles on C_1 : $\alpha = \pi/2$ and $\beta = (\mathbf{V}, \mathbf{Bt}) = (\mathbf{V}, \mathbf{B}) = \pi/2$; hence $\sin\alpha = \sin\beta = 1$.

Eq.(B.5) for $\mu > 1$ becomes $fm = f't - ft = (\mu - 1)\,q\,V\,B$: this is Eq. (B.1)

For dipoles on C_2 : $\beta = (\mathbf{V}, \mathbf{Bt}) = (\mathbf{V}, \mathbf{B}) + (\mathbf{B}, \mathbf{Bt}) = \pi/2 + (\pi/2 - \alpha) = \pi - \alpha$;
hence $\sin\beta = \sin(\pi - \alpha) = \sin\alpha$.

Eq.(B.5) for $\mu > 1$ becomes $fm = f't - ft = (\mu - 1)\,q\,V\,B \sin^2\alpha$: this is Eq. (B.2)

For dipoles on C_3 : $\beta = (\mathbf{V}, \mathbf{Bt}) = \pi/2$; and hence $\sin\beta = 1$.

Eq.(B.5) for $\mu > 1$ becomes $fm = f't - ft = (\mu - 1)\,q\,V\,B \sin\alpha$: this is Eq. (B.3)

Let γ be the angle between **fm** and the force-axis , we have $fm^* = fm \cos\gamma$;
and hence $F = \sum fm^* = \sum fm \cos\gamma$; where fm are given by (B.4) and (B.5)
- for $\mu < 1$: $F = \sum (1 - \mu)\, q\, V\, B \sin\alpha \sin\beta \cos\gamma$
- for $\mu > 1$: $F = \sum (\mu - 1)\, q\, V\, B \sin\alpha \sin\beta \cos\gamma$

Generally, **for $\mu < 1$** : $F = (1 - \mu)\, q\, V\, B \sum_{i=1}^{n} \sin\alpha_i \sin\beta_i \cos\gamma_i$ (B.6)

The index i indicates the surface dipole i ; n is the total number of surface dipoles .
F points to the left of the observer as shown in Fig.32 and is considered as a negative force ; and hence the sum $\sum_{i=1}^{n} \sin\alpha_i \sin\beta_i \cos\gamma_i < 0$

For $\mu > 1$: $\boxed{F = (\mu - 1)\, q\, V\, B \sum_{i=1}^{n} \sin\alpha_i \sin\beta_i \cos\gamma_i}$ (B.7)

F points to the right of the observer as shown in Fig.33 and is considered as a positive force ; and hence the sum $\sum_{i=1}^{n} \sin\alpha_i \sin\beta_i \cos\gamma_i > 0$ (B.8)

Appendix C : Determination of the magnetic field B' at the core ($-q_0$)

To determine the magnitude of **B'** at the core , we apply boundary conditions to a point A on the surface of the electron and the core O . (Since the electron is too small , we can consider A and O as lying on two sides of the interface which is the spherical surface of the electron) .

$$\mathbf{B} = \mathbf{Bn} + \mathbf{Bt} \qquad \text{(at A)} \qquad (C.1)$$

$$\mathbf{B'} = \mathbf{Bn} + \mu\, \mathbf{Bt} \qquad \text{(at O)} \qquad (C.2)$$

Because of the spherical symmetry of the structure of the electron , **B'** must be parallel to **B** **,** and hence we can write :

$$\mathbf{B'} = k\, \mathbf{B} \qquad (C.3)$$

where k is a positive number different from 1 : $0 < k \neq 1$.
k is positive because **B'** is parallel to **B** ; $k \neq 1$ because if $k = 1$, **B' = B** and this means **B'** is independent of the medium of the electron . But because **B'** is expected to depend on the medium of the electron , so , k must be different from 1 .
Eq.(C.3) can be rewritten as $\mathbf{B'} = k\,\mathbf{B} = k\mathbf{Bn} + k\,\mathbf{Bt}$ (C.4)

Comparing (C.2) and (C.4) we get

- $\mathbf{Bn} = k\,\mathbf{Bn} \rightarrow (k-1)\,\mathbf{Bn} = 0 \rightarrow \mathbf{Bn} = 0$ since $(k-1) \neq 0$
- $\mu\,\mathbf{Bt} = k\,\mathbf{Bt} \rightarrow \mu = k \neq 1$.

So , (C.1) gives : $\mathbf{B} = \mathbf{Bt}$ (at A) (C.5)

and (C.2) gives $\boxed{\mathbf{B'} = \mu\,\mathbf{Bt} = \mu\,\mathbf{B}}$ (at O) (C.6)

We conclude that when the electron is subject to the magnetic field B , the magnetic field **B'** parallel and equal to $\mu\,\mathbf{B}$ is produced at the core of the electron . And hence the magnetic force produced on the core is zero when the electron moves <u>parallel</u> to **B** and equal to $-\mu q_0 VB$ (Eq.(12) when it moves <u>normally</u> to **B** .
It is noteworthy that since $\mu > 1$, $\mathbf{B'} > \mathbf{B}$.

Note : From these results ($\mathbf{Bn} = 0$ and $\mathbf{B} = \mathbf{Bt}$) , the position of the point A is <u>not arbitrary</u> on the surface of the extended electron , it must lie on the equator of the electron because $Bn = B\cos\alpha = 0$ gives $\cos\alpha = 0$ or $\alpha = \pi/2$.
If A takes an <u>arbitrary position</u> on the surface of the extended electron , we cannot solve the equations given by the boundary conditions for the magnetic field **B'** at the core .