Reply to "Comments on 'There is No Axiomatic System for the Quantum Theory"'

Koji Nagata[1] and Tadao Nakamura[2]

[1] 1-17-107 West 19 South 3, Obihiro, Hokkaido 080-2469, Japan
[2] Department of Information and Computer Science, Keio University,
3-14-1 Hiyoshi, Kohoku-ku, Yokohama 223-8522, Japan

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Jose Acacio de Barros discusses as follows [Jose Acacio de Barros, Int. J. Theor. Phys. **50**, 1828 (2011)]. Nagata claims to derive inconsistencies from quantum mechanics [K. Nagata, Int. J. Theor. Phys. **48**, 3532 (2009)]. Jose Acacio de Barros considers that the inconsistencies do not come from quantum mechanics, but from extra assumptions about the reality of observables. Here we discuss the fact that there is a contradiction within the quantum theory. We discuss the fact that only one expected value in a spin-1/2 pure state $\langle\sigma_x\rangle$ rules out the reality of the observable. We do not accept extra assumptions about the reality of observables. We use the actually measured results of quantum measurements (raw data). We use a single Pauli observable. We stress that we can use the quantum theory even if we give up the axiomatic system for the quantum theory.

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I. INTRODUCTION

Jose Acacio de Barros discusses as follows [1]. Nagata claims to derive inconsistencies from quantum mechanics [2]. Jose Acacio de Barros considers that the inconsistencies do not come from quantum mechanics, but from extra assumptions about the reality of observables. More clearly, since quantum mechanics forbids the simultaneous measurements of non-commuting observables, as they do not commute, it does not allow us to simultaneously assign values to them. The contradiction does not come from quantum mechanics, but from the assumption that we can assign values to measurements that were not performed.

Here we discuss the fact that there is a contradiction within the quantum theory. We discuss the fact that only one expected value of a spin-1/2 pure state $\langle\sigma_x\rangle$ rules out the reality of the observable. We do not accept extra assumptions about the reality of observables. We use the actually measured results of quantum measurements (raw data). We use a single Pauli observable. We stress that we can use the quantum theory even if we give up the axiomatic system for the quantum theory.

II. THERE IS A CONTRADICTION WITHIN THE QUANTUM THEORY BY USING JOINT PROBABILITY

First we discuss an easy contradiction within the quantum theory as follows [3].

Matrix theory is not compatible with probability theory. Matrix theory has axioms. Probability theory has axioms. We consider joint set of such axioms. Does such joint set work as new set of axioms for matrix theory and probability theory?

Let us consider joint probability. A is an observable. B is an observable. a, b are actually measured results of quantum measurements in a quantum state, respectively. A and B are not commutative. Thus,

$$[A, B] \neq O. \tag{1}$$

We consider as follows: First we measure observable A and get a as the actually measured result. And next we measure observable B and get b as the actually measured result. This joint event is different if we exchange A to B, in general. Hence

$$P(\overbrace{A=a}^{\text{first}}\cap\overbrace{B=b}^{\text{second}}) \neq P(\overbrace{B=b}^{\text{first}}\cap\overbrace{A=a}^{\text{second}}). \tag{2}$$

On the other hand, the joint probability is depictured in terms of conditional probabilities:

$$\begin{aligned} P(A=a|B=b)P(B=b) &= P(A=a\cap B=b), \\ P(B=b|A=a)P(A=a) &= P(B=b\cap A=a). \end{aligned} \tag{3}$$

From axioms of probability theory, we have

$$P(A=a\cap B=b) = P(B=b\cap A=a). \tag{4}$$

We cannot assign truth value "1" for the proposition (2) and for the proposition (4), simultaneously. We are in a contradiction. It turns out that the joint set of axioms does not work as new set of axioms for matrix theory and probability theory. There is a contradiction within the quantum theory.

The first point is actually that, conventional Quantum Mechanics discussions typically do not employ conditional probabilities correctly if at all. This is the central issue with Bell's analysis leading to the idea that Quantum Mechanics requires non-locality or irreality and wave packet collapse and what not!

III. DOES PAULI OBSERVABLE IN A QUANTUM STATE HAVE A COUNTERPART IN PHYSICAL REALITY?

Recently, it is shown that the two expected values of a spin-1/2 pure state $\langle\sigma_x\rangle$ and $\langle\sigma_y\rangle$ rule out the existence of probability space of von Neumann's projective measurement [2, 4].

In this section, we discuss the fact that only one expected value of a spin-1/2 pure state $\langle\sigma_x\rangle$ rules out the existence of probability space of von Neumann's projective measurement.

We try to implement double-slit experiment. There is a detector just after each slit. Interference figure does not appear, and we do not consider such a pattern. Let (σ_z, σ_x) be the Pauli vector. We assume that a source of spin-carrying particles emits them in a state $|+_z\rangle$, which can be described as an eigenvector of Pauli observable σ_z.

We consider a quantum expected value $\langle\sigma_x\rangle$ as

$$\langle\sigma_x\rangle = \langle +_z|\sigma_x|+_z\rangle = 0. \tag{5}$$

We introduce a hidden-variables theory for the quantum expected value of the Pauli observable σ_x. Then, the quantum expected value given in (5) can be

$$\langle\sigma_x\rangle = \int d\lambda\rho(\lambda)f(\lambda). \tag{6}$$

where λ is some local hidden variable, $\rho(\lambda)$ is a probabilistic distribution, and $f(\lambda)$ is the predetermined "hidden" result of the measurement of the dichotomic observable σ_x. The possible values of $f(\lambda)$ are ± 1 (in $\hbar/2$ unit). If a particle passes one side slit, then the value of the result of measurement is $+1$. If a particle passes through another slit, then the value of the result of measurement is -1. It is von Neumann's projective measurement for σ_x

In what follows, we discuss the fact that we cannot assign the truth value "1" for the proposition (6). Assume the proposition (6) is true. We have the same proposition

$$\langle\sigma_x\rangle = \int d\lambda'\rho(\lambda')f(\lambda'). \tag{7}$$

An important note here is that the value of the right-hand-side of (6) is equal to the value of the right-hand-side of (7) because we only change a label.

We derive a necessary condition for the quantum expected value given in (6). We derive the possible value of the product $\langle\sigma_x\rangle^2\delta(\lambda-\lambda')$ of the quantum expected value and a delta function. The quantum expected value is $\langle\sigma_x\rangle$ given in (6). We have

$$\begin{aligned}
&\langle\sigma_x\rangle^2\delta(\lambda-\lambda') \\
&= \int d\lambda\rho(\lambda)f(\lambda) \times \int d\lambda'\rho(\lambda')f(\lambda')\delta(\lambda-\lambda') \\
&= \int d\lambda\rho(\lambda)\int d\lambda'\rho(\lambda')f(\lambda)f(\lambda')\delta(\lambda-\lambda') \\
&= \int d\lambda\rho(\lambda)(f(\lambda))^2 \\
&= \int d\lambda\rho(\lambda) = 1.
\end{aligned} \tag{8}$$

Here we use the fact

$$(f(\lambda))^2 = 1 \tag{9}$$

since the possible values of $f(\lambda)$ are ± 1. Hence we derive the following proposition if we assign the truth value "1" for a hidden-variables theory for the Pauli observable σ_x

$$\langle\sigma_x\rangle^2\delta(\lambda-\lambda') = 1. \tag{10}$$

We derive a necessary condition for the quantum expected value for the system in a pure spin-1/2 state $|+_z\rangle$ given in (5). We derive the possible value of the product

$$\langle\sigma_x\rangle \times \langle\sigma_x\rangle \times \delta(\lambda-\lambda') = \langle\sigma_x\rangle^2\delta(\lambda-\lambda'). \tag{11}$$

$\delta(\lambda-\lambda')$ is the delta function. $\langle\sigma_x\rangle$ is the quantum expected value given in (5). We have the following proposition since $\langle\sigma_x\rangle = 0$

$$\langle\sigma_x\rangle^2\delta(\lambda-\lambda') = 0. \tag{12}$$

We do not assign the truth value "1" for two propositions (10) and (12), simultaneously. We are in a contradiction. We have to give up a hidden-variables theory for the expected value of the Pauli observable σ_x. The measured observable σ_x in the state does not have a counterpart in physical reality.

IV. THERE IS A CONTRADICTION WITHIN THE QUANTUM THEORY BY USING A SINGLE PAULI OBSERVABLE

Next we discuss the fact that there is a contradiction within the quantum theory by using a single Pauli observable [5]. In this case, there is no need to argue that observables under consideration are commuting or noncommuting. Especially, we systematically describe our assertion based on more mathematical analysis using raw data (the actually measured results of quantum measurements). In this case, there is no need to argue the reality of observables. There exists raw data because we have seen it.

We consider the relation between double-slit experiment and projective measurement. We try to implement double-slit experiment. There is a detector just after each slit. Thus interference figure does not appear, and we do not consider such a pattern. The actually measured results of quantum measurements are ± 1 (in $\hbar/2$ unit). If a particle passes one side slit, then the value of the actually measured result of measurement is $+1$. If a particle passes through another slit, then the value of the actually measured result of measurement is -1.

A. A wave function analysis

Let (σ_z, σ_x) be the Pauli vector. We assume that a source of spin-carrying particles emits them in a state $|\psi\rangle$, which can be described as an eigenvector of Pauli observable σ_z. We consider a quantum expected value $\langle \sigma_x \rangle$ as

$$\langle \sigma_x \rangle = \langle \psi | \sigma_x | \psi \rangle = 0. \tag{13}$$

The above quantum expected value is zero if we consider only a wave function analysis.

We derive a necessary condition for the quantum expected value for the system in the pure spin-1/2 state $|\psi\rangle$ given in (13). We derive the possible value of the product $\langle \sigma_x \rangle \times \langle \sigma_x \rangle = \langle \sigma_x \rangle^2$. $\langle \sigma_x \rangle$ is the quantum expected value given in (13). We derive the following proposition

$$\langle \sigma_x \rangle^2 = 0. \tag{14}$$

B. Projective measurement

On the other hand, a mean value E admits projective measurement if it can be written as

$$E = \frac{\sum_{l=1}^{m} r_l(\sigma_x)}{m} \tag{15}$$

where l denotes a label and r is the actually measured result of projective measurement of the Pauli observable σ_x. We assume the actually measured value of r is ± 1 (in $\hbar/2$ unit).

Assume the quantum mean value with the system in an eigenvector ($|\psi\rangle$) of the Pauli observable σ_z admits projective measurement. One has the following proposition concerning projective measurement

$$\langle \sigma_x \rangle(m) = \frac{\sum_{l=1}^{m} r_l(\sigma_x)}{m}. \tag{16}$$

We can assume as follows by Strong Law of Large Numbers [6],

$$\langle \sigma_x \rangle(+\infty) = \langle \sigma_x \rangle = \langle \psi | \sigma_x | \psi \rangle. \tag{17}$$

In what follows, we show that we cannot assign the truth value "1" for the proposition (16) concerning projective measurement.

Assume the proposition (16) is true. By changing a label l into l', we have the same quantum mean value as follows

$$\langle \sigma_x \rangle(m) = \frac{\sum_{l'=1}^{m} r_{l'}(\sigma_x)}{m}. \tag{18}$$

An important note here is that the actually measured value of the right-hand-side of (16) is equal to the actually measured value of the right-hand-side of (18) because we only change the label. We have

$$\begin{aligned}
&\langle \sigma_x \rangle(m) \times \langle \sigma_x \rangle(m) \\
&= \frac{\sum_{l=1}^{m} r_l(\sigma_x)}{m} \times \frac{\sum_{l'=1}^{m} r_{l'}(\sigma_x)}{m} \\
&= \frac{\sum_{l=1}^{m} r_l(\sigma_x)}{\sum_{l=1}^{m}} \times \frac{\sum_{l'=1}^{m} r_{l'}(\sigma_x)}{\sum_{l'=1}^{m}} \times \frac{\delta_{ll'}}{\delta_{ll'}} \\
&= \frac{\sum_{l=1}^{m}}{m} \cdot (r_l(\sigma_x))^2 = \frac{\sum_{l=1}^{m}}{m} = 1.
\end{aligned} \tag{19}$$

Here $\delta_{ll'}$ is a delta function. We use the following fact

$$(r_l(\sigma_x))^2 = 1 \tag{20}$$

and

$$\frac{\delta_{ll'}}{\delta_{ll'}} = 1. \tag{21}$$

Thus we derive a proposition concerning the quantum mean value under an assumption that projective measurement is true (in a spin-1/2 system), that is

$$\langle \sigma_x \rangle(m) \times \langle \sigma_x \rangle(m) = 1. \tag{22}$$

From Strong Law of Large Numbers, we have

$$\langle \sigma_x \rangle \times \langle \sigma_x \rangle = 1. \tag{23}$$

Hence we derive the following proposition concerning projective measurement

$$\langle \sigma_x \rangle^2 = 1. \tag{24}$$

We do not assign the truth value "1" for two propositions (14) (concerning a wave function analysis) and (24) (concerning projective measurement), simultaneously. We are in a contradiction.

We cannot accept the validity of the proposition (16) (concerning projective measurement) if we assign the truth value "1" for the proposition (14) (concerning a wave function analysis). In other words, such projective measurement does not meet the detector model for spin observable σ_x. There is the contradiction within the quantum theory.

We note here that there is much nonsense in the Physics literature regarding the theoretical formality for Spin. The formalism is correct so long as only one dimension is under consideration—a restriction that is fully acceptable in view of the fact that to engage spin empirically a Magnetic (B) field is required and it can have only one direction at the point of interacting with a charge. All formal talk of the spin of a particle in both the Z and X or Y directions at the same instant is vacuous for lack of B-fields in two directions at once.

V. CONCLUSION

In conclusions, Jose Acacio de Barros has discussed as follows. Nagata has claimed to derive inconsistencies from quantum mechanics. Jose Acacio de Barros has considered that the inconsistencies do not come from quantum mechanics, but from extra assumptions about the reality of observables. Here we have discussed the fact that there is a contradiction within the quantum theory. We have discussed the fact that only one expected value of a spin-1/2 pure state $\langle \sigma_x \rangle$ rules out the reality of the observable. We do not have accepted extra assumptions about the reality of observables. We have used the actually measured results of quantum measurements (raw data). We have used a single Pauli observable. We have stressed that we can use the quantum theory even if we give up the axiomatic system for the quantum theory.

[1] Jose Acacio de Barros, Int. J. Theor. Phys. **50**, 1828 (2011).

[2] K. Nagata, Int. J. Theor. Phys. **48**, 3532 (2009).

[3] K. Nagata and T. Nakamura, Quantum Inf. Process. **12**, 3785 (2013).

[4] K. Nagata and T. Nakamura, Int. J. Theor. Phys. **49**, 162 (2010).

[5] K. Nagata and T. Nakamura, Precision Instrument and Mechanology, **2**, 104 (2013).

[6] In probability theory, the law of large numbers is a theorem that describes the result of performing the same experiment a large number of times. According to the law, the average of the results obtained from a large number of trials should be close to the expected value, and will tend to become closer as more trials are performed. The strong law of large numbers states that the sample average converges almost surely to the expected value.