The relation of colour charge to electric charge

Dirac has shown how the Klein-Gordon equation can be factored into two linear parts using 4x4 Dirac gamma matrices.

[Dirac, P.A.M., The Principles of Quantum Mechanics, 4th edition (Oxford University Press) ISBN 0-19-852011-5]

$$(\partial_t^2 - \partial_x^2 - \partial_y^2 - \partial_z^2 + m^2)\, I = (-i[s\gamma^0\partial_t + r\gamma^1\partial_x + g\gamma^2\partial_y + b\gamma^3\partial_z] - mI)\,(i[s\gamma^0\partial_t + r\gamma^1\partial_x + g\gamma^2\partial_y + b\gamma^3\partial_z] - mI)$$

where r,g,b and s equal +1 or -1.

For leptons r,g,b all equal -1 and for quarks two of r,g,b are equal to +1 and the third equals -1.

The polarities of r,g,b are all negated for anti-particles.

When s = +1, count the number of plus signs (say) for r,g,b which is 0 for leptons and 2 for quarks.

When s = -1, count the number of minus signs (say) for r,g,b which is 3 for leptons and 1 for quarks.

For material particles r,g,b are all equal, which is always true for leptons and true for three distinct quarks together or a quark and an appropriate anti-quark.

r	g	b	$s=+$	$s=-$
$-$	$-$	$-$	0	-3
$-$	$+$	$+$	$+2$	-1
$+$	$-$	$+$	$+2$	-1
$+$	$+$	$-$	$+2$	-1
$-$	$-$	$+$	$+1$	-2
$-$	$+$	$-$	$+1$	-2
$+$	$-$	$-$	$+1$	-2
$+$	$+$	$+$	$+3$	0

$$\begin{pmatrix}\gamma\\ ---\\ +0\end{pmatrix}\begin{pmatrix}\gamma\\ +++\\ -0\end{pmatrix} \qquad \begin{pmatrix}Z\\ ---\\ -3\end{pmatrix}\begin{pmatrix}Z\\ +++\\ +3\end{pmatrix}$$

$$\begin{pmatrix}W^-\\ ---\\ -3\end{pmatrix}\begin{pmatrix}W^-\\ +++\\ -0\end{pmatrix} \qquad \begin{pmatrix}W^+\\ ---\\ +0\end{pmatrix}\begin{pmatrix}W^+\\ +++\\ +3\end{pmatrix}$$

$$\begin{pmatrix}W^0\\ -++\\ -1\end{pmatrix}\begin{pmatrix}W^0\\ +--\\ +1\end{pmatrix} \qquad \begin{pmatrix}W^0\\ -++\\ +2\end{pmatrix}\begin{pmatrix}W^0\\ +--\\ -2\end{pmatrix}$$

$$\begin{pmatrix}W^0\\ +-+\\ -1\end{pmatrix}\begin{pmatrix}W^0\\ -+-\\ +1\end{pmatrix} \qquad \begin{pmatrix}W^0\\ +-+\\ +2\end{pmatrix}\begin{pmatrix}W^0\\ -+-\\ -2\end{pmatrix}$$

$$\begin{pmatrix}W^0\\ ++-\\ -1\end{pmatrix}\begin{pmatrix}W^0\\ --+\\ +1\end{pmatrix} \qquad \begin{pmatrix}W^0\\ -++\\ +2\end{pmatrix}\begin{pmatrix}W^0\\ +--\\ -2\end{pmatrix}$$

$$\begin{pmatrix}W^-\\ -++\\ -1\end{pmatrix}\begin{pmatrix}W^-\\ +--\\ -2\end{pmatrix} \qquad \begin{pmatrix}W^+\\ -++\\ +2\end{pmatrix}\begin{pmatrix}W^+\\ +--\\ +1\end{pmatrix}$$

$$\begin{pmatrix}W^-\\ +-+\\ -1\end{pmatrix}\begin{pmatrix}W^-\\ -+-\\ -2\end{pmatrix} \qquad \begin{pmatrix}W^+\\ +-+\\ +2\end{pmatrix}\begin{pmatrix}W^+\\ -+-\\ +1\end{pmatrix}$$

$$\begin{pmatrix}W^-\\ ++-\\ -1\end{pmatrix}\begin{pmatrix}W^-\\ --+\\ -2\end{pmatrix} \qquad \begin{pmatrix}W^+\\ ++-\\ +2\end{pmatrix}\begin{pmatrix}W^+\\ --+\\ +1\end{pmatrix}$$

$$\begin{pmatrix}d^{-1}\\ +-+\\ -1\end{pmatrix} \rightarrow \begin{pmatrix}u^{+2}\\ +-+\\ +2\end{pmatrix} + \begin{pmatrix}W^-\\ +-+\\ -1\end{pmatrix}\begin{pmatrix}W^-\\ -+-\\ -2\end{pmatrix}$$

$$\begin{pmatrix}W^-\\ +-+\\ -1\end{pmatrix}\begin{pmatrix}W^-\\ -+-\\ -2\end{pmatrix} \rightarrow \begin{pmatrix}W^-\\ ---\\ -3\end{pmatrix}\begin{pmatrix}W^-\\ +++\\ -0\end{pmatrix} \rightarrow \begin{pmatrix}e^-\\ ---\\ -3\end{pmatrix} + \begin{pmatrix}\bar{\nu}\\ +++\\ -0\end{pmatrix}$$