The mass of the electron is entirely electromagnetic in origin

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Abstract

Max Abraham (as a prominent advocate of the concept in 1903) thought that the electron mass was entirely of electromagnetic origin. This idea has since then been rejected in the framework of the present understanding in particle physics, wherein, how the mass (the so called bare mass) of electron arises, is still a mystery. As to the statement in the title of the paper (if not in all the details of his model in 1903), here we show that actually Abraham was right afterall! This, we show through a consistent study of the Standard Model (SM), where the group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ is spontaniously broken through the Higgs mechanism to the group $SU(3)_c \otimes U(1)_{em}$. It is shown that the electromagnetism obtained thereby in the SM, as a matter of self-consistency, generates the mass of the electron (and that of the other leptons and quarks) through Yukawa coupling. Thus Yukawa coupling is induced by $U(1)_{em}$ in the SM. Hence all the masses of the fermions (including the electron) in the SM, are fully electromgnnetic in origin.

Keywords: Standard Model, fermion masses, Yukawa coupling, Quantum Electrodynamics

Towards the end of the 19th century and during the early 20th century, there were several advocates of the idea that it was electromagnetism which generated partly or fully, the mass of the electron. Max Abraham was most active as to this idea [1] and which also included Poincare in its list [2]. With Einstein demostration that mass was equivalent to energy, the above idea started frittering away. With better handling of the concept of renormalization in modern quantum field theories, the structure of electron has been found to be very different than from what Abraham and his contemporaries had thought it to be. However still, one does not know whereform arises the mass (better the bare mass or the current mass) of the electron. The modern puzzlement on this issue can be best summarized by Wilczek statement that it may be arising from some "deeper theory", yet not known to us [3]. Hence, as per our best understanding of the issue, we have no clearcut idea as to how electron gains a mass of 0.51 MeV.

Let us go back to our work in 1989/1990 [4,5] wherein we showed that contrary to popular perception, electric charge was actually fully and consistently quantized in the Standard Model of particle physics. Below, we shall follow somewhat similar logic, except for one major change of mathematical and physical nature and which, here, will be shown to lead to a new and significant result.

Let us start by looking at the first generation of quarks and leptons (u, d, e,ν) and assign them to $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ representation as follows:

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, (3, 2, Y_q)$$

$$u_R; (3, 1, Y_u)$$

$$d_R; (3, 1, Y_d)$$

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}; (1, 2, Y_l)$$

$$e_R; (1, 1, Y_e) \tag{1}$$

To keep things as general as possible this brings in five unknown hypercharges.

Let us now define the electric charge in the most general way in terms of the diagonal generators of $SU(2)_L \otimes U(1)_Y$ as

$$Q = I_3 + bY \tag{2}$$

In the SM $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ is spontaniously broken through the Higgs mechanism to the group $SU(3)_c \otimes U(1)_{em}$. In this model the Higgs is assumed to be doublet ϕ with arbitrary hypercharge Y_ϕ. The isospin $I_3 = -\frac{1}{2}$ component of the Higgs develops a nonzero vacuum expectation value $< \phi >_o$. Since we want the $U(1)_{em}$ generator Q to be unbroken we require $Q < \phi >_o = 0$. This right away fixes b in eqn. (2) and we get

$$Q = I_3 + (\frac{1}{2Y_\phi})Y \tag{3}$$

To proceed further one imposes the anomaly cancellation conditions to establish constraints on the various hypercharges above. First $[SU(3)_c]^2 U(1)_Y$ gives $2Y_q = Y_u + Y_d$ and $[SU(2)_L]^2 U(1)_Y$ gives $3Y_q = -Y_l$. Next $[U(1)_Y]^3$ does not provide any new constraints. So the anomaly conditions themselves are not sufficient to provide quantization of electric charge in the SM. One has to provide new physical inputs to proceed further.

Note that the above definition of electric charge in eqn (3), is still chiral. The Spontaneous symmetry breaking only ensures that photon remains massless, but does not make the electric charge as chiral. But we know that the electric charge in electrodynamics is vector-like [4,5], which means that photon couples identically to the left handed and the right handed charges. Note that $U(1)_{em}$ as a priori condition, should be fully created in the SM to be able to further participate in the electro-weak interactions properly and consistently. Thus we have to impose a further unique condition to ensure this and that is to demand that $Q_L = Q_R$. Thus

$$\frac{1}{2}(1 + \frac{Y_q}{Y_\phi}) = \frac{1}{2}\frac{Y_u}{Y_\phi}; giving : Y_u = Y_q + Y_\phi$$

$$Q(d) = \frac{1}{2}(-1 + \frac{Y_q}{Y_\phi}) = \frac{1}{2}\frac{Y_d}{Y_\phi}; giving : Y_d = Y_q - Y_\phi$$

$$\frac{1}{2}(-1 + \frac{Y_l}{Y_\phi}) = \frac{1}{2}\frac{Y_e}{Y_\phi}; giving : Y_e = Y_l - Y_\phi \tag{4}$$

Let us repeat that this brings about the following constraints on the hypercharges:

$$Y_u = Y_q + Y_\phi; Y_d = Y_q - Y_\phi; Y_e = Y_l - Y_\phi \tag{5}$$

Note that $2Y_q = Y_u + Y_d$ from the anomaly cancellation condition for $[SU(3)_c]^2 U(1)_Y$ is automatically satisfied here. Now using $3Y_q = -Y_l$ from anomaly cancellation, along with the above constraints from the vector nature of QED above, plus $[U(1)_Y]^3$ does provide a new constrains of $Y_l = -Y_\phi$. Putting all these together one immediately gets charge quantization in the SM [4,5] as follows:

$$\begin{gathered}
q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, Y_q = \frac{Y_\phi}{3}, \\
Q(u) = \frac{2}{3}, Q(d) = \frac{-1}{3} \\
u_R, Y_u = \frac{3}{4} Y_\phi, Q(u_R) = \frac{2}{3} \\
d_R, Y_d = \frac{-2}{3} Y_\phi, Q(d_R) = \frac{-1}{3} \\
l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}, Y_l = -Y_\phi, Q(\nu) = 0, Q(e) = -1 \\
e_R, Y_e = -2Y_\phi, Q(e_R) = -1
\end{gathered} \tag{6}$$

Note that in the above quantization of the electric charge, Higgs hypercharge Y_ϕ always cancels out and hence remains unconstrained. A repetitive structure gives charges for the other generation of fermions as well.

Note that the above condition of obtaining the correct non-chiral charge for electromagnetism to be valid, is a fundamental and a basic step in the structure of the theory. It is essential for electromagnetism to arise fully and consistently in the SM as a sort of a priori condition, There are two things basic here. The first one is that photon be massless and that is ensured by the process of SSB itself. And the next one, that $U(1)_{em}$ be vector-like in interaction, and this gives out (as a consequence), hypercharge relationships in the form of eqn. (5) above. So $U(1)_{em}$ is providing empty slots and now something has to fit into it. As we show below, it is Yukawa coupling which steps in to fill these empty slots.

Now, it is important to note that the relationships on the hypercharges as arising from the demand of obtaining non-chiral charge in electrodynamics,

in the theory above, in eqn. (5), is exactly the same as what creation of mass through the Yukawa coupling requires (see eqn. (3) in refs. 4 and 5). So to say, if we ask for the mass generation for the first generation particles through Yukawa coupling, then it gives exactly the same relationships on the hypercharges as given in eqn. (5).

We should look at it this way. Creation of non-chiral charges as required by $U(1)_{em}$ demands specific relationships between the unknown hypercharges. Hence $U(1)_{em}$ actually creates these relationships. Now these relationships, next induce Yukawa couplings to give masses to quarks and leptons in the SM. That is, that the very creation of these relationships between hypercharges due to $U(1)_{em}$, generates masses consistently for the fermions through Yukawa couplings. There is no Yukawa coupling without the prior existence of the hypercharge relationships (as given in eqn. (5)) and as a consequence it is these that create the appropriate Yukawa couplings in the SM. Still another way, is to say, that the generation of a consistent $U(1)_{em}$ in SM, creates empty slots in terms of hypercharge relationships and which Yukawa coupling fits into directly. Therefore electrodynamics itself, is what is creating these masses. Hence the mass of the electron (and other quarks and leptons) is entirely electromagnetic in origin.

So in summary, here, we have employed spontaneous symmetry breaking of the Standard Model group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ to the group $SU(3)_c \otimes U(1)_{em}$ through an Higgs doublet and have ensured that the photon remains massless. This in itself does not ensure that the electric charges of the fermions that arise, are non-chiral. This is ensured through a non-trivial extra condition on the chiral charges and that is that $Q_L = Q_R$. This basic condition necessary to create non-chiral electric charges of the fermions, brings about specific relationships between the unknown hypercharges. We show that these same relationships then act as a source to generate masses of the fermions through unique Yukawa couplings as demanded by the electromgnetism. The whole idea is basically a consistency requirement on the theory. And thus we have shown that the bare masses of the electron (and the other fermions) are entirely of electromagnetic origin.

References

1. Max Abraham, "Prinzipien der dynamik des electrons", Annalen der Physik, 10 (1903) 105

2. H. Poincare, "Science and Method", (Published-France 1908), Dover, New York, 1952, p. 256

3. F. Wilczek, "Fantastic Realities", World Scientific, Singapore, 2006

4. A Abbas, "Anomalies and charge quantization in the Standard Model with arbitrary number of colours", Phys. Lett B 238 (1990) 344

5. A Abbas, "Spontaneous symmetry breaking, quantization of the electric charge and the anomalies", J. Phys. G 16 (1990) L163