Still about non-planar twistor diagrams

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February 21, 2013

Abstract

A question about how non-planar Feynman diagrams could be represented in twistor Grassmannian approach inspired a re-reading of the recent article by recent article by Nima Arkani-Hamed et al [B1]. This inspired the conjecture that non-planar twistor diagrams correspond to non-planar Feynman diagrams and a concrete proposal for realizing the earlier proposal that the contribution of non-planar diagrams could be calculated by transforming them to planar ones by using the procedure applied in knot theories to eliminate crossings by reducing the knot diagram with crossing to a combination of two diagrams for which the crossing is replaced with reconnection.

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1 Introduction

A question Krzysztof Bielas about how non-planar Feynman diagrams could be represented in twistor Grassmannian approach inspired a re-reading of the recent article by recent article by Nima Arkani-Hamed et al [B1].

This inspired the conjecture that non-planar twistor diagrams correspond to non-planar Feynman diagrams and a concrete proposal for realizing the earlier proposal [K2] that the contribution of non-planar diagrams could be calculated by transforming them to planar ones by using the procedure applied in knot theories to eliminate crossings by reducing the knot diagram with crossing to a combination of two diagrams for which the crossing is replaced with reconnection. The Wikipedia article about magnetic reconnection explains what reconnection means. More explicitly, the two reconnections for crossing line pair (AB, CD) correspond to the non-crossing line pairs (AD, BC) and (AC, BD).

In the article of Nima et al [B1] the twistor Grassmann program is discussed at rather detailed level and I found that I had moments of "I understand" feeling. A good test for whether this was just an illusion is to try to sum up up some basic ideas involved.

1. The crucial observation is that the on mass shell condition for n-particle vertex containing massless particles characterized by bi-spinors λ and $\tilde{\lambda}$ can be satisfied if either λ:s or $\tilde{\lambda}$:s are parallel. In the case of 3-vertices this dictates completely the dependence of the vertex on twistor variables for arbitrary helicities. There are therefore two vertices depending on the two manners to satisfy momentum conservation conditions. In $\mathcal{N} = 4$ theory different helicities belong to the same super multiplet and the dependence on helities disappears from the amplitude. There

are only two twistor 3-vertices : "black" and "white". From on mass shell 3-particle scattering amplitudes one can construct arbitrary planar scattering amplitudes. All virtual particles are on mass shell but complex momenta must be allowed. The physical interpretation of complex momenta in TGD framework is not quite clear: one possibility is that Euclidian regions of space-time surface (lines of generalized Feynman diagram indeed give imaginary contribution to four-momentum as the reality of $\sqrt{g_4}$ as compared to its imaginary value in Minkowskian regions suggests. Euclidian regions are indeed responsible for dissipation.

2. The diagrams have two basic symmetries. So called mergers and square moves generate twistor diagrams equivalent with the original one. Merger allows to transform a diagram involving n incoming particles and only black or white vertices to single n-vertex. The diagrams can be transformed to bipartite form in which black vertices *resp.* white vertices are lumped to single vertex are connected to each other. Square move rotates 4-particle twistor box diagram (counterpart of tree 4-particle tree diagrams) in which only black and white vertices are connected so that white and black vertices change positions. These equivalences reduce enormously the number of independent diagrams.

   These moves imply that in the case of $\mathcal{N} = 4$ SYM the amplitude assignable to the diagram is completely determined by the permutation assignable to it by the so called left-right rule stating that one starts from an external particle, call it "a", and moves along the diagram turning to the left if the vertex is white and to the right if it is black. Eventually one ends up to an external line - call it "b". The fate of "a" in permutation is $\sigma(a) = b$. It is difficult to exaggerate the importance of this result.

   These moves are analogous to something, which I proposed long time ago in [K1]. I however concluded that this is too crazy idea even from me and removed the chapter for several years from my homepage. During last year (2012) I returned to this idea from different point of view. The idea was that generalized Feynman diagrams could be seen as a sequences of algebraic operations in the generalization of arithmetic system including besides tensor product and direct sum also their inverse operations. Any fan of the Universe as quantum computer idea would be fascinated by this idea. Given sequence of arithmetic operations has infinite number of different representations: this would be the counterpart for the equivalence for infinite number of twistor diagrams.

3. The situation in $\mathcal{N} = 4$ theories is analogous to that in 1+1-D integral quantum field theories. Here the basic scattering event is $2 \rightarrow 2$ scattering: 4-vertex instead of 3-vertex. The sole effect of the scattering is permutation of the momenta and quantum numbers of the particle and phase lag. One can say that particle stops for a moment in the scattering vertex. The number of particles is conserved in the scattering. Yang-Baxter equations states that the scattering amplitude is characterized by a permutation (actually braiding that is element in the braid group defining the covering group of permutations).

2 Does TGD generalize $\mathcal{N} = 4$ SYM or 1+1-D integrable QFT?

What happens in TGD? To what alternative TGD corresponds to: $\mathcal{N} = 4$ SYM or 1+1-D integrable QFT?

1. Effective 2-dimensionality suggests that 1+1-D integrable QFTs might be the natural analog for TGD. In zero energy ontology fermions are the only fundamental particles and bosons emerge as fermion-antifermion pairs at opposite wormhole throat. This implies that 2+2-fermion vertex is the fundamental vertex. This vertex involves wormhole contact and throats as an additional topological ingredient. In TGD framework the conservation of particle numbers is replaced by fermion-number conservation which allows creation of pairs of fundamental fermions, in particular bosons. The essentially new element is the formation of bound states of massless bound states of fermions and anti-fermions which allows to solve the problems related to IR singularities since the theory itself generates the infrared cutoff in terms of mass scales of the bound states identifiable as p-adic mass scales.

2. Braiding is the key element of 1+1-D integrable QFTs and also in TGD generalized Feynman diagrams can be regarded as generalizations of braid diagrams allowing braids of braids. 3-D light-like orbits of wormhole throats carry braid strands carrying fermion number.

3. The proposal is that the 2-D plane M^2 carrying Feynman diagram - interpreted usually as a purely combinatorial auxiliary notion - is realized quite concretely as plane $M^2 \subset M^4$ to which the lines of the generalized Feynman diagram are projected. M^2 has several interpretations.

   In quantum measurement theory it corresponds to a plane spanned by the time axis of the rest system and spin quantization axis and characterizes given causal diamond (CD): note that quantum measurement has geometrization at the level of WCW ("world of classical worlds" defined as the space of 3-surfaces).

   At particle physics level M^2 corresponds to the plane of non-physical polarizations.

   M^2 has also number theoretic interpretation as (hyper)-complex plane of complexified octonions spanned by real unit and preferred imaginary unit. If TGD indeed relates closely to 1+1-D integrable QFT, one can ask whether the scattering is such that it represents just a permutation of incoming lines which in ZEO have either positive or negative energy: just this makes possible particle creation since particle number conservation is reduced to fermion number conservation.

4. One conjecture is that only the M^2 projections of massless 4-momenta of fermions appear as inverses of propagators assignable to the lines of generalized Feynman diagrams if they are actually twistor diagrams as ZEO strongly suggests (virtual fundamental fermions are on mass shell massless particles). Another possibility is that the virtual fermions have non-physical helicities so that the inverse of the massless propagator would not annihilate them.

5. Knotting and intersections associated with non-planarity would be both described in terms of generalized knot diagrams which are braids of braids ... Crossings would result as one projects the lines of generalized Feynman diagram to M^2. The conjecture is that generalized Feynman diagrams allow a generalization of the recursion process used to construct knot invariants to transform the diagrams to sums of planar diagrams to which twistor Grassmannian approach modified so that it applies to fermions applies. In algebraic knot theory one indeed allows also knot diagrams in which the intersection of the lines can be real rather than apparent (strand goes over or below the other one).

3   Could one understand non-planar diagrams in twistor approach?

Non-planar Feynman diagrams remain the technical challenge for the twistor Grassmannian approach (I have written something about this earlier in my blog). In ZEO all particles can be seen as bound states of massless fundamental fermions (leptons and quarks assignable to single generation with family replications described topologically). Hence twistor description is very natural in TGD framework

The vague idea that I try to make more precise in sequel is that non-planar diagrams could be reduced to planar ones by a procedure similar to construct knot invariants [K2]. Knots are generalized so that one allows also vertices. The crossings of lines could be reduced by to a combination of non-crossing lines (by reconnecting the four lines in crossing in two different non-crossing manners) and in this manner one would obtain eventually only planar diagrams.

In algebraic knot theory one considers also genuine crossings besides strand going over or below another one. I have discussed this from TGD point of view [K2](see also the blog posting). One should somehow eliminate the crossing. One could imagine of adding at each crossing a handle to the plane M^2 containing the diagram to obtain an imbedding to a higher genus surface. Knot theoretic approach suggests that the non-planar crossed amplitude is equal to a quantum superposition of two amplitudes without crossing obtained by reconnecting lines.

Also non-planar massless twistor diagrams make sense although only planar ones are discussed in the article by Nima et al [B1] . This raises some questions.

1. Could the non-planar twistor diagrams represent the contribution of the non-planar Feynman diagrams? This would mean an enormous simplification and perhaps the possibility to calculate

the non-planar contribution to the scattering amplitudes. That this should be the case is strongly suggested by the power and elegance of the twistor formalism itself.

2. Could the identification of the permutation associated with planar diagrams in terms of left-right paths generalize? The hope is that suitably defined right-left paths define a permutation also in the presence of crossings. The basic question is what happens at crossings? Should one go straight through or turn to the right or left in a given crossing? The straight-through option is the most natural one and allows to assign with each right-left path an arrow so that 2^n different arrow combinations are obtained: more details are discussed below.

3. Knot theory approach suggests that one recursively reduces non-planar amplitude to a super-position of planar amplitudes by replacing at each crossing the amplitude with a superposition of two "more planar" amplitudes obtained by reconnecting the crossing lines in two different manners. The simplest assumption is that one obtains either the sum or difference of the "more planar" amplitudes associated with the resulting two diagrams. How to choose between '+' and ' -'?

    It is known that non-planar contributions are negligible at large $\mathcal{N}$ limit for SUSYs. If the relative for "more planar" amplitudes is $'-'$, and the two reconnected amplitudes approach asymptotically the same amplitude, one can understand the dominance of the planar amplitudes at this limit. This suggests that '-' is the correct option. But which 'more planar' amplitude corresponds to '+' and which to '-'?

    In knot theory the overall sign would be fixed by whether the line that one is traversing goes over or below the crossing line. Now this option does not work. There is however an alternative possibility to fix the signs if one can assign to a given non-planar diagram the 2^n coverings with fixed arrows of right-left paths. Depending on how the reconnection is carried out, the right-left path continues in the same or opposite direction as the arrow assigned with the crossing line. It is natural to assign a positive sign with the "parallel" reconnected diagram and negative sign to the "antiparallel" one.

    One might of course argue that the arrows must be consistent so that one should actually allow only the "parallel" option. This would however produce only positive signs so that it does not look promising.

If this procedure works, it reduces non-planar twistor diagrams to a superposition of non-planar ones. One must however check that the procedure is well-defined. Consider first the problem of assigning right-left paths to a non-planar twistor diagram.

1. One must decide what happens sy the crossings and the simplest rule is that one just continues straight forward.

2. The possibility to assign freely an arrow with two possible directions to right-left path beginning from any external line is essential. Suppose that the notion of right-left paths based on the straight-through rule defines always a permutation also for non-planar diagrams. Suppose that one assign freely two possible arrows to right-left paths beginning from any external line. This would give 2^n assignments altogether.

3. If the right-left paths $a \to b$ and $b \to a$ are identical, this rule leads to inconsistency since the choice of the arrow for $a \to b$ would fix the arrow for $b \to a$ and the total number of independent choices would be reduced. Fortunately, this situation cannot occur since the right-left path beginning for b leaves the path coming from a at the first vertex.

4. Note that the notion of decorated permutation introduced by Nima et al also brings in 2^n-fold degeneracy by replacing the set of n external lines with its 2-fold covering space containing $2n$ lines and allowing besides permutation $a \to \sigma(a)$ also $a \to n + \sigma(a)$. Presumably these two descriptions are equivalent. A possible interpretation of the covering would be in terms of braid group representations defining a 2-fold covering of the permutation group.

The recursive elimination of crossings would proceed in the following manner.

1. One proceeds along right-left path in the direction of its arrow. If the movement in direction opposite to the arrow were allowed the resulting "more planar" amplitudes would sum up to zero. As one changes the direction of arrow, the elimination process begins from $\sigma(a)$ instead of a and proceeds along different path.

2. When a particular crossing on a given right-left path is eliminated the diagram with a superposition $A - B$ of "more planar" diagrams obtained by reconnection. The rule is that A corresponds to the reconnection for which the directions of the arrows are same and B to that for which they are opposite. One can continue for both resulting reconnected diagrams along the left-right path repeat the procedure at each crossing. k steps produces 2^k planar amplitudes with varying sign factors.

3. Eventually one ends up to an external line $b = \sigma(a)$: b is expected to depend on the particular "more planar" diagram that one is considering. The diagrams obtained in this manner can still contain crossings. One must continue to some direction and the natural choice is to turn around. The next turning point would be $c = \sigma(\sigma(a))$, where c again depends on the resulting "more planar" diagram. One can repeat the process and eventually end up to a situation in which one has returned back to a and there is no point to continue anymore since the process would repeat itselfw without eliminating crossings anymore.

4. Crossings could however still be present. What one can do is to repeat the reduction process by starting from some other external line not belonging to the path traversed. The hope is that eventually one has only planar twistor amplitudes reducible to their minimal form using the left-right rule assigning a unique permutation to each resulting planar diagram. One can also hope that the outcome is independent of the order in which one performs these wanderings around the diagrams rise to new diagrams. The similarity of the elimination process to that applied to knots gives hopes that the outcome does not depend on the order in which the right-left paths associated with external particles are treated in the process.

   Permutations can be decomposed to products of cycles in commuting cyclic subgroups Z_{n_i}, $\prod_{n_i} Z_{n_i} \subset S_n$ and $\sum_i n_i = n$. Therefore each cycle for a given final planar diagram defines one step in this process needed to obtain that particular planar diagram.

4 How stringy diagrams could relate to the planar and non-planar twistor diagrams?

What also popped up to my innocent mind was a question which any string theorist could probably answer immediately. Could it be that string world sheets with g handles could correspond in QFT description to non-planar diagrams imbeddable to a surface of genus g?

In TGD framework this would have a concrete meaning. In TGD Universe all fermions except right-handed neutrino are localized at string world sheets (sub-manifolds of the 4-surface of $M^4 \times CP_2$ representing space-time). The localization is forced by the condition that the modes of the induced spinor field are eigenstates of electric charge. The generalized Feynman diagrams involves a functional integration over WCW giving an expansion in terms of fermion propagators for fundamental fermions. By symmetry considerations the outcome is expected to give twistorial diagrams but with fermions as fundamental particles rather than super-symmetrized gauge fields. The conjecture is that Yangian symmetry forces twistorial Grassmann amplitudes.

In this framework the non-planar twistor diagrams could indeed correspond to the contributions of space-time surfaces for which string world sheets have handles. In Euclidian regions defining the lines of the generalized Feynman diagram higher genera should be possible although one does not have path integral but functional integral over preferred extremals of Kähler action [K3] for which also the dynamics of the string world sheets is fixed.

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