A conjecture about 2-Poulet numbers and a question about primes

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Abstract. To find generic formulas for Poulet numbers (beside, of course, the formula that defines them) was for long time one of my targets; I maybe found such a formula for Poulet numbers with two prime factors, involving the multiples of the number 30, that also is rising an interesting question about primes.

Conjecture:
Any Poulet number with two prime factors can be written as P = (q - 30*n)*(r + 30*n), where q and r are primes or are equal to 1 and n is positive integer, n >= 1.

Note: For a list of 2-Poulet numbers see the sequence A214305 that I submitted to OEIS.

Verifying the conjecture for the first few 2-Poulet numbers:

: P = 341 = 11*31 = (41 - 30*1)*(1 + 30*1) = (31 - 30*1)*(311 + 30*1);

: P = 1387 = 19*73 = (61 - 30*2)*(1327 + 30*2) = (79 - 30*2)*(13 + 30*2);

: P = 2047 = 23*89 = (31 - 30*1)*(2017 + 30*1) = (53 - 30*1)*(59 + 30*1) = (61 - 30*2)*(1987 + 30*2) = (83 - 30*2)*(29 + 30*2);

: P = 2701 = 37*73 = (31 - 30*1)*(2671 + 30*1) = (67 - 30*1)*(43 + 30*1) = (103 - 30*1)*(7 + 30*1) = (97 - 30*2)*(13 + 30*2) = (151 - 30*5)*(2551 + 30*5);

: P = 3277 = 29*113 = (59 - 30*1)*(83 + 30*1) = (89 - 30*2)*(53 + 30*2) = (211 - 30*7)*(3067 + 30*7) = (241 - 30*8)*(3037 + 30*8) = (421 - 30*14)*(2857 + 30*14) = (571 - 30*19)*(2707 + 30*19) = (601 - 30*20)*(2677 + 30*20) = (631 - 30*21)*(2647 + 30*21).

Note: It is remarkable in how many ways a 2-Poulet number can be written this way.

Note: The conjecture might probably be extended for all Poulet numbers not divisible by 3 or 5, not only with two prime factors.

Verifying the extended conjecture for first few Poulet numbers with more than two prime factors not divisible by 3 or 5:

: P = 1729 = 7*13*19 = (31 – 30*1)*(1699 + 30*1) = (43 – 30*1)*(103 + 30*1);

: P = 2821 = 7*13*31 = (31 – 30*1)*(2791 + 30*1) = (37 – 30*1)*(373 + 30*1) = (61 – 30*1)*(61 + 30*1);

: P = 6601 = 7*23*41 = (31 – 30*1)*(6571 + 30*1) = (53 – 30*1)*(257 + 30*1) = (71 – 30*1)*(131 + 30*1) = (191 – 30*1)*(11 + 30*1).

Note: This conjecture is rising the following question: which pairs of primes (x,y), at least one of them bigger than 30, have the property that can be written as (p – 30*n,q + 30*n), where p and q are primes or are equal to 1 and n is positive integer, n >= 1.