# Waves and Special Relativity

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The existence of the waves proves the existence of the special relativity.

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Let be a wave $\psi$ that it is propagating in the space, in the positive $x$ (coordinate) direction, with a constant speed $v$ with respect to a rest (reference) system $S$, and considering a moving system $S'$, also in the positive $x$ direction, with the same speed $v$; then, from the Galileo transformation, Galileo relativity, we have [1]:

$x' = x - vt$, where $t$ is the time
$\psi(x,t) = f(x') = f(x - vt)$, where $f$ is a function
$\partial\psi/\partial x = \partial f/\partial x = (\partial f/\partial x')(\partial x'/\partial x) = \partial f/\partial x'$, since $\partial x'/\partial x = 1$
$\partial\psi/\partial t = \partial f/\partial t = (\partial f/\partial x')(\partial x'/\partial t) = (\partial f/\partial x')(-v) = -v\partial f/\partial x'$
$\partial^2\psi/\partial x^2 = (\partial/\partial x)(\partial f/\partial x') = (\partial/\partial x')(\partial f/\partial x) = (\partial/\partial x')(\partial f/\partial x') = \partial^2 f/\partial x'^2$
$\partial^2\psi/\partial t^2 = (\partial/\partial t)(-v\partial f/\partial x') = -v(\partial/\partial x')(\partial f/\partial t) = -v(\partial/\partial x')(-v\partial f/\partial x') = v^2\partial^2 f/\partial x'^2$
$\partial^2\psi/\partial x^2 = \partial^2 f/\partial x'^2 = (1/v^2)\partial^2\psi/\partial t^2$
$\partial^2\psi/\partial x^2 - (1/v^2)\partial^2\psi/\partial t^2 = 0$

which is the wave equation in one dimension.

And, from the Lorentz transformation, Einstein special relativity (SR), we have:

$x' = \gamma(x - vt)$, where $\gamma = (1 - v^2/c^2)^{-1/2}$, and $c$ is the speed of the light in the vacuum
$\psi(x,t) = f(x') = f(\gamma(x - vt))$
$\partial\psi/\partial x = \partial f/\partial x = (\partial f/\partial x')(\partial x'/\partial x) = (\partial f/\partial x')\gamma = \gamma\partial f/\partial x'$
$\partial\psi/\partial t = \partial f/\partial t = (\partial f/\partial x')(\partial x'/\partial t) = (\partial f/\partial x')(-\gamma v) = -\gamma v\partial f/\partial x'$
$\partial^2\psi/\partial x^2 = (\partial/\partial x)(\gamma\partial f/\partial x') = \gamma(\partial/\partial x')(\partial f/\partial x) = \gamma(\partial/\partial x')(\gamma\partial f/\partial x') = \gamma^2\partial^2 f/\partial x'^2$
$\partial^2\psi/\partial t^2 = (\partial/\partial t)(-\gamma v\partial f/\partial x') = -\gamma v(\partial/\partial x')(\partial f/\partial t) = -\gamma v(\partial/\partial x')(-\gamma v\partial f/\partial x') = \gamma^2 v^2\partial^2 f/\partial x'^2$
$\partial^2\psi/\partial x^2 = \gamma^2\partial^2 f/\partial x'^2 = (1/v^2)\partial^2\psi/\partial t^2$
$\partial^2\psi/\partial x^2 - (1/v^2)\partial^2\psi/\partial t^2 = 0$

that it is again the wave equation in one dimension, but now by virtue of the SR it is $v \leq c$, which is the restriction observed in the nature.

Therefore, the existence of the waves proves the existence of the SR.

[1] Eugene Hecht and Alfred Zajac, Óptica, pp. 12-14, Fondo Educativo Interamericano, 1977. Original edition, Optics, Addison-Wesley, Reading, Massachusetts, USA, 1974.