Circularly polarized beam carries the double angular momentum

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It is shown that there are two different types of angular momentum of electromagnetic radiation: 1) Spin; an elliptic polarization causes the spin density in any point (without a moment arm). 2) Moment of linear momentum; it is an orbital angular momentum. A circularly polarized light beam with plane phase front and the radiation from a rotating dipole carry angular momenta of both types, contrary to the standard electrodynamics, and these two types of angular momentum are spatially separated. Thus the angular momentum splits into spin and orbital angular momenta unambiguously.

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I. INTRODUCTION. SPIN OF LIGHT

It was suggested as early as 1899 by Sadowsky [1] and as 1909 by Poynting [2] that any circularly polarized light has angular momentum *density*. That is the angular momentum is present in any point of the light. We illustrate this phenomenon in this Section.

Poynting [2] considers a beam of monochromatic circularly polarized light with plane phase front in terms of densities. He completely ignores the wall layer of the beam and wall effects. So, this consideration is valid for plane waves, and we let:

j_z [J.s/m^3] is the z-component of the angular momentum volume density in the light,
w [J/m^3] is the energy volume density,
μ_z [J/m^2] is the z-components of the angular momentum flux density, i.e. torque per unit area
f_z [W/m^2] is the z-components of energy flux density, i.e. of the Poynting vector.
Then, according to Poynting [2][1], in vacuum,

$$\frac{j_z}{w} = \frac{\mu_z}{f_z} = \frac{1}{\omega} \tag{1.1}$$

(the Poynting vector is denoted by $\mathbf{f}$, this designation is used in [3, p.96]; ω is angular frequency of the light).

To demonstrate the angular momentum density, let a body absorbs at least a part of the light or/and changes its polarization state. Then a torque τ [J] acts on the body, and the z-component of the torque, which acts on an area a of the body, is $\tau_z = \int \mu_z da$. That is $\tau / a = \mu$.

Carrara [4] wrote: "If a circularly polarized wave is absorbed by a screen or is transformed into a linearly polarized wave, the angular momentum vanishes. Therefore the screen must be subjected to a torque per unit surface equal to the variation of the angular momentum per unit time. The intensity of this torque is $\pm f / \omega$".

The volume density of the torque in the medium of the absorbing body, according Beth [5][2], is $\tau / V = \mathbf{P}\times\mathbf{E}$, and torque per unit area is $\tau / a = \mu = \int (\mathbf{P}\times\mathbf{E}) dz$.

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[1] Equation (1.1) in Poynting's designations is $G = E\lambda / 2\pi$, where G is the torque per unit area (our μ_z), E is energy in unit volume (our w).

[2] Beth wrote: "The moment of force or torque exerted on a doubly refracting medium by a light wave passing through it arises from the fact that the dielectric constant is a tensor. Consequently the electric intensity $\mathbf{E}$ is not parallel to the electric polarization $\mathbf{P}$ in the medium. The torque per unit volume produced by the action of the electric field on the polarization of the medium is $\tau / V = \mathbf{P}\times\mathbf{E}$".

Beth used a half-wave plate, which changed the handedness of the circular polarization into the reversed one, so that the plate experienced the torque [5]. But this torque can be determined also in the Righi experiment (1882) [6,7]. Namely, if the half-wave plate is rotated in its own plane, work is in progress. This amount of work must reappear as an alteration in the frequency of the light (in the energy of the photons), which will result in moving interference fringes in an interference experiment. A variant of such an experiment was suggested in [8].

A calculation of a torque acting on a dielectric body, in which a circularly polarized plane wave died down, was performed in [9] by the Beth's formula.

The wave is:

$$\begin{gathered}\breve{\mathbf{E}} = \exp[i(\breve{k}z-\omega t)](\mathbf{x}+i\mathbf{y})E_0, \quad \breve{\mathbf{B}} = -i\sqrt{\breve{\varepsilon}}\,\breve{\mathbf{E}}, \\ \breve{\varepsilon} = \breve{k}^2/\omega^2, \quad \breve{k} = k'+ik'', \quad \breve{k}^2 = k'^2 - k''^2 + 2ik'k'', \quad c = \varepsilon_0 = \mu_0 = 1\end{gathered} \tag{1.2}$$

(here the mark brave indicates complex numbers). According to Beth,

$$\begin{aligned}\tau^z/V &= (\mathbf{P}\times\mathbf{E})^z = e^{xyz}\Re\{\breve{P}_{[x}\bar{E}_{y]}\} = \Re\{(\breve{\varepsilon}-1)\breve{E}_{[x}\bar{E}_{y]}\} = \exp(-2k''z)\Re\{(\breve{\varepsilon}-1)(-i-i)\}E_0^2/2 \\ &= \exp(-2k''z)\Im(\breve{\varepsilon}-1)E_0^2 = \exp(-2k''z)2k'k''E_0^2/\omega^2\end{aligned} \tag{1.3}$$

(the mark bar indicates conjugate complex numbers, $e^{xyz} = 1$ is the Levi-Civita symbol). Then the torque per unit surface $z = 0$ is

$$\tau/a = \mu = \int_0^\infty \exp(-2k''z)2k'k''E_0^2 dz/\omega^2 = k'E_0^2/\omega^2 \tag{1.4}$$

Spin nature of angular momentum under consideration was shown by Feynman [10, 17–4]. He explained that circularly polarized light carries an angular momentum and energy in proportion to $1/\omega$ because photons carry **spin** angular momentum $\hbar$ and energy $\hbar\omega$. So, the angular momentum volume density j_z is the spin volume density, $s_z = j_z$; $S_z = \int j_z dV$ ($\mathbf{S}$ is spin); and μ_z is z-component of the spin torque per unit area. Therefore we may rewrite (1.1) as

$$\frac{s_z}{w} = \frac{\mu_z}{f_z} = \frac{1}{\omega}. \tag{1.5}$$

We noted [11] that the spin torque density μ_z produced a specific mechanical stress in the absorbing screen. and this effect may be tested experimentally [8].

To confirm spin nature of the angular momentum we recalculated result (1.4) in [11] by the use of the *spin tensor density* in vacuum [12,13]:

$$\Upsilon^{\lambda\mu\nu} = (A^{[\lambda}\partial^{|\nu|}A^{\mu]} + \Pi^{[\lambda}\partial^{|\nu|}\Pi^{\mu]}), \quad dS^{\lambda\mu} = \Upsilon^{\lambda\mu\nu}dV_\nu\,, \tag{1.6}$$

where A^λ and Π^λ are magnetic and electric vector potentials, which satisfy $2\partial_{[\mu}A_{\nu]} = F_{\mu\nu}$, $2\partial_{[\mu}\Pi_{\nu]} = -e_{\mu\nu\alpha\beta}F^{\alpha\beta}$. The sense of a spin tensor is represented by $dS^{xy} = \Upsilon^{xyt}dV$, $d\tau^{xy} = \Upsilon^{xyz}da_z$.

The torque per unit area μ_z (or rather $\mu^{xy} = e^{xyz}\mu_z$) can be expressed now in terms of the electromagnetic fields of the light wave as components of spin tensor (1.6):

$$\mu^{xy} = \Upsilon^{xyz} = -A^{[x}\partial_z A^{y]} - \Pi^{[x}\partial_z\Pi^{y]} \tag{1.7}$$

(we take into account that $\partial_z = -\partial^z$ because of the signature $+---$).

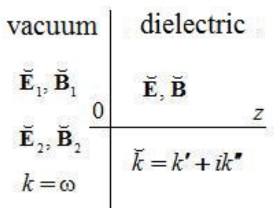

Figure 1.

The result (1.4) for angular momentum flux density may be repeated as spin flux density (1.7) if we will consider vacuum at $z \le 0$ [9]. The wave (1.2) is provoked by incident and reflected waves (Fig. 1):

$$\breve{\mathbf{E}}_1 = (1+\breve{k}/\omega)\exp[i(\omega z-\omega t)](\mathbf{x}+i\mathbf{y})E_0/2, \quad \breve{\mathbf{B}}_1 = -i\breve{\mathbf{E}}_1, \tag{1.8}$$

$$\breve{\mathbf{E}}_2 = (1-\breve{k}/\omega)\exp[i(-\omega z-\omega t)](\mathbf{x}+i\mathbf{y})E_0/2, \quad \breve{\mathbf{B}}_2 = i\breve{\mathbf{E}}_2\,. \tag{1.9}$$

However, it may be shown that the two addends in (1.7) are equal to each other for this case. Thus

$$\mu^{xy} = \Upsilon^{xyz} = -2A^{[x}\partial_z A^{y]}. \tag{1.10}$$

Substituting $A^x = -\int E^x dt = -\int (E_1^x + E_2^x)dt, \quad A^y = -\int E^y dt = -\int (E_1^y + E_2^y)dt$ into $\Upsilon^{xyz} = -\Re(\overline{A}^x \partial_z \breve{A}^y - \overline{A}^y \partial_z \breve{A}^x)/2$ gives

$$\overline{A}^x \partial_z \breve{A}^y = -\overline{A}^y \partial_z \breve{A}^x = i\,\overline{A}^x \partial_z \breve{A}^x, \text{ and } \Upsilon^{xyz} = -\Re(i\,\overline{A}^x \partial_z \breve{A}^x) \qquad (1.11)$$

because $\breve{E}^y = i\breve{E}^x$ and consequently $\breve{A}^y = i\breve{A}^x$. As a result we have sequentially:

$$\breve{A}^x = -i\breve{E}^x/\omega = -i(\breve{E}_1^x + \breve{E}_2^x)/\omega, \quad \partial_z \breve{A}^x = (\breve{E}_1^x - \breve{E}_2^x). \qquad (1.12)$$

$$\begin{aligned}\Upsilon^{xyz} &= \Re\{(\overline{E}_1^x + \overline{E}_2^x)(\breve{E}_1^x - \breve{E}_2^x)\}/\omega = [|E_1^x|^2 - |E_2^x|^2]/\omega + \Re(-\overline{E}_1^x \breve{E}_2^x + \overline{E}_2^x \breve{E}_1^x)/\omega \\ &= [|(1+\breve{k}/\omega)|^2 - |(1-\breve{k}/\omega)|^2]E_0^2/4\omega \\ &= [(1+k'/\omega)^2 + (k''/\omega)^2 - (1-k'/\omega)^2 - (k''/\omega)^2]E_0^2/4\omega = k'E_0^2/\omega.\end{aligned} \qquad (1.13)$$

You see (1.13) coincides with (1.4). This confirm spin nature of the angular momentum densities.

But there is another evidence that the angular momentum volume density j_z is spin density. Namely, the well known expression

$$\varepsilon_0 \mathbf{E}\times\mathbf{A} \qquad (1.14)$$

is widely used as spin volume density:

Jackson [14]: "The term $\varepsilon_0 \int \mathbf{E}\times\mathbf{A} d^3x$ is identified with the 'spin' of the photon".

Ohanian [15]: "The term

$$\mathbf{S} = \varepsilon_0 \int \mathbf{E}\times\mathbf{A}\, d^3x \qquad (1.15)$$

represents the spin".

Friese et al. [16]: "The angular momentum of **a plane electromagnetic wave** can be found from the electric field $\mathbf{E}$ and its complex conjugate $\mathbf{E}^*$ by integrating over all spatial elements d^3r giving $\mathbf{J} = (\varepsilon_0/(2i\omega))\int d^3r\, \mathbf{E}^* \times \mathbf{E}$".

Crichton & Marston [17]: "The spin angular momentum density, $s_i = E_j^*(-i\varepsilon_{ijk})E_k/(8\pi\omega)$, is appropriately named in that there is **no moment arm**".

It is remarkable that $\varepsilon_0 \mathbf{E}\times\mathbf{A}$ is our $\Upsilon^{xyt} = 2A^{[x}\partial^{|t|}A^{y]}$. Really, if we take into account the dimensions, A^x [V.s/m], ε_0 [C/V.m], and that $E^y = -\partial_t A^y$, we will obtain $\Upsilon^{xyt}\varepsilon_0 = 2A^{[x}\partial^{|t|}A^{y]}\varepsilon_0 = -2A^{[x}E^{y]}\varepsilon_0$ [J.s/m^3]

But, in the same time, a strange opinion is widely spread that circularly polarized plane waves have no angular momentum:

Heitler**: "**A plane wave travelling in z-directionand with infinite extension in the xy-directions can have no angular momentum about the z-axis, because $\mathbf{f}$ is in the z-direction and $(\mathbf{r}\times\mathbf{f})_z = 0$" [18].

Of course, Heitler is right, the plane wave has no moment of momentum, $(\mathbf{r}\times\mathbf{f})_z/c^2 = 0$. But spin $\varepsilon_0 \mathbf{E}\times\mathbf{A}$ is not a moment of momentum.

II. ORBITAL ANGULAR MOMENTUM OF A LIGHT WITH PLANE PHASE FRONT

However, an angular momentum of another nature exists at the lateral surface of a circularly polarized wave, i.e. at the surface of a circularly polarized beam. The point is that there are longitudinal components of electromagnetic fields near the lateral surface of a wave because the field lines are closed loops [15]. It entails a rotary mass-energy flow and, correspondingly, an orbital angular momentum volume density $\mathbf{l} = (\mathbf{r}\times\mathbf{f})/c^2$, which is determined by the moment arm $\mathbf{r}$.

$$\mathbf{L} = \frac{1}{c^2}\int (\mathbf{r}\times\mathbf{f})dV \qquad (2.1)$$

is the *orbital* angular momentum of the beam.

Heitler: "It can be shown that the wall of a wave packet gives a finite contribution to $\mathbf{L}$" [18].

Simmonds and Guttmann: “The electric and magnetic fields can have a nonzero z-component only within the skin region of this wave. Having z-components within this region implies the possibility of a nonzero z-component of angular momentum within this region” [19].

The cylindrical beam has the form [14]

$$\mathbf{E} = \exp(ikz - i\omega t)[\mathbf{x} + i\mathbf{y} + \frac{\mathbf{z}}{k}(i\partial_x - \partial_y)]E_0(r), \quad r^2 = x^2 + y^2, \quad \mathbf{B} = -i\mathbf{E}/c, \tag{2.2}$$

and z-component of the orbital angular momentum volume density was found to be [20,21]

$$l_z = -\varepsilon_0 r\partial_r E_0^2(r)/2\omega. \tag{2.3}$$

Energy volume density in the beam (2.2) is

$$w = \varepsilon_0 E_0^2. \tag{2.4}$$

Therefore the ratio between the densities,

$$\frac{l_z}{w} = -\frac{r\partial_r E_0^2(r)}{2\omega E_0^2(r)}, \tag{2.5}$$

has a sharp maximum near the beam boundary[3], in contrast to (1.1), (1.5).

However, despite of the difference in the distributions, spin (1.15) and orbital angular momentum (2.1) of a piece of the beam are equal to each other:

$$\mathbf{S} = \mathbf{L}, \quad \varepsilon_0 \int (\mathbf{E} \times \mathbf{A})dV = \frac{1}{c^2}\int (\mathbf{r} \times \mathbf{f})dV. \tag{2.6}$$

Integrating of energy density (2.4) over the same piece gives

$$W = \omega S = \omega L. \tag{2.7}$$

Thus the total angular momentum is

$$J = S + L = 2W/\omega. \tag{2.8}$$

This result contradicts standard paradigm.

III. MOMENT OF LINEAR MOMENTUM IS NOT SPIN

Famous equality (2.6) is usually referred to as a Humblet equality [24]. On the ground of this equality, an inference was made that spin (1.15) and orbital angular momentum (2.1) are the same *matter* and $J = W/\omega$ in spite of the fact that they are spatially separated.
Ohanian: “This angular momentum (2.1) is the spin of the wave” [15].

To confirm this inference, Jackson [14] and Becker [3, p.320] tried to extend equation (2.6) to a free electromagnetic radiation produced by a source localized in a finite region of space. They applied the Humblet transformation with the integration by parts for fields produced a finite time in the past and obtained the same equality (2.6).

But they were mistaken! This integration by parts cannot be used when radiating into space. A straight calculation presented in [25] for the radiation of a rotating dipole gives

$$2\mathbf{S} = \mathbf{L}, \quad 2\varepsilon_0 \int \mathbf{E} \times \mathbf{A}\, dV = \frac{1}{c^2}\int (\mathbf{r} \times \mathbf{f})dV, \tag{3.1}$$

instead of $\mathbf{S} = \mathbf{L}$ (2.6). Somewhat such result must be expected because when radiating into space photons are variously directed, and their spins are not parallel to each other as in a beam. As a result, equality (3.1) proves that the moment of momentum is not the spin, and so there are two different types of angular momentum of electromagnetic radiation: spin and moment of linear momentum. And equality $J = 2W/\omega$ (2.8) is true for a circularly polarized beam.

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[3] Allen et al. wrote:
“This means inevitably that the ratio changes from place to place” [20].
“At a particular local point the z-component of angular momentum flux divided by energy flux does not yield a simple value” [22].
“A different amount of angular momentum might be expected to be transferred at different positions in the wavefront” [23].

The spatial separation of spin $\varepsilon_0 \mathbf{E}\times\mathbf{A}$ and moment of momentum $(\mathbf{r}\times\mathbf{f})/c^2$ is obvious for a light beam. The analogous separation for the radiation of a rotating dipole is depicted in Figure 2 (partly from [26]). In this case moment of momentum, $(\mathbf{r}\times\mathbf{f})/c^2$, is radiated mainly near the plane of rotating of the dipole (Fig. 2a), while spin, $\varepsilon_0 \mathbf{E}\times\mathbf{A}$, exists near the axis of rotation (Fig. 2c), where the radiation is circularly or elliptically polarized [27].

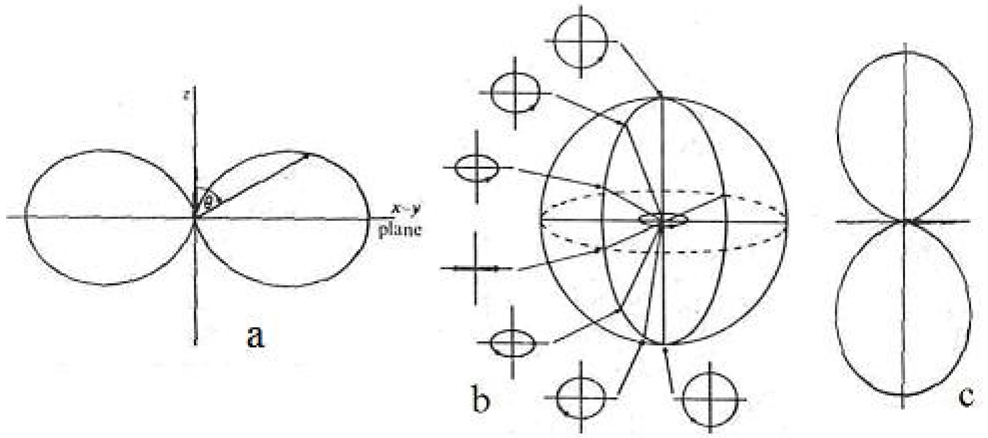

Figure 2.
(a) Angular distribution of z-component of the moment of momentum flux, $dL_z / dtd\Omega \propto \sin^2\theta$.
(b) Polarization of the electric field seen by looking from different directions at the rotating dipole.
(c) Angular distribution of z-component of the spin flux, $dS_z / dtd\Omega \propto \cos^2\theta$.

It is remarkable that our result, $dS_z / dtd\Omega \propto \cos^2\theta$, for the angular distribution of z-component of the spin flux was obtained by Feynman [7] beyond the standard electrodynamics. Really, the amplitudes that a RHC photon and a LHC photon are emitted in the direction θ into a certain small solid angle $d\Omega$ are [7, (18.1), (18.2)]

$$a(1+\cos\theta)/2 \quad \text{and} \quad -a(1-\cos\theta)/2. \tag{3.2}$$

So, in the direction θ, the spin flux density is proportional to

$$[a(1+\cos\theta)/2]^2 - [a(1-\cos\theta)/2]^2 = a^2\cos\theta. \tag{3.3}$$

The projection of the spin flux density on z-axis is

$$dS_z / dtd\Omega \propto a^2\cos^2\theta. \tag{3.4}$$

Thus, according to Feynman, spin (3.3), (3.4) is not a moment of momentum as well.

There is another important circumstance, which prevents the interpretation of moment of momentum $(\mathbf{r}\times\mathbf{f})/c^2$ as spin density of a *radiation*. The Poynting vector of a radiation $\mathbf{f}$ is parallel to the wave vector $\mathbf{k}$ and to the position vector $\mathbf{r}$, $(\mathbf{E}\times\mathbf{H})\times\mathbf{k} = \mathbf{f}\times\mathbf{k} = \mathbf{f}\times\mathbf{r} = 0$. Therefore $\mathbf{E}\,\&\,\mathbf{H}$ - fields, which used in $\mathbf{L} = \int(\mathbf{r}\times\mathbf{f})dV/c^2$, must be non-radiative fields. Really, they are proportional to $1/r^2$ in the case of a radiation into space. This indicate non-radiative nature of the moment of momentum while spin is an attribute of a radiation and must be calculated by the use of fields, which are proportional to $1/r$ only. Heitler, when defending the spin nature of the moment of momentum, refers to a subtle interference effect on this subject [18]. But this explanation seems to be not convincing.

CONCLUSION

The spatial separation of spin from moment of momentum means that total angular momentum splits into spin and orbital angular momenta unambiguously.

Simmonds and Guttmann [19] claimed: “A classical quantity associated with the electromagnetic field does not necessarily indicate the value of that quantity which will be measured. The angular momentum density of the wave was zero at the center, yet when we attempted to measure it there the classical field adjusted themselves and produced a nonzero measurement". We explain this magic trick.

ACKNOWLEDGMENTS

I am deeply grateful to Professor Robert H. Romer for valiant publishing of my question [28] (submitted on 7 October 1999) and to Professor Timo Nieminen for valuable discussions (Newsgroups: sci.physics.electromag).

Notes

This paper was rejected by Journal of Modern Optics without review:
- "September 13, 2013. Our editorial team have now considered your paper but feel the the topic discussed is not best suited to the Journal of Modern Optics. Editorial Office"

This decision was strange because JMO published my paper on this topic: Khrapko R.I. "Mechanical stresses produced by a light beam", *J. Modern Optics* 55, 1487-1500 (2008). So, the decision required an explanation. And I found the explanation. The explanation was in my message.

I hope Prof. Jonathan Marangos Editor in Chief, remembers that his Reviewer-2007 understood the conclusion of my paper "Mechanical stresses produced by a light beam". He wrote:
- "There is an additional spin angular momentum for the photon, that is not present in standard (Maxwell-based) theory".

Nevertheless the Reviewer admitted publishing of the paper because he was sure that the paper being in error and would not damage the interests of the physical authorities. He wrote:
- "This is a difficult paper to judge. It attempts to clarify and correct some questions in one of the 4 or so century-old controversies in classical electrodynamics, perhaps the major one of interest in modern optics. I think the paper, almost in the present form, would be a useful addition to the research literature on the topic, and I'm willing to recommend publication with minor changes.
This is despite the paper being in error, in my opinion. The paper is on a topic where the literature is literately riddled with error, confusion, and dispute. The topic is of interest in practical issues in optical micromanipulation and of theoretical interest in the foundations of field theory and classical electrodynamics. Given the confused situation of the literature on this topic, I'm prepared to recommend the paper for publication despite the errors - it won't make things worse, and does make, in my opinion, a positive contribution.
The main error in the paper, in my opinion, is one of double-counting. The angular momentum transport by a light beam can be deal with, in most cases, either in terms of the moment of the Poynting vector, or by the spin + orbital angular angular momenta, as done by Humblet. For example, there is a page of problems in Jackson, 3rd ed, devoted to this point. The author adds the two together, which is wrong. However, I don't think this will lead readers into error, so I don't see this as a real obstacle to publication".

And Prof. Jonathan Marangos wrote to me:
- "September 9, 2007. We are pleased to accept your paper in its current form and we look forward to receiving further submissions from you.

Reviewer-2009, when considering my next paper *), also understood the conclusion, and was sure that the paper being in error, but, unfortunately, as opposed to Reviewer-2007, he believed that "the conventional (Maxwell and Poynting - based) theory of optical angular momentum is in excellent agreement with all recent experiments and there is no need nor evidence for any correction of the type envisaged by the author". And paper *) was rejected.

Now an anonymous Editorial team has recognized that the conclusion presented in my new paper (this paper) is true. The team requested a translation of my old paper **). The team could give no objections against this paper. And then the team rejected this paper because this paper would damage the interests of the physical authorities.

This is a shame!

*) Khrapko R.I. "Experiments for Determination of Angular Momentum Flux Density". This paper is now published: "On the possibility of an experiment on 'nonlocality' of electrodynamics", *Quantum Electron*, 2012, 42 (12), 1133 http://khrapkori.wmsite.ru/ftpgetfile.php?id=34&module=files, viXra:1307.0110 . See also http://khrapkori.wmsite.ru/ftpgetfile.php?id=46&module=files (replies of journals are presented), http://www.mai.ru/science/trudy/published.php?ID=28833 (2012).

**) Khrapko R.I. "Circularly polarized beam carries the double angular momentum. (2003)" http://www.mai.ru/science/trudy/published.php?ID=34422 (in Russian). See http://viXra.org/abs/1309.0090

New Journal of Physics rejected the paper without consideration:
- "October 03, 2013. We do not publish this type of article in any of our journals and so we are unable to consider your article further. Kryssa Roycroft and Joanna Bewley".

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