Precise formulation and proof of Dirac large numbers hypothesis

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Abstract

In our first calculations using dimensional analysis, we found the approximate value of a large number (the order of 10^{61}) connecting mass, age, size, minimal measurable temperature and density of the observable universe with Planck mass, time, length, temperature and density, respectively.

In separate calculations, we have recalculated Planck units with a second definition of Planck mass as a mass whose Compton wavelength and gravitational radius are equals. In result, exact equation of the large number $N_V = \sqrt{c^5/(2G\hbar H^2)} \approx 5.73 \times 10^{60}$ has been found, connecting cosmological parameters (mass, age, size, minimal measurable temperature and density of the observable universe) and fundamental microscopic properties of the matter (Planck mass, time, length, temperature and density). Thus, a precise formulation and proof of Dirac Large Numbers Hypothesis (*LNH*) has been found, connecting the microworld and the macroworld. Besides, it has been found that the Planck mass represents the geometric mean of Hubble mass and mass of the observable universe. Finally, a hypothesis has been suggested for superhot superdense primordial Planck state of the matter (PSM) with Planckian density and temperature from where the universe emerged after the Big Bang.

Keywords: Dirac large numbers hypothesis; Planck units; dimensional analysis; Hubble mass; mass of the observable universe; Planck state of matter

1. Introduction

Dirac [1] suggested the Large Numbers Hypothesis (*LNH*) pointing out that the ratio of the age of the universe H^{-1} and the atomic unit of time $\tau = e^2/(m_e c^3) \sim 10^{-23} s$ is a large number of the order of 10^{40}. Besides, the ratio of electrostatic e^2/r^2 and gravitational forces $Gm_e m_p/r^2$ between proton and electron in a hydrogen atom is of the order of 10^{39} and the ratio of mass of the observable universe *M* and nucleon mass roughly is of the order of 10^{80}. That is to say:

$$\frac{H^{-1}}{\tau} \sim \frac{e^2}{Gm_e m_p} \sim \sqrt{\frac{M}{m_p}} \sim N_D \sim 10^{40} \qquad (1)$$

where *e* is the charge of the electron, m_e is the electron mass, m_p is the proton mass and $N_D \sim 10^{40}$ is the Dirac large number.

Relying on the ratios (11), he proposed that as a consequence of causal connections between macro and micro physical world, gravitational constant *G* slowly decreases with time.

Many other interesting ratios have been found approximately relating some cosmological parameters and microscopic properties of the matter. For example, Narlikar [2] shows that the ratio of radius of the observable universe and classical radius of the electron $e^2/(m_e c^2)$ is of the order of 10^{40}. Also, the ratio of the electron mass and Hubble (mass) parameter $\hbar H/c^2$ approximates to 10^{39} [3]. Jordan [4] noted that the mass ratio for a typical star and an electron is of the order of 10^{60}. The ratio of mass of the observable universe and Planck mass is of the order of 10^{61} [5]. Peacock [6] points out that the ratio of Hubble distance and Planck length is of the order of 10^{60}. Finally, the ratio of Planck density ρ_P and recent critical density of the universe ρ_c is of the order of 10^{121} [7]. Most of these large numbers are rough ratios of astrophysical parameters and microscopic properties of the matter determined with accuracy of the order of magnitude.

The Planck mass m_P has been derived in [8] by dimensional analysis using three fundamental constants – the speed of light in vacuum (*c*), the gravitational constant (*G*), and the reduced Planck constant ($\hbar$):

$$m_P \sim \sqrt{\frac{\hbar c}{G}} \approx 2.17\times 10^{-8}\ \text{kg} \tag{2}$$

Also, the Planck mass can be derived by setting it as a mass, whose Compton wavelength and gravitational radius are equal [9]. Analogously, formulae for Planck length l_P, Planck time $t_P = l_P/c$ and Planck density ρ_P were derived by dimensional analysis, too [8]. The energy equivalent of Planck mass $E_P = m_P c^2 \sim 10^{19}$ GeV represents unification energy of the fundamental interactions [10].

The Planck temperature T_P is defined as:

$$T_P \sim \frac{m_P c^2}{k_B} = \sqrt{\frac{\hbar c^5}{G k_B^2}} \approx 1.42\times 10^{32}\ K, \tag{3}$$

where k_B is the Boltzmann constant.

Although, the deep nature of Planck units yet is unrevealed, they are a subject of theoretical research of modern quantum cosmology string theory and quantum gravity. Apparently, the Planck length sets the fundamental limits on the accuracy of length measurement. In some forms of quantum gravity, the Planck length is the length scale at which the structure of spacetime becomes dominated by quantum effects, and it is impossible to determine the difference between two locations less than one Planck length apart. The precise effects of quantum gravity are unknown, but it is theorized that spacetime might have a discrete or foamy structure at a Planck length scale.

The dimensional analysis is a conceptual tool often applied in physics to understand physical situations involving certain physical quantities [11-14]. When it is known that quantities should be connected, but the form of this connection is unknown, a dimensional equation is formulated. Most often, dimensional analysis is applied in mechanics and other fields of modern physics, where problems have few determinative quantities. Many interesting and important problems related to the fundamental constants have been considered in [15-18].

The discovery of the linear relationship between recessional velocity of distant galaxies, and distance *v* = *Hr* [19] introduces new fundamental quantity in physics and cosmology – the famous Hubble "constant" (parameter) *H*. The Hubble parameter determines the age of the universe H^{-1} ~ 13.8 *Gyr*, the Hubble distance cH^{-1} ~ 13.8 *Glyr*, and the critical density of the universe $\rho_c = 3H^2/(8\pi G)$ ≈ 9.47×10^{-27} kg/m^3 [20]. According to the recent cosmology, the Hubble "constant" slowly decreases with the age of the universe, but there are indications that other constants, especially gravitational and fine structure constants also vary with a comparable rate [1, 21, 22]. That is why, the Hubble parameter deserves being treated on an equal level with the other three used constants.

2. Approximate estimation of the large number *N* relating cosmological parameters and Planck units by means of dimensional analysis

Because of the importance of the Hubble constant, we have included *H* in the dimensional analysis together with *c*, *G* and $\hbar$, and thus three new triads of constants besides (*c*, *G*, $\hbar$) have been created – (*c*, $\hbar$, *H*), (*c*, *G*, *H*) and (*G*, $\hbar$, *H*) [23]. There we have shown that a unique mass m_i can be deduced from every mentioned triad. The first Valev mass m_1 has been identified with the so called Hubble mass m_H [24, 25]:

$$m_1 \sim \frac{\hbar H}{c^2} = m_H \sim 10^{-33}\ eV \tag{4}$$

This exceptionally small mass coincides with the minimal measurable gravitational self-energy of a particle [26] which is accepted as minimum quantum of energy $E_{\min} = \hbar H \sim 10^{-33}$ *eV* in [27]. This energy takes substantial place in the estimations of total information and entropy of the observable universe [28-30]. Thus, the mass m_1 seems close to the graviton mass obtained by different methods [31-34]. The mass m_1 is in several orders of magnitude smaller than the upper limit of graviton mass, obtained by astrophysical constraints [35].
The presence of a small nonzero mass of the graviton should involve Yukawa type potential of gravitational field $V(r) = -\frac{Gm}{r}\exp(-\frac{m_H c}{\hbar}r)$ that set a finite range of the gravity close to the Hubble distance cH^{-1}. Therefore, the Hubble distance $cH^{-1} \approx 1.38\times10^{10}$ light years is the size of gravitationally connected (observable) universe for an arbitrary observer.
Evidently, the minimum quantum of energy $E_{\min} = \hbar H$ set a lowest limit of measurable temperature T_H:

$$T_H = \frac{\hbar H}{k_B} \approx 1.75\times10^{-29}\ K \tag{5}$$

This temperature is of the order of inverse temperature of the universe $T_u \sim 10^{-30}$ *K* found in [36] by the quantum tunneling between the observable universe and the rest, coinciding with Hawking temperature $T = \frac{\hbar c^3}{8\pi G k_B M}$ [37] for a black hole having mass of the observable universe.
The second mass m_2 derived in [23] is close to the Hoyle-Carvalho formula [38, 39] for the mass of the observable universe and to the mass of the Hubble sphere $M_H = c^3/(2GH)$, having radius cH^{-1} and density ρ_c:

$$m_2 \sim \frac{c^3}{GH} = M \sim 10^{53}\ kg \tag{6}$$

The third mass $m_3 \sim \sqrt[5]{H\hbar^3 / G^2} \sim 10^7\ GeV$ is not identified at present time. It could be a heuristic prediction of an unknown very heavy particle or fundamental energy scale. Besides, the approximate equation for total density of the universe $\tilde{\rho} \sim \rho_c$ has been deduced by means of dimensional analysis in [40]:

$$\tilde{\rho} \sim \frac{H^2}{G} \approx 7.93\times10^{-26}\ kg/m^3 \tag{7}$$

Thus, the Hubble mass (4), the mass of the observable universe (6) and the approximate equation for total density of the universe (7) have been derived by dimensional analysis with the fundamental constants c, G, $\hbar$ and H. The Planck mass (1), Planck temperature (2), Planck length (8), Planck time (9) and Planck density (10) also have been deduced by dimensional analysis by means of constants c, G and $\hbar$:

$$l_P \sim \sqrt{\frac{G\hbar}{c^3}} \approx 1.61\times10^{-35}\ m \tag{8}$$

$$t_P = l_P / c \sim \sqrt{\frac{G\hbar}{c^5}} \approx 5.37\times10^{-44}\ s \tag{9}$$

$$\rho_P \sim \frac{c^5}{\hbar G^2} \approx 5.2\times10^{96}\ kg/m^3 \tag{10}$$

Taking into account equations (2 - 10), as well as Hubble distance cH^{-1} and Hubble time (age of the universe) H^{-1} we find remarkable ratios:

$$\sqrt{\frac{M}{m_H}} = \frac{M}{m_P} = \frac{m_P}{m_H} = \frac{cH^{-1}}{l_P} = \frac{H^{-1}}{t_P} = \frac{T_P}{T_H} = \sqrt{\frac{\rho_P}{\tilde{\rho}}} = \sqrt{\frac{c^5}{G\hbar H^2}} = N \approx 8.1\times10^{60} \tag{11}$$

Therefore, the ratio of the mass of the observable universe M and the Planck mass m_P is equal to the large number N defined from the equation $N = \sqrt{c^5 /(G\hbar H^2)} \approx 8.1\times10^{60}$. Besides, the large number N defines the ratio of Planck mass m_P and the Hubble mass m_H, the ratio of the Hubble distance cH^{-1} and the Planck length l_P, the ratio of Hubble time (age of the universe) H^{-1} and the Planck time t_P, the ratio of Planck temperature T_P and minimal measurable temperature T_H, and the square root of the ratio of the Planck density ρ_P and the approximate density of the universe $\tilde{\rho}$. These ratios are very important because they connect cosmological parameters (mass, age, size, minimal measurable temperature and density of the observable universe) and the fundamental microscopic properties of the matter (Planck mass, Planck time, Planck length, Planck temperature, Planck density and Hubble mass). In recent quantum gravity models, the Planck units imply quantization of spacetime at extremely short range. Thus, the ratios (11) represent connection between cosmological parameters and quantum properties of spacetime. Obviously, the ratios (11) represent an approximate formulation of *LNH* because according recent *CMB* observations [41-43] the total density of the universe $\overline{\rho}$ is close to the critical one:

$$\overline{\rho} = \rho_c = \frac{3H^2}{8\pi G} \approx 9.47\times 10^{-26}\ kg/m^3 \tag{12}$$

Replacing experimental density of the universe $\overline{\rho}$ instead $\widetilde{\rho}$ and mass of the Hubble sphere $M_H = c^3/(2GH)$ instead M in ratios (11) latter become approximate. In Section 3, we show that the reasons of these small discrepancies of ratios (11) are approximate values of Planck units obtained by dimensional analysis.

3. Precise determination of the large number *N* and proof of Dirac *LNH* by recalculation of Planck units

It is known that the dimensional analysis allows findings unknown quantities with accuracy of the dimensionless parameter *k*, unit order of magnitude [12]. Below, we recalculate the ratios (11) using experimental value of total density of the universe $\overline{\rho}$, mass of the Hubble sphere M_H and recalculated values of the Plank units by means of the second definition of Planck mass, namely as a mass whose Compton wavelength and gravitational radius are equal. We mark these values of Planck mass and the rest Planck units by asterisk to differentiate them from approximate "classical" Planck units derived by dimensional analysis. Therefore, the recalculated value of Planck mass m_P^* is the mass, whose reduced Compton wavelength $\lambdabar$ and gravitational (Schwarzschild) radius r_S are equal:

$$\lambdabar = \frac{\hbar}{mc} = r_S = \frac{2Gm}{c^2} \tag{13}$$

From (13) we find the recalculated value of Planck mass:

$$m_P^* = \sqrt{\frac{\hbar c}{2G}} = m_P/\sqrt{2} \approx 1.54\times 10^{-8}\,kg \tag{14}$$

The recalculated value of Planck length l_P^* follows from (13) and (14):

$$l_P^* = r_S = \frac{2G}{c^2} m_P^* = \sqrt{\frac{2G\hbar}{c^3}} = \sqrt{2}l_P \approx 2.28\times 10^{-35}\ m. \tag{15}$$

Clearly, the recalculated value of Planck time is:

$$t_P^* = l_P^*/c = \sqrt{\frac{2G\hbar}{c^5}} = \sqrt{2}t_P \approx 7.59\times 10^{-44}\,s \tag{16}$$

The recalculated value of Planck density ρ_P^* is determined as the density of a sphere possessing mass m_P^* and radius l_P^*:

$$\rho_P^* = \frac{3m_P^*}{4\pi l_P^{*3}} = \frac{3}{16\pi}\frac{c^5}{\hbar G^2} = \frac{3}{16\pi}\rho_P \approx 3.1\times 10^{95}\, kg/m^3 \tag{17}$$

Finally, the recalculated value of Planck temperature T_P^* is:

$$T_P^* = \frac{m_P^* c^2}{k_B} = \sqrt{\frac{\hbar c^5}{2Gk_B^2}} \approx 10^{32}\, K \tag{18}$$

Taking into account equations (4), (5), (12), (14 - 18) and $M_H = c^3/(2GH)$, as well as Hubble distance cH^{-1} and Hubble time H^{-1}, we find the exact ratios (19):

$$\sqrt{\frac{M_H}{m_H}} = \frac{M_H}{m_P^*} = \frac{m_P^*}{m_H} = \frac{cH^{-1}}{l_P^*} = \frac{H^{-1}}{t_P^*} = \frac{T_P^*}{T_H} = \sqrt{\frac{\rho_P^*}{\overline{\rho}}} = \sqrt{\frac{c^5}{2G\hbar H^2}} = N_V = N/\sqrt{2} \approx 5.73\times 10^{60} \tag{19}$$

Clearly, ratios (19) where Planck units are obtained by definition of Planck mass as a mass whose Compton wavelength and gravitational radius are equal perfectly fits with experimental value of total density of the universe $\overline{\rho} \approx \rho_c$ and mass of the Hubble sphere M_H. That reinforces the trust in the recalculated (corrected) Planck units by means of this approach. Since, the total density of the universe $\overline{\rho} \approx \rho_c = \dfrac{3H^2}{8\pi G}$ ≈ 9.47×10^{-27} *kg/m*^{3} is experimentally determinate by experiment *WMAP* with relative error < 0.4 % [44], this experiment should be considered as crucial evidence of the found formulation of Dirac *LNH* (19). Evidently, the large number N_V determines the age, size and mass of the observable universe in whole number Planck units. On the other hand, N_V^2 determines the mass of the observable universe in whole number graviton masses and the dilution of density of the universe from Planck time to now.

Therefore, the recalculated equations (2-6b) for the Planck mass, length, time and density are exact whereas Planck units obtained by dimensional analysis are approximate. Besides, the large number $N_V = \sqrt{c^5/(2G\hbar H^2)} \approx 5.73\times 10^{60}$ is not simply ratio of two quantities but it is an exact formula expressed by means of the fundamental constants *c*, *G*, $\hbar$ and *H*. Therefore, the ratios (19) represent a precise formulation and proof of Dirac *LNH*.

The following thought experiment shows that the Planck length sets the fundamental limits on the accuracy of length measurement: Suppose we want to determine the position of an object using electromagnetic radiation (photons). The greater is the energy of photons, the shorter is their wavelength and the more accurate the measurement. When the wavelength reaches $\lambda/(2\pi) = l_P^* = \sqrt{2G\hbar/c^3}$ the photon has enough energy $E = h\nu = \hbar c/(\lambda/2\pi) = \sqrt{\hbar c^5/(2G)} = m_P^* c^2$ to measure objects the size of the Planck length l_P^*. But the photon would collapse into a black hole having mass $m = E/c^2 \equiv m_P^*$ and Schwarzschild radius $r_S = 2Gm_P^*/c^2 = \sqrt{2G\hbar/c^3} \equiv l_P^*$, and the measurement would be impossible.

It is very interesting that the Planck mass represents the geometric mean of graviton mass and mass of the Hubble sphere:

$$\sqrt{m_H M_H} = \sqrt{\frac{\hbar H}{c^2}\frac{c^3}{2GH}} = \sqrt{\frac{\hbar c}{2G}} \equiv m_P^* \tag{20}$$

Besides, the ratios (21) take place:

$$v_0 = \frac{M_H}{\rho_P^*} = \frac{m_H}{\overline{\rho}} = \frac{8\pi}{3}\frac{G\hbar}{Hc^2} \approx 2.83\times10^{-43} m^3 \tag{21}$$

The radius of the sphere having volume v_0 is $r_0 \approx 4.1\times10^{-15}$ m, i.e. of the order of size of the atomic nucleus. Therefore, the equation (21) shows that when the size of the universe was of the order of atomic nucleus its density was close to the Planck density ρ_P^*. Besides, the volume v_0 of the recent universe having average density $\overline{\rho} \approx \rho_c \sim 10^{-26} kg/m^3$ holds matter and energy equivalent to the Hubble mass $m_H \sim 10^{-33}$ eV.
It follows from equations (21) and (20):

$$v_0 v_0 = v_0^2 = \frac{M_H}{\rho_P^*}\frac{m_H}{\overline{\rho}} = \frac{m_P^{*2}}{\rho_P^* \overline{\rho}} \tag{22}$$

Therefore, $m_P^* = v_0\sqrt{\overline{\rho}\rho_P^*}$, i.e. the atomic nucleus volume v_0 having geometric mean density $\rho_{gm} = \sqrt{\overline{\rho}\rho_P^*} \approx 5.4\times10^{34} kg/m^3$ contains mass equal to the Planck mass m_P^*.
Finally, taking in account equations (15) and (17), as well as $\overline{\rho} \approx \rho_c = 3H^2/(8\pi G)$ we find that a sphere having Planck volume $v_P^* = (4/3)\pi l_P^{*3} \approx 5\times10^{-104} m^3$ and density ρ_{gm} holds matter equal to the Hubble mass m_H :

$$m = (4/3)\pi l_P^{*3}\sqrt{\overline{\rho}\rho_P^*} = \frac{\hbar H}{c^2} \equiv m_H \tag{23}$$

As the large number N_V is inverse proportional to H, the former increases during cosmological expansion (if c, $\hbar$ and G don't vary with time). Apparently, the total density of the universe $\overline{\rho} \approx \rho_c = 3H^2/(8\pi G)$ and the Hubble (graviton) mass $m_H = \hbar H/c^2$ decrease with the age of the universe H^{-1}, whereas the mass of the observable universe $M \sim M_H = c^3/(2GH)$ increases. Nevertheless, the equations (19) and (20) continue to be in force during the extension. Furthermore, the time variations of these quantities are negligible:

$$\frac{\dot{M}}{M} = -\frac{\dot{m}_H}{m_H} = -\frac{1}{2}\frac{\dot{\overline{\rho}}}{\overline{\rho}} = \frac{\dot{N}}{N} \sim H \approx 7.26\times10^{-11} yr^{-1} \tag{24}$$

Clearly, the large number N and Dirac large number N_D are connected by the approximate formula (25):

$$N_D \sim N_V^{2/3} = \sqrt[3]{\frac{c^5}{2G\hbar H^2}} \approx 3.2\times10^{40} \tag{25}$$

4. Discussions and conclusions

At present, we are far away from understanding of Planck state of matter (PSM) featuring of enormous temperature and density of the order of Planckian ($\rho_P \sim 10^{96}$ kg/m^3 and $T_P \sim 10^{32}$ K). Probably, this extreme state is primordial latent (unobservable) state of matter (Planck quantum vacuum) from where the universe emerged 13.8 billion years ago because of equilibrium violations of PSM in a microscopic volume about the size of an atomic nucleus. In Section 3 we have shown that when the observable (gravitationally connected) universe had the size of an atomic nucleus (4×10^{-15} m), the density was close to the Planckian. Besides, contemporary Lambda-CDM model [45, 46] states that in moment close to the Planck time ~ 10^{-43} s after the Big Bang, the universe density and temperature had been close to the Planckian. The temperature and density of PSM are close to the temperature and density of 'false vacuum' [47-49]. We suppose PSM as an eternal infinite medium that unstoppably creates and supplies with matter and energy new and new bubbles-universes. This state of matter coincides with 'strongly symmetric matter' existing till $t_P \sim 10^{-43}$ s after the Big Bang when the gravity freezes out and the symmetry of forces breaks up.

The hypothesis for superhot superdense PSM (quantum vacuum) could explain why the early universe had had extremely high density and temperature close to the Planckian. Because, these are the properties of primordial PSM and the universe had emerged (detached) from this state after Big Bang. Besides, the suggested hypothesis avoids the question of creation out of nothing ("creatio ex nihilo") of the universe – the latter emerges from primordial superhot superdense PSM and the singularity has no place. Possibly, the found ratios between Planck units and cosmological parameters are the result of detachment of the microscopic area from the PSM after Big Bang. In the result, the cosmological parameters of this expanding area conserve their relation with initial condition of the area in the moment of detachment from the PSM after Big Bang.

In the two calculations, the ratio of the Hubble sphere mass M_H and the Planck mass m_P^* was found equal to the substantial large number N_V definite from the equation $N_V = \sqrt{c^5/(2G\hbar H^2)} \approx 5.73\times10^{60}$. Besides, the large number N_V defines the ratio of Planck mass m_P^* and the Hubble mass m_H, the ratio of the Hubble distance cH^{-1} and the Planck length l_P^*, the ratio of Hubble time (age of the universe) H^{-1} and the Planck time t_P^*, the ratio of Planck temperature T_P^* and minimal measurable temperature T_H, and the square root of the ratio of the Planck density ρ_P^* and actual total density of the universe $\bar{\rho}$. Therefore, the large number N_V connects cosmological parameters (mass, age, size, minimal measurable temperature and density, of the observable universe) and fundamental microscopic properties of the matter (Planck mass, Planck time, Planck length, Planck temperature, Planck density and Hubble mass). Thus, a precise formulation and proof of Dirac *LNH* has been found and a new fundamental physical law has been established connecting the microworld and the macroworld, although the deep nature of Planck units and Hubble mass is not sufficiently clear. It is worth noting that the derived ratios (19) are not simply numbers of the same order of magnitude but a single large number N_V, represented by an exact equation by means of fundamental constants – c, G, $\hbar$ and H. Besides, it has been found that the Planck mass represents the geometric mean of Hubble mass and mass of the observable universe $m_P^* = \sqrt{m_H M_H}$.

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