Calculation of the Hubble Parameter from Geometry

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It is shown that the field equations of Einstein gravity sourced by a real massless scalar inflaton field φ, with inflaton potential identically equal to zero, cast on an eight-dimensional pseudo-Riemannian manifold $\mathbb{X}_{4,4}$ (a spacetime of four space dimensions and four time dimensions) admit a solution that exhibits temporal exponential **deflation of three of the four time dimensions** and temporal exponential inflation of three of the four space dimensions. [The signature and dimension of $\mathbb{X}_{4,4}$ are chosen because its tangent spaces satisfy a triality principle [1] (Minkowski vectors and spinors are equivalent).] Comoving coordinates for the two **unscaled** dimensions are chosen to be ($x^4 \leftrightarrow$ time, $x^8 \leftrightarrow$ space). The x^4 coordinate corresponds to our universe's observed physical time dimension. The x^8 coordinate corresponds to a compact spatial dimension with circumference C_8. C_8 determines the initial value of the Hubble parameter H. Most importantly, this model describes an initially inflating/deflating Universe created with inflaton potential identically equal to zero, which is an initial condition that is exponentially more probable than an initial condition that assumes an initial inflaton potential of order of the Planck mass.

This model predicts that the Hubble parameter H during inflation is $H = \frac{\pi}{3\,C_8}$.

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1. INTRODUCTION

Recent Planck 2013 data analysis [2] is in remarkable accord with a flat ΛCDM model with inflation, based upon a spatially flat, expanding Universe whose dynamics are governed

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by General Relativity and dominated by cold dark matter and a cosmological constant Λ, and sourced by a slow-roll scalar inflaton field [3–5]. Planck 2013 reports that the seeds of cosmology structure have Gaussian statistics and form an almost scale-invariant (scalar spectral index $n_s = 0.9603 \pm 0.0073$, ruling out exact scale invariance at over 5σ) spectrum of adiabatic fluctuations and establishes an upper bound on the tensor-to-scalar ratio at $r < 0.11$ (95% CL). "The *Planck* data shrink the space of allowed standard inflationary models, preferring potentials with $V'' < 0$. Exponential potential models, the simplest hybrid inflationary models, and monomial potential models of degree $n \geq 2$ do not provide a good fit to the data. ... We also present a direct reconstruction of the observable range of the inflaton potential. Unless a quartic term is allowed in the potential, we find results consistent with second-order slow-roll predictions."[6]

These observations are not contradicted by many inflationary cosmology models. The main predictions of inflationary cosmology are also consistent [7–10] with other recent observational data from important experiments such as WMAP [11] and the Sloan Digital Sky Survey [12–14], to name only two.

However, standard inflationary cosmology may not represent fundamental physics. It has been shown that if one employs a canonical measure for inflation [15] [16] [17], then the probability for the existence of the initial conditions that are required for slow-roll chaotic inflation is extremely unlikely [15].

Here we discuss a new model of inflation/deflation that overcomes this problem. This model is based on the idea that our universe has as many time dimensions as space dimensions, and also that the **inflaton potential is identically zero**. This model makes the semiclassical prediction that the Hubble parameter H during pure exponential inflation is related to the inverse scale of the compact spatial dimension according to $H = \frac{\pi}{3\,C_8}$, where C_8 denotes the circumference of the compact spatial dimension. Moreover, after "inflation" the observable physical macroscopic world appears to a classical observer to have three space dimensions and one time dimension.

1.1. Notation and conventions

Let $\mathbb{X}_{4,4}$ denote an eight-dimensional pseudo-Riemannian manifold that admits a spin structure, and whose local tangent spaces are isomorphic to flat Minkowski spacetime $\mathbb{M}_{4,4}$.

$\mathbb{X}_{4,4}$ is a spacetime of four space dimensions, with local comoving coordinates (x^1, x^2, x^3, x^8), and four time dimensions, with local comoving coordinates (x^4, x^5, x^6, x^7), employing the usual component notation in local charts. The x^8 coordinate corresponds to a compact spatial dimension with circumference denoted C_8. All coordinates have dimension of length [time coordinates are scaled with a normalized speed parameter $c = 1$ that represents the speed of gravitational waves in vacuum, so that they have appropriate units]. The domains of the comoving coordinates x^α (Greek indices run from 1 to 8) are $-\infty < x^\alpha < \infty$; when $\alpha = 8$ this is understood to be modulo C_8. Please see Sub-section [4.1] for a discussion of the allowed general coordinate transformations for this problem; this topic is not important for the calculations that follow. Let $\mathbf{g}$ denote the pseudo-Riemannian metric tensor on $\mathbb{X}_{4,4}$. The signature of the metric $\mathbf{g}$ is $(4,4) \leftrightarrow (+++----+)$. The covariant derivative with respect to the symmetric connection associated to the metric g is denoted by a double-bar. $\mathbf{g} \leftrightarrow g_{\alpha\beta} = g_{\alpha\beta}(x^\mu)$ is assumed to carry the Newton-Einstein gravitational degrees of freedom. It is moreover assumed that the ordinary Einstein field equations (on $\mathbb{X}_{4,4}$)

$$G^\mu{}_\nu \; = \; 8\,\pi\,\mathbb{G}\,T^\mu{}_\nu \tag{1}$$

are satisfied. We employ the Landau-Lifshitz spacelike sign conventions [18]: here $G_{\mu\nu}$ denotes the Einstein tensor; $\mathbb{G}$ denotes the Newtonian gravitational constant, and the reduced Planck mass is $M_{Pl} = [\,8\pi\,\mathbb{G}\,]^{-1/2}$. Lastly, if $f = f(x^4, x^8)$ then

$$\begin{array}{lll} f^{(1,0)} = \frac{\partial}{\partial x^4} f(x^4, x^8), & f^{(0,1)} = \frac{\partial}{\partial x^8} f(x^4, x^8), & f^{(1,1)} = \frac{\partial^2}{\partial x^4\,\partial x^8} f(x^4, x^8), \\ f^{(2,0)} = \frac{\partial^2}{\partial x^{4\,2}} f(x^4, x^8), & f^{(0,2)} = \frac{\partial^2}{\partial x^{8\,2}} f(x^4, x^8), & \text{etc.} \end{array} \tag{2}$$

2. PROBLEM FORMULATION AND SOLUTION

We study a solution to the Einstein field equations on $\mathbb{X}_{4,4}$ that exhibits inflation/deflation and describes a Universe that is spatially flat throughout the inflation era. During "inflation", the scale factor $a = a(x^4, x^8)$ for the three space dimensions (x^1, x^2, x^3) exponentially inflates, and the scale factor $b = b(x^4, x^8)$ for the three time dimensions (x^5, x^6, x^7) exponentially deflates; moreover, x^4 and x^8 do not scale. For brevity this phenomenon is sometimes simply called "inflation."

The line element for inflation/deflation is assumed to be given by

$$\begin{aligned}
\{ds\}^2 \;&=\; \left\{a(x^4,x^8)\right\}^2\left[\left(dx^1\right)^2+\left(dx^2\right)^2+\left(dx^3\right)^2\right]-\left(dx^4\right)^2\\
&-\;\left\{b(x^4,x^8)\right\}^2\left[\left(dx^5\right)^2+\left(dx^6\right)^2+\left(dx^7\right)^2\right]+\left(dx^8\right)^2\\
&=\; a^2\left[\left(dx^1\right)^2+\left(dx^2\right)^2+\left(dx^3\right)^2\right]-b^2\left[\left(dx^5\right)^2+\left(dx^6\right)^2+\left(dx^7\right)^2\right]\\
&-\;\left(dx^4\right)^2+\left(dx^8\right)^2\;\;; \qquad (3)
\end{aligned}$$

where $a=a(x^4,x^8)$ and $b=b(x^4,x^8)$ carry the metric degrees of freedom in this model. The real massless scalar inflaton field is $\varphi=\varphi(x^4,x^8)$. The action for the metric and inflaton degrees of freedom is assumed to be given by

$$S=\int\left(\frac{1}{8\,\pi\,\mathbb{G}}\right)^2 d^8x\,\sqrt{det(g_{\alpha\,\beta})}\left[\frac{1}{16\,\pi\,\mathbb{G}}\,(R-2\Lambda)\;-\;\frac{1}{2}g^{\mu\,\nu}\partial_\mu\varphi\,\partial_\nu\varphi\right]. \qquad (4)$$

Here Λ is the cosmological constant. The inflaton potential is zero; its action is purely kinematic, although $\varphi^{(0,1)}(x^4,x^8)^2$ may sometimes be regarded as an effective inflaton potential.

2.1. Canonical stress-energy tensor

The canonical stress-energy tensor for the real massless scalar inflaton field is $T_{\mu\,\nu} = -g^{\mu\,\alpha}\frac{2}{\sqrt{\det(g_{\alpha\beta})}}\frac{\partial}{\partial\,g_{\alpha\,\nu}}L_\varphi$, $L_\varphi=\sqrt{det(g_{\alpha\,\beta})}\left[-\,\frac{1}{2}g^{\mu\,\nu}\partial_\mu\varphi\,\partial_\nu\varphi\,\right]$. The distinct components are

$$T_{3\,3}\;=\;\frac{1}{2}\mathbf{a}(x^4,x^8)^2\left[\,\varphi^{(1,0)}(x^4,x^8)^2-\varphi^{(0,1)}(x^4,x^8)^2\right] \qquad (5)$$

$$T_{4\,4}\;=\;\frac{1}{2}\left[\,\varphi^{(1,0)}(x^4,x^8)^2+\varphi^{(0,1)}(x^4,x^8)^2\right] \qquad (6)$$

$$T_{5\,5}\;=\;\frac{1}{2}\mathbf{b}(x^4,x^8)^2\left(\,\varphi^{(0,1)}(x^4,x^8)^2-\varphi^{(1,0)}(x^4,x^8)^2\right) \qquad (7)$$

$$T_{8\,8}\;=\;\frac{1}{2}\varphi^{(0,1)}(x^4,x^8)^2+\frac{1}{2}\varphi^{(1,0)}(x^4,x^8)^2 \qquad (8)$$

$$T_{4\,8}=T_{8\,4}=\varphi^{(0,1)}(x^4,x^8)\varphi^{(1,0)}(x^4,x^8), \qquad (9)$$

which is non-zero, in general. Due to the (x^4,x^8) dependence of the metric the $(4,8)$ and $(8,4)$ components of the Einstein tensor $G_{4\,8}=G_{8\,4}$ are also, in general, non-zero.

2.2. Field Equations

$$G_{\mu\nu} \;=\; 8\,\pi\,\mathbb{G}\,T_{\mu\nu} \tag{10}$$

The distinct field equation components may be written as

$$G_{4\,8} = G_{8\,4} = -\frac{3a^{(1,1)}}{a} - \frac{3b^{(1,1)}}{b} = 8\pi\mathbb{G}\;\varphi^{(0,1)}\varphi^{(1,0)} \tag{11}$$

$$\begin{aligned} G_{3\,3} \;&=\; \frac{1}{b^2}\left[3ab\left(2a^{(0,1)}b^{(0,1)} - 2a^{(1,0)}b^{(1,0)} + a\left(b^{(0,2)} - b^{(2,0)}\right)\right)\right. \\ &+\; \left.\left(a^{(0,1)2} - a^{(1,0)2} + 2a\left(a^{(0,2)} - a^{(2,0)}\right)\right)b^2 + 3a^2\left(b^{(0,1)2} - b^{(1,0)2}\right)\right] \\ &=\; 4\pi\mathbb{G}a^2\left(-\varphi^{(0,1)2} + \varphi^{(1,0)2}\right) \end{aligned} \tag{12}$$

$$\begin{aligned} G_{4\,4} \;&=\; -\frac{3}{a^2b^2}\left[b^2\left(a^{(0,1)2} - a^{(1,0)2} + aa^{(0,2)}\right) + ab\left(3a^{(0,1)}b^{(0,1)} - 3a^{(1,0)}b^{(1,0)} + ab^{(0,2)}\right)\right. \\ &+\; \left.a^2\left(b^{(0,1)2} - b^{(1,0)2}\right)\right] \\ &=\; 4\pi\mathbb{G}\left(\varphi^{(0,1)2} + \varphi^{(1,0)2}\right) \end{aligned} \tag{13}$$

$$\begin{aligned} G_{5\,5} \;&=\; \frac{1}{a^2}\left[2ab\left(-3a^{(0,1)}b^{(0,1)} + 3a^{(1,0)}b^{(1,0)} + a\left(b^{(2,0)} - b^{(0,2)}\right)\right)\right. \\ &+\; \left.3\left(-a^{(0,1)2} + a^{(1,0)2} + a\left(a^{(2,0)} - a^{(0,2)}\right)\right)b^2 + a^2\left(b^{(1,0)2} - b^{(0,1)2}\right)\right] \\ &=\; 4\pi\mathbb{G}b^2\left(\varphi^{(0,1)2} - \varphi^{(1,0)2}\right) \end{aligned} \tag{14}$$

$$\begin{aligned} G_{8\,8} \;&=\; -\frac{3}{a^2b^2}\left[a^2\left(b^{(1,0)2} - b^{(0,1)2}\right) + \left(-a^{(0,1)2} + a^{(1,0)2} + aa^{(2,0)}\right)b^2\right. \\ &+\; \left.ab\left(-3a^{(0,1)}b^{(0,1)} + 3a^{(1,0)}b^{(1,0)} + ab^{(2,0)}\right)\right] \\ &=\; 4\pi\mathbb{G}\left(\varphi^{(0,1)2} + \varphi^{(1,0)2}\right) \end{aligned} \tag{15}$$

The components of $T^{\mu}_{||\mu\,\alpha}$ that are not identically zero must satisfy

$$
\begin{aligned}
a\,b\,T^{\mu}_{||\mu\,4} \;&=\; 0 \;=3a\left(-b^{(1,0)}\left(\varphi^{(1,0)2}\right)+b^{(0,1)}\varphi^{(0,1)}\varphi^{(1,0)}+b^{(0,1)}\right)\\
&+\; b\left(-3a^{(1,0)}\left(\varphi^{(1,0)2}\right)+3a^{(0,1)}\varphi^{(0,1)}\varphi^{(1,0)}\right.\\
&+\; a\left(\varphi^{(1,0)}\left(\varphi^{(0,2)}-\varphi^{(2,0)}\right)\right)\Big)\\
a\,b\,T^{\mu}_{||\mu\,8} \;&=\; 0 \;=3a\left(b^{(0,1)}\left(\varphi^{(0,1)2}\right)-b^{(1,0)}\varphi^{(0,1)}\varphi^{(1,0)}\right)\\
&-\; b\left(-3a^{(0,1)}\varphi^{(0,1)2}+3a^{(1,0)}\varphi^{(1,0)}\varphi^{(0,1)}\right.\\
&-\; a\left(\varphi^{(0,1)}\left(\varphi^{(0,2)}-\varphi^{(2,0)}\right)\right)\Big)\\
&.
\end{aligned}
\tag{16}
$$

Let $H_1=\frac{\mathbf{a}^{(1,0)}(x^4,x^8)}{\mathbf{a}(x^4,x^8)}$, $H_3=\frac{\mathbf{a}^{(0,1)}(x^4,x^8)}{\mathbf{a}(x^4,x^8)}$, $H_2=\frac{\mathbf{b}^{(1,0)}(x^4,x^8)}{\mathbf{b}(x^4,x^8)}$ and $H_4=\frac{\mathbf{b}^{(0,1)}(x^4,x^8)}{\mathbf{b}(x^4,x^8)}$. The Euler-Lagrange equation for the inflaton field yields

$$
\begin{aligned}
\varphi^{(2,0)}(x^4,x^8) \;&-\; \varphi^{(0,2)}(x^4,x^8)+3\varphi^{(1,0)}(x^4,x^8)\left(H_1+H_2\right)-3\varphi^{(0,1)}(x^4,x^8)\left(H_3+H_4\right)\\
&=\; 0,
\end{aligned}
\tag{17}
$$

or, equivalently,

$$
\begin{aligned}
\varphi^{(2,0)}(x^4,x^8) \;&+\; 3\varphi^{(1,0)}(x^4,x^8)\left(H_1+H_2\right)+M^2\,\varphi(x^4,x^8)\\
&=\; \varphi^{(0,2)}(x^4,x^8)+3\varphi^{(0,1)}(x^4,x^8)\left(H_3+H_4\right)+M^2\,\varphi(x^4,x^8),
\end{aligned}
\tag{18}
$$

where M^2 is arbitrary.

Lastly, the field equations imply that

$$
\begin{aligned}
H_1^2+3H_1H_2+H_2^2 \;&-\; \frac{4}{3}\pi\mathbb{G}\,\varphi^{(1,0)}(x^4,x^8)^2\\
-\left[H_3^2+3H_3H_4+H_4^2\right. \;&-\; \left.\frac{4}{3}\pi\mathbb{G}\,\varphi^{(0,1)}(x^4,x^8)^2\right] \;=\; \frac{2}{9}\Lambda,
\end{aligned}
\tag{19}
$$

which identifies a first integral of the motion, and

$$
\frac{a^{(2,0)}(x^4,x^8)}{a(x^4,x^8)}+\frac{b^{(2,0)}(x^4,x^8)}{b(x^4,x^8)}+\frac{8\pi\mathbb{G}}{3}\varphi^{(1,0)}(x^4,x^8)^2=\frac{\Lambda}{9}. \tag{20}
$$

In the next section we present an exact analytical solution to these field equations that predicts a Universe that is spatially flat throughout the "inflation era", meaning a time interval during which the scale factor $a=a(x^4,x^8)$ for the three space dimensions (x^1,x^2,x^3) exponentially inflates, and the scale factor $b=b(x^4,x^8)$ for the three time dimensions (x^5,x^6,x^7) exponentially deflates; moreover the x^4 and x^8 dimensions do not scale.

We do not detail the solutions of the field equation for the cases of a radiation dominated phenomenological stress-energy tensor and a matter dominated phenomenological stress-energy tensor, because after "inflation/deflation" the observable physical macroscopic world, to zeroth approximation, appears to have three space dimensions and one time dimension; the solutions to the field equations for this model look like the textbook solutions. An exact analytical solution to these field equations for the case representing the end of inflation/deflation has not yet been obtained.

3. EXACT EXPONENTIAL INFLATION/DEFLATION

Let H denote the Hubble parameter; we seek a zero-curvature separable solution to the field equations of the form

$$\begin{aligned} a &= a(x^4, x^8) = \exp\left(H\, x^4\right) F(x^8) \\ b &= b(x^4, x^8) = \exp\left(-H\, x^4\right) F(x^8) \\ \varphi &= \varphi(x^4, x^8) = \phi_8(x^8). \end{aligned} \tag{21}$$

We find the following solution:

$$\begin{aligned} H^2 &= \frac{1}{18}\,\Lambda \\ \varphi &= \frac{1}{8}\sqrt{\frac{5}{3\pi\,\mathbb{G}}}\,\ln\left[\left(\tan\left[3H(x^8 - x^8_0)\right]\right)^2\right], \end{aligned} \tag{22}$$

where x^8_0 is a constant phase. The scale factors are

$$\begin{aligned} a &= a(x^4, x^8) = a_0 \exp\left(\, H\, x^4\right) \sqrt[12]{\sin^2(6H\,(x^8 - x^8_0))} \\ b &= b(x^4, x^8) = b_0 \exp\left(-\, H\, x^4\right) \sqrt[12]{\sin^2(6H\,(x^8 - x^8_0))}, \end{aligned} \tag{23}$$

where a_0 and b_0 are constants. The scale factors (a, b) have a spatial period equal to $\frac{\pi}{6H}$, while the three functions (a, b, φ) possess a common spatial periodicity in the x^8 coordinate whose nonzero minimum value is equal to $\frac{\pi}{3H}$, which we hypothesize is equal to $n_1 \times C_8,\ n_1 \in \mathbf{N}$, the natural numbers. This relationship may be expressed in terms of the the initial value of the comoving Hubble radius $R_H = (aH)^{-1}$, as $\frac{\pi}{3H} = \frac{\pi}{3}\, a\, R_H = n_1\, C_8$. One may quantize the initial value of the comoving Hubble radius semiclassically by asking that

$$n_2\,\frac{\pi}{3H} = n_2\,\frac{\pi}{3}\,a\,R_H = n_1\,C_8, \quad n_1, n_2 \in \mathbf{N} \text{ , the natural numbers.} \tag{24}$$

The initial value of the Hubble parameter H and the circumference C_8 of the compact x^8 dimension satisfy a semiclassical quantization condition that must await a quantum theory of gravity for correct interpretation. The choices $n_1 = 1 = n_2$ are, perhaps, the most natural, and lead to the prediction that

$$H = \frac{\pi}{3\,C_8}. \tag{25}$$

C_8 determines the initial value of the Hubble parameter during inflation in this particular solution of the field equations for this model.

4. CONCLUSION

As noted above, the solutions (a, b, φ) of the field equations are **periodic** in x^8 with a common spatial periodicity $\frac{\pi}{3H} = \pi\sqrt{\frac{2}{\Lambda}}$. This common spatial period has been associated with the circumference C_8 of the compact x^8 dimension. This identification assumes that quantum effects that occur in the physics after the Planck scale do not shift the value of the common spatial period. Notwithstanding this caveat, this model makes a semiclassical prediction that the initial value of the Hubble parameter H during inflation is $H = \sqrt{\frac{\Lambda}{18}} = \frac{\pi}{3\,C_8}$.

This model describes an initially inflating/deflating Universe in which the inflaton potential is identically equal to zero. The concomitant initial condition for this inflaton potential model is exponentially more probable than the corresponding initial condition for a model in which the initial inflaton potential is non-zero and on the order of (in order for the inflationary period to persist for approximately 60-efolds) the Planck mass, give or take a few factors of 10. Note that in Eq.[17], for a general separable solution $\varphi = \varphi(x^4, x^8) = \phi_4(x^4)\phi_8(x^8)$, the term $-\frac{1}{\phi_8(x^8)}\frac{\partial^2}{\partial x^{8\,2}}\phi_8(x^8)$ acts as an effective mass-squared term in the $\frac{\partial^2}{\partial x^{4\,2}}\phi_4(x^4)$ equation. In this case the effective mass of the inflaton comes from geometry: the magnitude of the x^8 component of the inflaton momentum plays the role of an effective mass.

Examination of the inflaton wave equation Eq.[18] shows that in this model accelerated expansion will be sustained until φ develops a x^4 time dependence and begins to oscillate, and then $H_1 + H_2$ becomes positive, at which time the the x^4 oscillations of the inflaton begin to dampen out. The details of this transition are under investigation. A salient feature of this model is that after "inflation/deflation" the observable physical macroscopic world appears to possess three space dimensions and one time dimension, at the classical level.

4.1. Inflationary fluctuations

In order to calculate the contribution of quantum fluctuations to the power spectrum of curvature fluctuations at horizon crossing one must identify the gauge invariant variables following, for example, [19] [20] [21], for the full eight dimensional theory, modified by the following considerations: It makes little physical sense to mix a compact dimension with non-compact dimensions under a general coordinate transformation (or even a special Lorentz transformation), because the domain of the image of the transformation (i.e., the new coordinate) is ill-defined. Either the idea that the x^8 dimension is compact should be abandoned, or it must be recognized that the x^8 dimension plays a distinguished role in the physics. We arrive at a model of the universe with a time "degree of freedom," coordinatized by x^4, which **carries** observers along its axis in the direction of the "arrow of time," plus a second distinguished dimension that is spatial and compact. At this point in our understanding it seems that allowed coordinate transformations should preserve the x^8 distinguished compact spatial dimension. Once the allowed coordinate transformations have been restricted this metric theory of gravity resembles an "induced-matter interpretation" of a Kaluza-Klein theory [22] [23], except that all physical fields do *actually* depend on the x^8 coordinate of the compact spatial dimension. Cosmological perturbation theory proceeds from here.

4.2. Observing the extra dimensions

Typically and approximately, inflation scenarios inflate a scale of the size of one billionth the present radius of a proton to the size of the present radius of a marble or a grapefruit in about 10^{-32} seconds. In virtue of the Heisenberg Uuncertainty Principle, and because the comoving (x^1, x^2, x^3) dimensions have undergone inflation while the x^8 dimension has not, present epoch quantum fields that are functions of $(x^1, x^2, x^3, x^4, x^8)$ are expected to almost uniformly sample the region of the x^8 dimension that they occupy, since the x^8 dimension is compact. The average of functions of x^8 may be expected to appear in effective four dimensional spacetime theories. For example, relevant for the inflationary epoch but not relevant today, the average of $\sqrt[12]{\sin^2(6H(x^8 - x^8_0))}$ over a spatial period is $\frac{\Gamma\left(\frac{7}{12}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{13}{12}\right)} \approx 0.90003$, and the average of its cube, the spatial contribution to the volume element, is

$\frac{2\sqrt{2}\,\Gamma\left(\frac{3}{4}\right)^2}{\pi^{3/2}} \approx 0.76279.$

Both the fourth space dimension, whose associated co-moving coordinate is x^8, as well as the extra time dimensions, evidently pose a challenge to observe, if they exist. Relative to the first three space dimensions, whose co-moving coordinates are $(x^1, x^2, x^3) \in \mathbb{R}^3$, the distance ΔX between two points $(x_0, x_0+\Delta X)$ on the x^8-axis is expected to be exponentially smaller by about 60 e-folds than the distance between two points in $\mathbb{R}^3$ separated by the same coordinate difference ΔX, but lying on, say, the x^3-axis; the distance between two points $(x_0, x_0 + \Delta X)$ lying on, say, the x^7-axis is even smaller, since the extra time dimensions experience deflation.

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