A Unified Field Theory
Peter Hickman

Abstract

In this paper solutions to the nature of Matter, Dark matter, Dark energy, Inflation, Matter-Antimatter asymmetry and Higgs Hierarchy problem.are proposed. Vector spaces (p,q,ℝ) with p+q complex dimensional scalar fields and an asymmetric metric are considered. The dimension p+q is calculated to be 6. Matter and Dark Matter energy is bounded [7.1e-3/g,1.7e28/*h*] eV.where g,h are functions of the degeneracy. The Higgs VEV is dependent on the Cosmological constant, using Planck Mission data the Higgs VEV is 246.3GeV. The gauge group SU(2,2) spontaneously breaks to SU(3) or SU(2)xU (1). Its predicted that Matter has 48 degrees of freedom which after symmetry breaking results in the 6 quarks and 6 leptons.The normalisation of a quaternion 2-vector results in a lower bound for r and consequently the Schwarzschild physical singularity is non-existent. Its predicted that Space-Time inflated by 89.2 e-foldings. Cosmological density ratios are predicted (Planck Mission): Dark matter 0.268(0.268), Baryonic 0.05((0.049), and Dark energy 0.682(0.683).

Introduction

Cosmological observations has elucidated the need for dark matter and dark energy to explain the rotation curves of galaxies and the accelerating expansion of the Universe respectively.
Within the Λ-CDM model of cosmology, the Planck Mission **[1a]** has constrained the ratios of Dark matter, Dark energy, Baryonic matter and has also found support for the inflation hypothesis. However the inflation hypothesis does have issues of its own with the inflation potential and fine-tuning.**[2]**.
Particle physics seeks explanation for the Matter-Antimatter asymmetry, the origin of the Lie gauge groups of the Standard Model of particle physics, the 3 generations of quarks and leptons and the Higgs Hierarchy problem. Quantum Field Theory and General Relativity both break down as $r \to 0$.

This theory is based on real vector spaces (p,q) pq $\neq$ 0 with p+q dimension complex fields and including an asymmetric metric.

Affine Connection of (p,q) spaces with Asymmetric metric

The invariant interval between two points on a metric space (p,q) is

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \tag{3.1}$$

The invariance of ds, requires

$$D_{\sigma} g_{\mu\nu} = 0 \tag{3.2}$$

where D_σ indicates covariant differentiation.
For the affine connection to be determined by a metric tensor only, two cases arise:
Case I: The metric and affine connection are both symmetric

$$g_{\mu\nu} = g_{\nu\mu} \; \Gamma_{\sigma\mu\nu} = \Gamma_{\sigma\nu\mu} \tag{3.3}$$

With the conditions **(3.3)** and **(3.2)** the affine connection are the Christoffel Symbols **[3]**

$$\Gamma_{\sigma\mu\nu} = \frac{1}{2}\left(\partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\nu\mu} + \partial_\mu g_{\sigma\mu}\right) \tag{3.4}$$

Case 2: The metric and affine connection are both asymmetric:

$$\bar{g}_{\mu\nu} = -\bar{g}_{\nu\mu} \qquad \bar{\Gamma}_{\sigma\mu\nu} = -\bar{\Gamma}_{\sigma\nu\mu} \tag{3.5}$$

With the conditions **(3.5)** and **(3.2)** the asymmetric affine connection is

$$\bar{\Gamma}_{\sigma\mu\nu} = \frac{1}{2}\left(\partial_\mu \bar{g}_{\sigma\nu} + \partial_\sigma \bar{g}_{\mu\nu} + \partial_\nu \bar{g}_{\mu\sigma}\right) \tag{3.6}$$

A general affine connection can be formed from equations **(3.4)** and **(3.6)**

$$A_{\sigma\mu\nu} = \Gamma_{\sigma\mu\nu} + i\left(\bar{\Gamma}_{\sigma\mu\nu} + h_{\sigma\mu\nu}\right) \tag{3.7}$$

Where the imaginary part of the connection is asymmetric in μ and v; it can be shown that using **(3.2)** and **(3.7)** affine connection reduces to

$$A_{\sigma\mu\nu} = \Gamma_{\sigma\mu\nu} + i\widehat{\Gamma}_{\sigma\mu\nu} \tag{3.8}$$

where

$$\widehat{\Gamma}_{\sigma\mu\nu} = \left(\partial_\sigma \widehat{g}_{\mu\nu} + \partial_\nu \widehat{g}_{\sigma\mu} + \partial_\mu \widehat{g}_{\nu\sigma}\right) \tag{3.9}$$

and the asymmetric affine connection is completely asymmetric

The following identities hold where f=3 to N

$$\widehat{\Gamma}_{\alpha\mu}\,{}^{\beta}\widehat{\Gamma}^{\alpha}_{\sigma\beta} = 2\widehat{\Gamma}_{f\mu}\,{}^{\beta}\widehat{\Gamma}^{f}_{\sigma\beta} \tag{3.10}$$

$$\widehat{\Gamma}_{\alpha}\,{}^{\nu\beta}\widehat{\Gamma}^{\alpha}_{\nu\beta} = 3\widehat{\Gamma}_{f\nu}\,{}^{\beta}\widehat{\Gamma}^{f}_{\nu\beta} \tag{3.11}$$

Curvature of (p,q) spaces with Asymmetric metric

The Riemann curvature tensor can be calculated using the commutator of the 2 covariant derivatives of a real vector **[4]**.

Similarly, taking the commutator of 2 covariant derivatives of a real vector using **(3.8)** for the affine connection gives

$$\left[D_\sigma, D_\rho\right]B_\mu = G^{\alpha}_{\nu\rho\sigma}B_\alpha + iH^{\alpha}_{\nu\rho\sigma}B_\alpha + i2\widehat{\Gamma}^{\alpha}_{\rho\sigma}\partial_\alpha B_\alpha \tag{4.1}$$

where

$$G^{\alpha}_{\nu\rho\sigma} = R^{\alpha}_{\nu\rho\sigma} + S^{\alpha}_{\nu\rho\sigma} \tag{4.2}$$

$$R^{\alpha}_{\nu\rho\sigma} = \partial_\sigma \Gamma^{\alpha}_{\nu\rho} - \partial_\rho \Gamma^{\alpha}_{\nu\sigma} + \Gamma^{\beta}_{\nu\rho}\Gamma^{\alpha}_{\beta\sigma} - \Gamma^{\beta}_{\nu\sigma}\Gamma^{\alpha}_{\beta\rho} \tag{4.3}$$

$$S^{\alpha}_{\nu\rho\sigma} = \widehat{\Gamma}^{\beta}_{\nu\rho}\widehat{\Gamma}^{\alpha}_{\beta\sigma} + \widehat{\Gamma}^{\beta}_{\nu\rho}\widehat{\Gamma}^{\alpha}_{\beta\sigma} + 2\widehat{\Gamma}^{\beta}_{\rho\sigma}\widehat{\Gamma}^{\alpha}_{\nu\beta} \tag{4.4}$$

$$H^{\alpha}_{\nu\rho\sigma} = \partial_\sigma \widehat{\Gamma}^{\alpha}_{\nu\rho} - \partial_\rho \widehat{\Gamma}^{\alpha}_{\nu\sigma} + \Gamma^{\beta}_{\nu\rho}\widehat{\Gamma}^{\alpha}_{\beta\sigma} + \Gamma^{\alpha}_{\beta\sigma}\widehat{\Gamma}^{\beta}_{\nu\rho} - \Gamma^{\beta}_{\nu\sigma}\widehat{\Gamma}^{\alpha}_{\beta\rho} - \Gamma^{\alpha}_{\beta\rho}\widehat{\Gamma}^{\beta}_{\nu\sigma} + 2\Gamma^{\alpha}_{\nu\beta}\widehat{\Gamma}^{\beta}_{\alpha\rho} \tag{4.5}$$

The curvature scalar is G

$$G = g^{\nu\rho}g^{\alpha\sigma}G_{\alpha\nu\rho\sigma} = g^{\nu\rho}g^{\alpha\sigma}\left(R_{\alpha\nu\rho\sigma} + S_{\alpha\nu\rho\sigma}\right) = R + S \tag{4.6}$$

Field Equations

By adding 2 $A^{\alpha}_{\rho\sigma}P_\alpha B_\nu$ to both sides of **(4.1)** equating LHS to zero, and lowering α results in the following 2 equations

$$G_{\beta\nu\rho\sigma}B_{\alpha}+2\Gamma_{\beta\rho\sigma}P_{\alpha}B_{\nu}=0 \tag{5.1}$$

$$2\widehat{\Gamma}_{\beta\rho\sigma}\left(\partial_{\alpha}+P_{\alpha}\right)B_{\nu}+H_{\beta\nu\rho\sigma}B_{\alpha}=0 \tag{5.2}$$

Contracting **(5.2)** by $\nu=\beta$

$$\left(\partial_{\alpha}+P_{\alpha}\right)B_{\nu}-\Gamma^{\mu}_{\mu\nu}B_{\alpha}=0 \tag{5.3}$$

Contracting **(5.1)** with $\beta=\sigma$ and $\nu=\rho$ gives

$$\left(GB_{\alpha}+2\,\Gamma^{\mu\nu}_{\mu}P_{\alpha}B_{\nu}\right)=0 \tag{5.4}$$

Case I

The vectors P_{α} and B_{ν} can be eliminated by using the following relations

$$P_{\alpha}=B_{\alpha}=\left(\Phi^{\dagger}\Phi\right)_{\alpha} \tag{5.5}$$

Where $\left[\Phi^{\dagger}\Phi\right]=L^{-1}$

Equation **(5.3)** reduces to

$$\partial_{\alpha}\left(\Phi^{\dagger}\Phi\right)^{\alpha}+\left(\Phi^{\dagger}\Phi\right)_{\alpha}\left(\Phi^{\dagger}\Phi\right)^{\alpha}-\Gamma^{\mu}_{\mu\alpha}\left(\Phi^{\dagger}\Phi\right)^{\alpha}=0 \tag{5.6}$$

The dark energy potential $V\left(\Phi^{\dagger}\Phi\right)$ is

$$V\left(\Phi^{\dagger}\Phi\right)=\left(\Phi^{\dagger}\Phi\right)_{\alpha}\left(\Phi^{\dagger}\Phi\right)^{\alpha}-\Gamma^{\mu}_{\mu\alpha}\left(\Phi^{\dagger}\Phi\right)^{\alpha} \tag{5.7}$$

Equation **(5.4)** reduces to

$$G+2\,\Gamma^{\sigma\nu}_{\sigma}\left(\Phi^{\dagger}\Phi\right)_{\nu}=0 \tag{5.8}$$

Case II

Eliminating the symmetric connection from **(5.3)** and **(5.4)** gives

$$\left[\gamma^{\mu}\partial_{\mu}+\left(\frac{1}{\lambda}-\frac{1}{2}\lambda\,G\right)\beta\right]B_{\alpha}=0 \tag{5.9}$$

where $\gamma^{\mu}P_{\mu}=1/\lambda\beta$ and $P^{\mu}P_{\mu}=-1/\lambda^{2}$

The matrices γ^{μ}, β are (p+q) x (p+q) so the vector P_{α} is not the 4-momentum vector.

The Lagrangian is

$$\mathcal{L}=B^{\alpha}\left[\gamma^{\mu}\partial_{\mu}+\left(\frac{1}{\lambda}-\frac{1}{2}\lambda\,G\right)\beta\right]B_{\alpha} \tag{5.10}$$

The reduced Compton wavelength is λ and the Energy is

$$E=\left(\frac{1}{\lambda}-\frac{1}{2}G\lambda\right)\hbar c \tag{5.11}$$

The Dimension of (p,q) Space

The quanta of the asymmetric metric are massless hence $Tr\left(T_{\mu\nu}\right)=0$ where $T_{\mu\nu}$ is

$$T_{\mu\nu}=S_{\mu\nu}-\frac{1}{N}g_{\mu\nu}S \tag{6.1}$$

In 4d Energy-Momentum conservation gives $div\,T_{\mu\nu}=0$

hence in p+q dimensions p+q-divergence of T vanishes:

$$D^{\mu}\left(S_{\mu\nu}-\frac{1}{N}g_{\mu\nu}S\right)=0 \tag{6.2}$$

Using the identities **(3.10)** and **(3.11)** equation **(6.2)** reduces to

$$\left(1-\frac{3}{N}\right)\widehat{\Gamma}_{f}^{\nu\beta}D_{\nu}\widehat{\Gamma}_{\sigma\beta}^{f}+\frac{3}{N}\widehat{\Gamma}_{f}^{\nu\beta}D_{\beta}\widehat{\Gamma}_{\sigma\nu}^{f}=0 \tag{6.3}$$

where f=3 to N
Hence it follows that N=p+q=6. The vector spaces (p,q), pq≠0 with asymmetric metric are $(p,q)\in\{(3,3),(4,2),(5,1)\}$

Quaternion 2-Vector

Isomorphism $SO(5,1)\cong SL(2,\mathrm{H})$
The quaternion vector H is 2d. The interpretation of H is similar to the Born Interpretation of the wavefunction.
$\mathrm{H}^{\dagger}\mathrm{H}$ is the co-ordinate probability density function and
$\mathrm{H}^{\dagger}\mathrm{H}$ is the scalar product of $\mathrm{H}^{\dagger}$ and H and $\mathrm{H}^{\dagger}=e_j^{\dagger}\mathrm{H}$ where $e_j^{\dagger}$ is defined as

$$e_j^{\dagger}=\{e_0,\ e_i : i\in[1,2,3]\} \tag{7.1}$$

The normalisation of the quaternion vector is

$$\int H_h^{\dagger}H^h\, d\Omega=1 \tag{7.2}$$

where h=1,2. and Ω is the 3-volume. The dimension $\left[\mathrm{H}^{\dagger}\mathrm{H}\right]=L^{-3}$

Particle Phenomenology

SU(2) spin matrices are 2s+1 x 2s+1, where s is the spin.
(p,0) and (0,q) are spin spaces with dimension p and q respectively.

$$(p,q)=(2s+1,\ 2t+1) \tag{8.1}$$

The irreducible spinors on these (p,q) vector spaces are 4d **[5]**
Hence the spin of the 4d spinors $\chi_{p,q}$ are determined by the relations

$$\left(s=\frac{p-1}{2},\ t=\frac{q-1}{2}\right) \tag{8.2}$$

Table 1
Summary of the spin of the 4d spinors

(p,q)	Spin (s,t)
$(3,3)$	$(1,1)$
$(4,2)$	$(3/2,1/2)$
$(5,1)$	$(2,0)$

Particle wavefunction
A particle wavefunction Ψ is the product of the 4d spinor χ, and the quaternion 2-vector H

$$\Psi=\chi H \tag{8.3}$$

Space-Time and Dark Energy

The intersection of the vector spaces forms a (3,1) vector space, Space-Time.

$$(3,3) \bigcap (4,2) \bigcap (5,1) \rightarrow (3,1) \tag{8.4}$$

Dark energy is the gravitational waves from the $\mathbb{C}^6$ scalars, Higgs bosons and gravitons. There are 4 6d complex scalar fields 1 for each (p.q) vector space and the resultant 6d complex scalar field from the intersection of these (p,q) vector spaces.

Matter and Dark Matter
Matter and Dark matter emerge when the spin states of the 4d spinor pairs entangle. Entangled pairs can be formed by inner or outer multiplication of the spin states of the 4d spinors to form 4d spinors with spin states $s \in [0,2]$

Table 2
Matter & Dark Matter Spinors

Spin	Spinor	No of particles
1/2	(1/2,0),(1,1/2),(1/2,1),(1.3/2),(3/2,1),(3/2,2)	6
0	(0.0),(1,1)	2
1	(1,0),(0,1),(1,2),(2,1),(3/2,1/2)	5
3/2	(3/2,0),(1,1/2),(1/2,1),(1/2,2)	4
2	(1,1),(3/2,1/2),(2,0)	3

The gauge group is SU(2,2) from the isomorphism SO(4,2)≅SU(2,2).

The primary quantum field has the following states
1. Matter 48 fermionic
2. 64 fermionic and 128 bosonic
3. Massless SU(2,2) 30 bosonic
4. 4 6d complex scalar + conjugates 8 bosonic
5. Graviton 2 bosonic
6. 1 Higgs Boson

The s=1/2 fermions have 6×2×4=48 degrees of freedom and constitutes the matter sector. The remaining spinors and bosons constitute the dark matter sector with 64 fermionic and 169 bosonic degrees of freedom.

Matter-Antimatter Asymmetry

Particle-Antiparticle pairs can be produced from the decay of gauge bosons as a consequence of U(1) electric charge conservation.
The single spin 1/2 4d spinor has chiral U(1) electric charges, that is the spin 1/2 particle has 4 U(1) electric charge states. It follows that the spin 1/2 field can only form matter and no antimatter.

Particle mass energy bounds

The equations of General Relativity with cosmological constant Λ are **[6]**

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - g_{\mu\nu}\Lambda = -\kappa T_{\mu\nu} \qquad \textbf{(10.1)}$$

Contracting equation **(10.1)** gives

$$-R - 4\Lambda = -\kappa T = -\kappa\rho \Rightarrow R = \kappa\rho - 4\Lambda \qquad \textbf{(10.2)}$$

Let S = 0 and since energy E $\geq$ 0 equation **(5.11)** reduces to

$$E = \frac{1}{\lambda} - \frac{1}{2} R\lambda \geq 0 \Rightarrow \frac{1}{\lambda^2} \geq \frac{1}{2} R = \frac{1}{2}(\kappa\rho - 4\Lambda) \qquad \textbf{(10.3)}$$

The energy density of fermions and or bosons in equilibrium is given by Fermi-Dirac and Bose-Einstein statistics.

$$\rho = \frac{\beta\hbar^4 c^4}{\lambda^4} \qquad \textbf{(10.4)}$$

where $\beta = \frac{\gamma}{\hbar^3 c^3} = \left(\frac{7\pi^2}{240} f + \frac{\pi^2}{30} b\right)\frac{1}{\hbar^3 c^3}$

and f, b are the degeneracy of the fermions and bosons. Substituting **(10.4)** into **(10.3)**

$$\frac{1}{\lambda^2} \geq \frac{\kappa\beta\hbar^4 c^4}{2\lambda^4} - 2\Lambda \Rightarrow 4\Lambda\lambda^4 + 2\lambda^2 - \kappa\beta\hbar^4 c^4 \geq 0 \qquad \textbf{(10.5)}$$

Note that $\Lambda\kappa\beta\hbar^4 c^4 \ll 1$

$$\lambda^2 \geq \frac{-2 \pm 2\left(1 + 2\Lambda\kappa\beta\hbar^4 c^4\right)}{8\Lambda} \qquad \textbf{(10.6)}$$

$$\lambda^2 \geq \frac{1}{2}\kappa\beta\hbar^4 c^4 \Rightarrow \lambda \geq \left(\frac{1}{2}\kappa\beta\hbar^4 c^4\right)^{½} \qquad \textbf{(10.7)}$$

Hence states have a minimum Compton wavelength.
Equation **(10.3)** also implies

$$\frac{1}{\lambda^2} \geq 0 \Rightarrow \kappa\rho \geq 4\Lambda \Rightarrow \frac{\kappa\beta\hbar^4 c^4}{\lambda^4} \geq 4\Lambda \Rightarrow \lambda \leq \left(\frac{\kappa\beta\hbar^4 c^4}{4\Lambda}\right)^{¼} \qquad \textbf{(10.8)}$$

Hence states have a maximum Compton wavelength.
Combining the 2 inequalities gives the state mass energy bounds

$$\hbar c\left(\frac{4\Lambda}{\kappa\beta\hbar^4 c^4}\right)^{¼} \leq E \leq \hbar c\left(\frac{2}{\kappa\beta\hbar^4 c^4}\right)^{½} \qquad \textbf{(10.9)}$$

The upper bound simplifies to

$$E = (4\pi\gamma)^{-½} E_p \qquad \textbf{(10.10)}$$

For $\Lambda \sim 10^{-52}$ the bounds are

$$7.1 \times 10^{-3}(7f + 8b)^{-¼}\, eV \leq E \leq 1.70 \times 10^{19}(7f + 8b)^{-½}\, GeV \qquad \textbf{(10.11)}$$

Hence states of Matter and/or Dark Matter, the mass energy is bounded.

Gravitation range lower bound

Expanding the normalisation integral **(7.2)** gives

$$\int H^{\dagger} H \,d\Omega = \int g_{ij} \mathrm{H}^{\dagger}_{hi} \mathrm{H}^{hj} + g_{00} \mathrm{H}^{\dagger}_{h0} \mathrm{H}^{h0} \,d\Omega \leq 1 \quad \textbf{(11.1)}$$

Let $\int H^{\dagger}_{hi} H^{h}_{i} \,d\Omega = \delta_{1i}$ Let $\int H^{\dagger}_{h0} H^{h}_{0} \,d\Omega = 1$

Equation **(11.1)** results in the following inequality

$$g_{11} + g_{00} \leq 1 \quad \textbf{(11.2)}$$

The Schwarzschild metric in the [-+++] convention is **[7]**

$$-ds^2 = \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - \left(1 - \frac{r_s}{r}\right) c^2 dt^2 + r^2 \left(d\theta^2 + \sin^2\theta d\Phi^2\right) \quad \textbf{(11.3)}$$

where $r_s = 2\,GM \big/ c^2$ is the Schwarzschild radius and mass M>0

Substituting $g_{11} = \left(1 - r_s \big/ r\right)^{-1}$ and $g_{00} = -\left(1 - r_s \big/ r\right)$ into **(11.2)** gives the inequality

$$\frac{1}{\left(1 - \frac{r_s}{r}\right)} - \left(1 - \frac{r_s}{r}\right) \leq 1 \quad \textbf{(11.4)}$$

Hence r has a minimum

$$r_m \geq \frac{2\,r_s}{3 + \sqrt{5}} \quad \textbf{(11.5)}$$

Since M>0 it follows that $r_m > 0$ hence the physical singularity at r = 0 does not exist.
Consider a boson in thermal equilibrium of energy E in a spherical volume $r = r_m$. Thus it follows that the upper energy density satisfies the equation

$$\frac{3E}{4\pi\, a^3 r_s^3} = \frac{g_s}{\hbar^3 c^3} \frac{\pi^2}{30} E^4 \quad \textbf{(11.6)}$$

Substituting $r_s = 2\,GE \big/ c^4$ into **(11.6)** and solving for E gives

$$E = \sqrt[6]{\frac{45}{16\,\pi^3 a^3 g_s}} \left(\frac{\hbar c^5}{G}\right)^{\frac{1}{2}} \quad \textbf{(11.7)}$$

Which evaluates to

$$E = 1.486 \left(g_s\right)^{-\frac{1}{6}} \times 10^{19}\, GeV \quad \textbf{(11.8)}$$

For massless spin 2 field $g_s = 2$ the upper bound is $E_p \sim 1.324 \times 10^{19}$GeV, which will be referenced as the Planck scale.
Particles crossing the event horizon of a Black Hole do not fall to a singularity, but accumulate in a shell of radius $r_s - r_m \sim 0.618\, r_s$

SU(2,2) Gauge coupling strength and the Electro-weak vacuum.

The SM is valid upto $\Lambda_{UV} = 2.5 \times 10^{18}$GeV. The gravitational coupling strength at the upper energy bound is

$$\alpha_G = \frac{60}{\pi^3 (7f + 8b)} \tag{12.1}$$

It follows that $\alpha_G(\Lambda_{UV}) = 0.042$ Gravitation in equilibrium with SU(2,2) gauge bosons, hence coupling strength of SU(2,2) is 0.042.

Write the lower $E_l(x)$ and upper bound $E_u(x)$ states as Fourier integrals

$$E_l(x) = \frac{1}{(2\pi)^{½}} \int_{-\infty}^{\infty} \Phi_l(p) e^{ipx} \, dp \tag{12.2}$$

$$E_u(x) = \frac{1}{(2\pi)^{½}} \int_{-\infty}^{\infty} \Phi_u(p) e^{ipx} \, dp \tag{12.3}$$

The gravitational coupling strength is

$$\alpha_G = \frac{E_l E_u}{2\pi E_p^2} \tag{12.4}$$

where E_l and E_u are given by **(10.9)**

The energy scale $E = \sqrt{\frac{E_l E_u}{2\pi}}$

$$E = \left(\frac{27000\, \hbar^5 c^{17} \Lambda}{\pi^{13} G^3 (7f + 8b)^3} \right)^{\frac{1}{8}} \tag{12.5}$$

The primary quantum field f=112, b=169, evaluating **(12.5)** with $\Lambda \sim 1.1 \times 10^{-52}$ **[1b]** results in $E \sim 246.3$ GeV

The energy bounds are $E_u \sim 3.7 \times 10^{17}$GeV and $E_l \sim 1.04 \times 10^{-3}$eV

Thus the gravitational coupling strength of the primary quantum fields of the lower and upper energy bounds results in a energy scale of 246GeV, the Higgs VEV.

Inflation

The energy bounds **(10.11)** and the emergence of Space-Time **(8.4)** at the Planck scale implies that the density of dark matter, dark energy and matter is zero.

The Friedmann-Robertson-Walker-Equations are **[8]**

$$\frac{\ddot{a}(t)}{a(t)} = - \frac{4\pi G (\rho + 3p)}{c^2} + \frac{\Lambda_{eff} c^2}{3} \tag{13.1}$$

$$\frac{\dot{a}(t)^2}{a(t)^2} + \frac{kc^2}{a(t)^2} = \frac{8\pi G \rho}{3c^2} + \frac{\Lambda_{eff} c^2}{3} \tag{13.2}$$

where Λ_{eff} is the effective cosmological term and ρ is the total matter plus dark matter energy density. With ρ=0 and k=0, **(13.1)** is

$$\frac{\dot{a}(t)^2}{a(t)^2} = \frac{\Lambda_{eff} c^2}{3} \tag{13.3}$$

The only contribution to the energy density is from the 3 asymmetric metrics which constitutes the inflation field. Each asymmetric metric field has energy density given by

$$T_{00} = \frac{1}{\kappa}\left(S_{00} - \frac{1}{6}g_{00}S\right) \tag{13.4}$$

Using **(4.4)** equation **(13.4)** is

$$T_{00} = \frac{1}{\kappa}\left(2\widehat{\Gamma}^{\beta}_{0\alpha}\widehat{\Gamma}^{\alpha}_{0\beta} - \frac{1}{3}g_{00}\widehat{\Gamma}_{\mu\rho\sigma}\widehat{\Gamma}^{\mu\rho\sigma}\right) \tag{13.5}$$

Assuming spatial derivatives of the asymmetric metric are zero, and $g_{00} = 1$ it follows that $S_{00} = 0$ and the inflation density simplifies to

$$T_{00} = -\frac{2}{3\kappa}\left[2\left(\partial_0\widehat{g}_{0\mu}\right)\left(\partial^0\widehat{g}^{0\mu}\right) + \left(\partial_0\widehat{g}_{\beta\mu}\right)\left(\partial^0\widehat{g}^{\beta\mu}\right)\right] \tag{13.6}$$

Thus the energy density is negative and acts in a similar way to a positive cosmological constant. Each asymmetric metric field is in thermal equilibrium, hence the inflation density is $3\,T_{oo}$

Equation **(13.2)** is

$$\frac{\dot{a}(t)^2}{a(t)^2} = \kappa\rho_i c^2 \tag{13.7}$$

where $\rho_i = T_{00}$

which for constant density has the general solution

$$a(t) = a(0)\exp\left(\sqrt{\kappa\rho_i}\,ct\right) \tag{13.8}$$

The number of e-foldings χ is

$$\chi = \sqrt{\kappa\rho_i}\,\tau \tag{13.9}$$

where τ is the duration of inflation.

Substituting $\rho_i = \frac{g\pi^2\hbar}{30\,c^3}\left(\frac{E_i}{\hbar c}\right)^4$ with the degeneracy of the spin 2 asymmetric massless field is $g = 2$ and the duration of inflation is $\tau = \hbar\,/\,E$

$$\chi = \sqrt{\frac{8\,\pi^3 G E_i^4}{30\,\hbar c^5}}\,\frac{1}{E} \tag{13.10}$$

Substituting **(11.7)** into **(13.10)** gives the number of e-foldings

$$\chi = \sqrt[3]{\frac{45}{32\,\pi^3 a^3}}\cdot\sqrt{\frac{8\,\pi^3}{30}}\cdot\frac{E_p}{E} \tag{13.11}$$

Inflation ends when matter and dark matter are in thermal equilibrium with s=0 state. Substituting equation **(10.1)**

$$\chi = \frac{\pi^2}{15\,a}\left(\frac{45}{32}\right)^{\frac{1}{3}}(7\,f + 8\,b)^{½} \tag{13.12}$$

$$\chi \sim 1.9299(7\,f + 8\,b)^{½} \tag{13.13}$$

Using f=112 and b=169 for the primary quantum field gives the average energy per particle state after inflation 3.7×10^{17}GeV and the number of e-foldings 89.2. The duration of Inflation is 1.8×10^{-42}s

Dark Energy and the Cosmological Constant

Since N=6 Equation (5.8) gives

$$\Lambda = \frac{1}{3}\Gamma^{\sigma\nu}_{\sigma}\left(\Phi^{\dagger}\Phi\right)_{\nu} \tag{14.1}$$

After inflation, the dark energy potential $V\left(\Phi^{\dagger}\Phi\right)$ has a minimum in (3,1)d when $\left(\Phi^{\dagger}\Phi\right)_{\alpha} = \Gamma^{\mu}_{\mu\alpha}$

Hence Dark energy minimum is

$$\Lambda = \frac{1}{3}\Gamma^{\sigma\nu}_{\sigma}\Gamma^{\beta}_{\beta\nu} \tag{14.2}$$

Ignoring spatial derivatives and using (3.4) $\Gamma^{\sigma}_{\sigma 0} = \frac{1}{2}\, g^{\mu\sigma}\partial_0 g_{\mu\sigma}$ equation (14.2) is

$$\Lambda = \frac{1}{12}\left(g^{\mu\sigma}\partial^0 g_{\mu\sigma}\right)\left(g^{\mu\beta}\partial_0 g_{\mu\beta}\right) \tag{14.3}$$

Using $h_{\mu\nu} = \eta_{\mu\nu} - g_{\mu\nu}$ [9] results in

$$\Lambda = \frac{1}{3\lambda^2} \tag{14.4}$$

where λ is the reduced wavelength.
The natural frequency of gravitational waves is **[10]**

$$\omega = \sqrt{\frac{\pi G\rho}{c^2}} \tag{14.5}$$

It follows that Λ is

$$\Lambda = \frac{\pi G\rho}{3c^4} \tag{14.6}$$

where ρ is the mean mass-energy density of the complex scalars, Higgs Bosons and Gravitons in the ground state.

Cosmological Density Ratios

(In this section $\hbar$=c=1)

Using Bose-Einstein and Fermi-Dirac statistics, the density of dark matter ρ_d at the end of inflation is

$$\rho_d = \left(\frac{7\pi^2}{240}f + \frac{\pi^2}{30}b\right)T^4 \tag{15.1}$$

where T is the temperature. Substituting f=64, b=169, the density of dark matter is

$$\rho_d = \frac{15\pi^2}{2}\,T^4 \tag{15.2}$$

The density of dark energy iρ_Λ is

$$\rho_\Lambda = \left(\frac{7\pi^2}{240} f + \frac{\pi^2}{30} b \right) T^4 \tag{15.3}$$

Dark energy is due to the 4d spinors, the complex scalars, the Higgs and the graviton.
The 4d spinors fermionic 24, bosonic 48.
Each of the 4 6d complex scalars can form

$$\mathbb{C}^6 \rightarrow \{6 \times \mathbb{C}^1, 15 \times \mathbb{C}^2, 20 \times \mathbb{C}^3, 15 \times \mathbb{C}^4, 6 \times \mathbb{C}^5, 1 \times \mathbb{C}^6\} \tag{15.4}$$

including the conjugate complex scalars, 2×63 = 126 bosonic degrees of freedom with conjugate complex spinors
Hence for dark energy f=24, b=48+4×126=552 and ρ_Λ

$$\rho_\Lambda = \frac{191\,\pi^2\, T^4}{10} \tag{15.5}$$

The degeneracy of spin 1/2 matter is 48, hence density of baryonic matter ρ_b is

$$\rho_b = \frac{7\pi^2}{5} T^4 \tag{15.6}$$

In thermal equilibrium the ratio of baryonic to dark matter is

$$\frac{\Omega_b}{\Omega_d} = \frac{\rho_b}{\rho_d} \tag{15.7}$$

Hence it follows that

$$\frac{\Omega_b}{\Omega_d} = \frac{14}{75} \tag{15.8}$$

At the end of inflation the Universe is close to critical density

$$\Omega_\Lambda + \Omega_d + \Omega_b = 1 \tag{15.9}$$

Solving ??,(15.8), (15.9) gives the following cosmological density ratios

$$\Omega_b = 0.05,\ \Omega_d = 0.268,\ \Omega_\Lambda = 0.682 \tag{15.10}$$

which are in agreement with the Planck mission results **[1c]** ($\Omega_b = 0.049$, Ω_d=0.268, Ω_Λ= 0.683)

Galaxy Rotation Curves

Assuming Euler-Lagrange equations of motion for dark matter bosonic spinors (ignoring spin) is

$$\partial_\mu \partial^\mu \zeta + a^2 \zeta = 0 \tag{16.1}$$

The general solution is

$$\zeta(r, t) = \frac{R}{r} e^{-kr} e^{ift} \tag{16.2}$$

where $a^2 = k^2 + f^2$
The Weak Gravitational field approximation to Einstein's Field equations is the Poisson equation

$$\nabla^2\Phi = -\kappa\rho = -\partial^0\zeta^\dagger\partial_0\zeta = -\left(\partial_0\zeta\right)^2 \tag{16.3}$$

where Φ is the gravitational potential per unit mass. For centrally symmetric field dominated by Dark matter,**(16.3)** is

$$\frac{1}{r^2}\partial_r\left(r^2\frac{\partial\Phi}{\partial r}\right) = \left(\frac{R^2}{r^2}e^{-2kr}\right)f^2 e^{i2ft} \tag{16.4}$$

which has a general solution

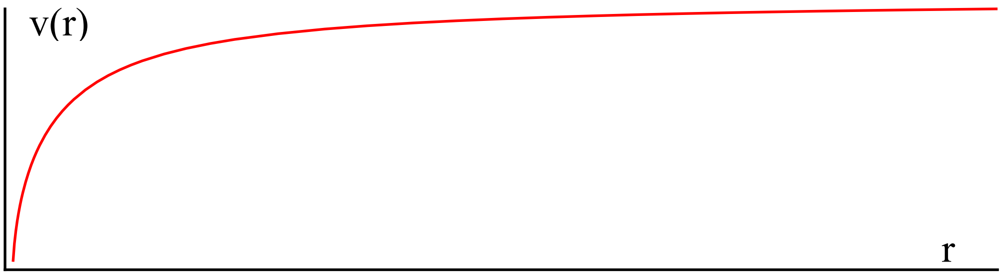
$$\Phi(r) = \frac{R^2 e^{-2kr}}{12kr} - \frac{R^2\,\mathrm{Ei}_1(2kr)}{6} - \frac{A}{r} + B \tag{16.5}$$

where A and B are constants and Ei_1 is the exponential integral.

It follows that the speed of rotation v(r,t) is

$$v(r,t) = \left[\frac{R^2 e^{-2kr}}{12kr} - \frac{R^2\,\mathrm{Ei}_1(2kr)}{6} - \frac{A}{r} + B\right]^{½} fe^{ift} \tag{16.6}$$

A particular graph is



r

which is similar to the general profile for a galaxy rotation curve **[11]** with limiting rotational speed With A=0 the speed approaches a constant when r is a solution of the equation

$$\frac{e^{-2kr}}{2kr} - \mathrm{Ei}(2kr) = 0 \tag{16.7}$$

Which has solution $r \sim 0.32\,\lambda$

Hence it follows that the wavelength of these dark matter particles is of the order of the distance where $v(r \sim 0.32\,\lambda) \rightarrow$constant.

Higgs Hierarchy Problem

The radiative corrections to the Higgs mass are [**12**]

$$\Delta m_H^2 = -\frac{1}{8\pi^2}\sum_{f=1}^{n} g_f^2\left[\Lambda_{UV}^2 + \ldots\right] + \frac{1}{16\pi^2}\sum_{b=1}^{m} g_b^2\left[\Lambda_{UV}^2 + \ldots\right] \tag{17.1}$$

where g_f and g_b are the couplings of fermion and bosons to the Higgs field..

The Standard Model (SM) thus predicts quadratically divergent corrections to the mass of the Higgs boson.

The radiative corrections are due to the set of fermions and bosons given by Table 1, 2 fermions and 4 bosons.These couple to the Higgs field with $g_f = g_b$

It follows that with n=2 and m=4 the radiative corrections $\Delta m_H = 0$

Using **(10.11)** with f=2, b=4 the cutoff is $\Lambda_{UV} = 2.5 \times 10^{18}$GeV and therefore the SM is valid below this energy scale.

SU(2,2) Spontaneous Symmetry Breaking SSB

The number of Higgs is given by **[13]**

$$H = n - N + M \tag{18.1}$$

where n is dimension of real scalar field, N is the dimension of the gauge group SU(N) and M is the dimension of the sub-group.
(3,3)∪(5,1)=(8,4) and (3,3)∪(4,2)∪(5,1)=(12,6)
The sub-spaces {(2,2),(4,4),(6,6)} are complex spaces $\{\mathbb{C}^2, \mathbb{C}^4, \mathbb{C}^6\}$
with {2,4,6} dimension complex scalar multiplets.
The complex scalar multiplets have real dimension n∈{4,8,12}
Assuming only 1 Higgs Boson, it is found that SSB of SU(2,2) leads to only 2 sub-groups

$$SU(2,2) \to \begin{pmatrix} SU(3) \\ SU(2) \times U(1) \end{pmatrix} \tag{18.2}$$

The $\mathbb{C}^3$ complex vector does not lead to any sub-group.
Electro-weak group SU(2)×U(1).is broken by the $\mathbb{C}^2$ scalar doublet.
After SU(2,2) symmetry breaking, the 48 matter states partitions into
6 SU(3) fermions and 6 singlets, ie 6 quarks and 6 leptons.

Conclusion

Extending Riemann geometry on a (p,q) $pq \neq 0$ space by including an asymmetric metric results in $(p,q) \in \{(3,3),(4,2),(5,1)\}$ and the dimension p+q=6
The 4d spinors on these 6d spaces have spin (1,1,1/2,3/2,2,0) and have 4 additional degrees of freedom.

Space-Time (3,1) emerges from the intersection of the 3 vector spaces (3,3),(4,2),(5,1).
It is found that the mass-energy bounds of boson and/or fermion fields is below the Planck scale.

The energy density of matter,dark matter and dark energy is zero at the Planck scale, the inflation field of negative energy density inflated Space-Time by 89.2 e-foldings.
Inflation ended with the emergence of matter, dark matter in thermal equilibrium.

The source of dark energy is the gravitational waves emitted from the 4d massless spinors and complex scalars.

Matter emerged with 48 degrees of freedom which after symmetry breaking of SU(2,2) to the Standard Model gauge groups, partition into 36 for 6 quarks and 12 for 6 leptons.

The Higgs Hierarchy problem is resolved by the summation of the radiative corrections to the Higgs boson mass over the set of 4 bosonic and 2 fermionic 4d spinors.

The gravitational coupling strength of the primary quantum fields of the lower and upper energy bounds results in a energy scale of 246GeV, the Higgs VEV.

This theory is UV-divergent free in the gravitational sector and the matter, dark matter and dark energy sectors.

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