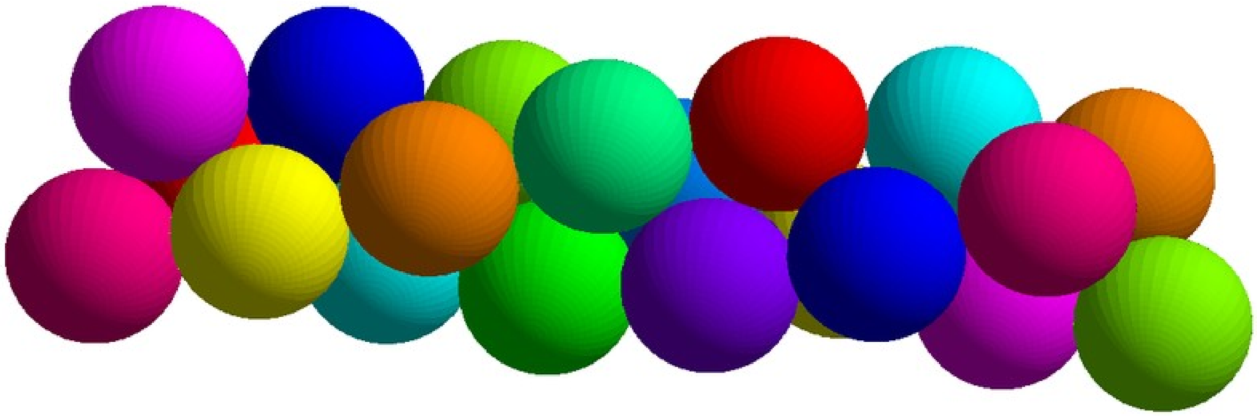
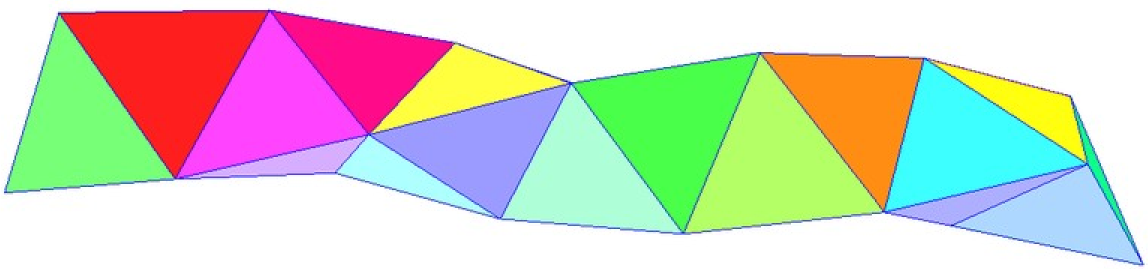
The Boerdijk-Coxeter Helix and the Qi Men Dun Jia Model





By John Frederick Sweeney

Abstract

The final element of the Qi Men Dun Jia Model is the Boerdijk-Coxeter Helix, since this brings matter up to the level of DNA strings or lattices. Composed of Octonions, Twisted Octonions and Sedenions, the author examines the Boerdijk-Coxeter Helix from various perspectives to illustrate how BC – Helices play an important role in the formation of matter.

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Introduction

The Qi Men Dun Jia Model has been described in this series of papers published on Vixra, and at this point the author has shown that all matter begins in the invisible substratum of "black hole" material at the natural logarithm of e, (2.718) from whence it rises through the substratum via a combinatorial process of beats or counts, perhaps similar to the number of breaths a human takes per minute, for example.

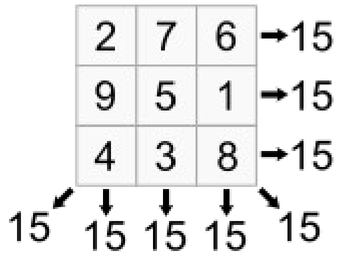
When approaching the border with visible matter, the new-forming matter takes on a spherical form and can be measured with the use of Pi. The newly - forming matter will take on the characteristics of 8 x 8 stable Satva matter, or the 9 x 9 character of the more dynamic Raja matter.

Upon passing the threshold of visible matter, a series of structures await the newly - emergent matter, starting with a 3 x 3 Magic Square, of the Luo Shu, or He Tu (Book of Luo, River Diagram) varieties or in a related order. The Magic Square functions to provide the "ribbon," in terms of Buckminster Fuller, which will ultimately bind the tetrahedra at the opposite end of the series of structures.

One possibility remains, in addition to the Magic Square, of Magic Cayley Graphs, and the author shall explore this topic in a future paper.

Magic Squares / Wikipedia

It is possible to construct a normal magic square of any size except 2 x 2 (that is, where n = 2), although the solution to a magic square where $n = 1$ is trivial, since it consists simply of a single cell containing the number 1. The smallest nontrivial case, shown below, is a 3 x 3 grid (that is, a magic square of order 3).



The constant that is the sum of every row, column and diagonal is called the magic constant or magic sum, M. Every normal magic square has a unique constant determined solely by the value of n.

Lo Shu square (3×3 magic square)

Main article: Lo Shu Square
Chinese literature dating from as early as 650 BCE tells the legend of Lo Shu or "scroll of the river Lo".[2] According to the legend, there was at one time in ancient China a huge flood. While the great king Yu (禹) was trying to channel the water out to sea, a turtle emerged from it with a curious figure / pattern on its shell: a 3x3 grid in which circular dots of numbers were arranged, such that the sum of the numbers in each row, column and diagonal was the same: 15, which is also the number of days in each of the 24 cycles of the Chinese solar year. According to the legend, thereafter people were able to use this pattern in a certain way to control the river and protect themselves from floods.

4	9	2
3	5	7
8	1	6

The Lo Shu Square, as the magic square on the turtle shell is called, is the unique normal magic square of order three in which 1 is at the bottom and 2 is in the upper right corner. Every normal magic square of order three is obtained from the Lo Shu by rotation or reflection.
The Square of Lo Shu is also referred to as the Magic Square of Saturn.

Forming a circle around the center of the Magic Square lies the Spinorial Clock or the Clifford Clock, comprised of the real, complex numbers and Quarternions. The Clifford Clock lies embedded in the Clock of Complex Spaces, while forgetful functors circulate in the anti-clockwise direction, limiting the function of the higher numbers.

Forgetful functors: The functor U : **Grp** → **Set** which maps a group to its underlying set and a group homomorphism to its underlying function of sets is a functor.[7] Functors like these, which "forget" some structure, are termed *forgetful functors*. Another example is the functor **Rng** → **Ab** which maps a ring to its underlying additive abelian group. Morphisms in **Rng** (ring homomorphisms) become morphisms in **Ab** (abelian group homomorphisms). (See Appendix IV for details on forgetful functors).

As the sphere of matter emerges into the realm of visible matter, it takes on certain characteristics based on the current arrangement of numbers in the Magic Square as well as the arrangement of Clifford Algebras in the Clifford Clock, with minor aspects imprinted from the Five Element aspects of each Clifford Algebra and its complex space. The characteristics imprinted from the Magic Square program the particle wave which determines the winding of the “ribbon” (in Buckminster Fuller’s term) that connects to the Tetrahelix, with its icosahedral and tetrahedral elements.

Of the eight figures putatively rotating about the face of the Clifford Clock, one of them always turns up missing, which leaves the actual count of numbers at seven – the number of Octonions in the Fano Plane. In a later paper the author will describe the related aspects of the Qi Men Dun Jia Cosmic Board, suffice it here to state that there may exist as many as seven additional dimensions associated with the Clifford Algebras, each of which possesses unique Five Element associations, and which add additional qualities to each Clifford Algebra’s informational content.

Next, the mole of matter receives imprint from a rotating set of 60 Stellated Icosahedra, in the form of double icosahedra, which provide information in terms of frequency and Time. The double icosahedra represent the 60 Na Yin and 60 Jia Zi of Chinese metaphysics, which is a Base 60 cycle and have Period 60, given Pisano Periodicity. Pisano Periodicity limits the number of permutations in nature to 60.

In relation to Pisano Periodicity, Fibonacci Numbers and the Golden Ratio enter the process of material formation, along with a series of Platonic Solids, from the Triangle and Square to the Icosahedron and Dodecahedron. The latter two play critical roles in the formation of matter, especially that of DNA strands. The author speculates here that the role of the Golden Ratio in the Qi Men Dun Jia Model is to separate matter in the 8 x 8 Satva State from matter in the 9 x 9 Raja State. In the discussion of the Boerdijk-Coxeter Helix, where both states form part of the structure.

The next essential structure of the Qi Men Dun Jia Model is the Poincare Dodecahedral Space, which the author described in detail in an earlier paper in this series. Then, associated with the Poincare

Dodecahedral Space is the Hopf Fibration, some 15 copies of which float about the structure of the Boerdijk-Coxeter Helix, apparently helping to rotate the helix.

In fact, the literature on the Hopf Fibration is quite confusing, some of which leads the author to consider the Hopf Fibration as a micro element of the Qi Men Dun Jia Model, and some articles which suggest the opposite. It is the author's intention that the logic of the Qi Men Dun Jia Model will help to solve this dilemma.

With regard to the Boerdijk – Coxeter Helix, only a minimum amount of research has been done, given the extreme importance of the structure in the formation of matter. Worse, the helix goes under different names in different disciplines, as R. Buckminster Fuller referred to this as the Tetrahelix, while physicists refer to the same structure as the Bernal Spiral.

In fact, the BC – Helix appears as one among a group of similar structures which appear across disciplines and have a variety of uses. Given the isolation of one academic discipline to another, it appears that the BC – Helix crosses these artificial boundaries and wears many faces.

Insofar as the Qi Men Dun Jia Model is concerned, it appears that matter, once having passed through the Poincare Dodecahedral Space, may pass through either the BC – Helix or the Polytope (3,3,5), while there may exist additional similar structures. The Polytope (3,3,5) appears to produce matter of the 9 x 9 Raja state, while the BC – Helix appears to produce both Satva 8 x 8 and 9 x 9 Raja states of matter, interwoven upon the same structure. Other candidates include the y-brass and the bcc, which the author will explore in future papers.

This paper attempts to assemble and display the known, relevant information concerning the Boerdijk – Coxeter Helix for the purpose of conducting an autopsy – analysis, to illuminate and elucidate its parts, to demonstrate its workings and to show how it fits into the QMDJ Model.

Sedenions, Twisted Octonions and Octonions

Sir Roger Penrose has described them as “the lost cause” of physics, but the Octonions enter the structure of the Qi Men Dun Jia Model at this point, along with the Twisted Octonions and the Sedenions. At the same time, the model moves from Binary or Yin – Yang numbers towards ternary quadratic equations, since it proves necessary to engage the function of triples in the transition from Octonions to Sedenions.

From the work of Sultan Catto and Donald Chesley, we learn about the subtleties of the Octonion – Sedenion connection:

Preliminary Classification of Sedenion Types

Testing each of the 2^{35} values of signmask in the XOR-based multiplication tables and analyzing the associators $(e_ie_j)e_k - e_i(e_je_k)$ shows that there are 9 broad classes of sedenions, classified by the nature of the heptads: of the 15 heptads, anywhere from 0 to 8 are true octonions, with the balance being twisted. Below, counts[N] shows how many signmask values give N true octonionic heptads in the corresponding multiplication table:

```
counts[0] = 4699455488
counts[1] = 9688596480
counts[2] = 10254827520
counts[3] = 6041190400
counts[4] = 2582200320
counts[5] = 817152000
counts[6] = 248299520
counts[7] = 25804800
counts[8] = 2211840
counts[9] = 0
```

Adding these up gives 2^{35}, establishing the fact that (at least for representations derived *via* permutation from the XOR-based multiplication tables) all sedenion types must include at least 7 twisted octonion subalgebras.

2.2 Refinements in Classification

When embedded in the sedenions, heptads have more subtle properties than simply whether they are twisted or not, and a more refined classification of the sedenion types must take this into account. Each twisted heptad has a distinguished triad, and that triad occurs in 2 other heptads as well, which might be untwisted, or twisted with a different distinguished triad, or twisted with the same distinguished triad. Any heptad has 7 triads - how many of them are distinguished in some other heptad? A partial analysis based on these questions has so far revealed more than 52 types of sedenions. Several different types will emerge in constructions presented below. Moreover, any quaternionic groupings not based on permutations of XOR indices would (if they exist) add whole new families of sedenion types to the classification scheme. Although classification is still an ongoing process and the inventory is far from complete, enough is known to inform observations on properties like zero divisors, *etc.* in the following sections.

2.3 Zero Divisors in Sedenions

Since all sedenion multiplication tables contain at least some twisted octonion multiplication tables embedded in them, there are at least the zero divisors already described for the twisted octonions. Moreover, sharing of distinguished triads can induce complicated interactions among the sets of zero divisors induced by different heptads. A general survey awaits completion of the classification scheme sketched above.

23 144: (ac - bd*, ad + c*b)

Twisted octonions XOR/10

	e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_0	e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_1	e_1	$-e_0$	e_3	$-e_2$	e_5	$-e_4$	e_7	$-e_6$
e_2	e_2	$-e_3$	$-e_0$	e_1	e_6	$-e_7$	$-e_4$	e_5
e_3	e_3	e_2	$-e_1$	$-e_0$	e_7	e_6	$-e_5$	$-e_4$
e_4	e_4	$-e_5$	$-e_6$	$-e_7$	$-e_0$	e_1	e_2	e_3
e_5	e_5	e_4	e_7	$-e_6$	$-e_1$	$-e_0$	e_3	$-e_2$
e_6	e_6	$-e_7$	e_4	e_5	$-e_2$	$-e_3$	$-e_0$	e_1
e_7	e_7	e_6	$-e_5$	e_4	$-e_3$	e_2	$-e_1$	$-e_0$

Wikipedia

The **Boerdijk–Coxeter helix**, named after H. S. M. Coxeter and A. H. Boerdijk, is a linear stacking of regular tetrahedra, arranged so that the edges of the complex that belong to a single tetrahedron form three intertwined helices. There are two chiral forms, with either clockwise or counterclockwise windings. Contrary to any other stacking of Platonic solids, the Boerdijk–Coxeter helix is not rotationally repetitive. Even in an infinite string of stacked tetrahedra, no two tetrahedra will have the same orientation. This is because the helical pitch per cell is not a rational fraction of the circle. Buckminster Fuller named it a *tetrahelix* and considered them with regular and irregular tetrahedral elements.[1]

Higher dimensional geometry

The 600-cell partitions into 20 rings of 30 tetrahedra, each a *Boerdijk–Coxeter helix*. When superimposed onto the 3-sphere curvature it becomes periodic, with a period of ten vertices, encompassing all 30 cells. The collective of such helices in the 600-cell represent a discrete Hopf fibration. While in 3 dimensions the edges are helices, in the imposed 3-sphere topology they are geodesics and have no torsion. They spiral around each other naturally due to the Hopf fibration.

Octahedron / Wolfram

Octahedron

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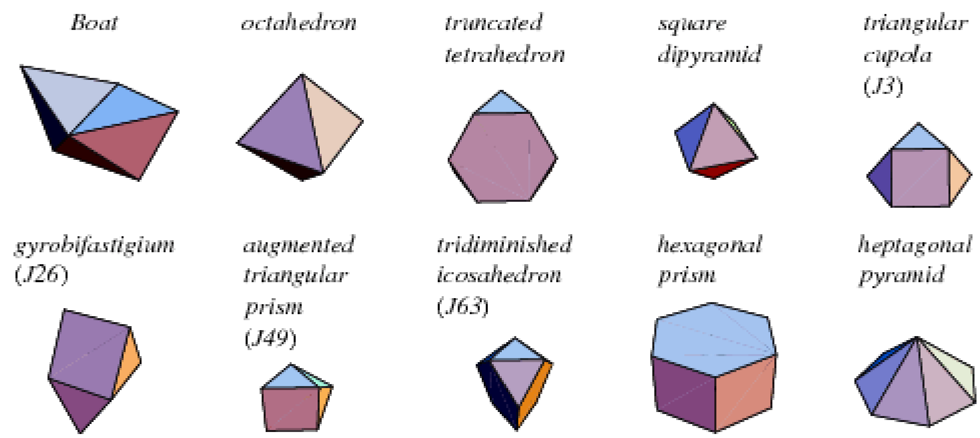
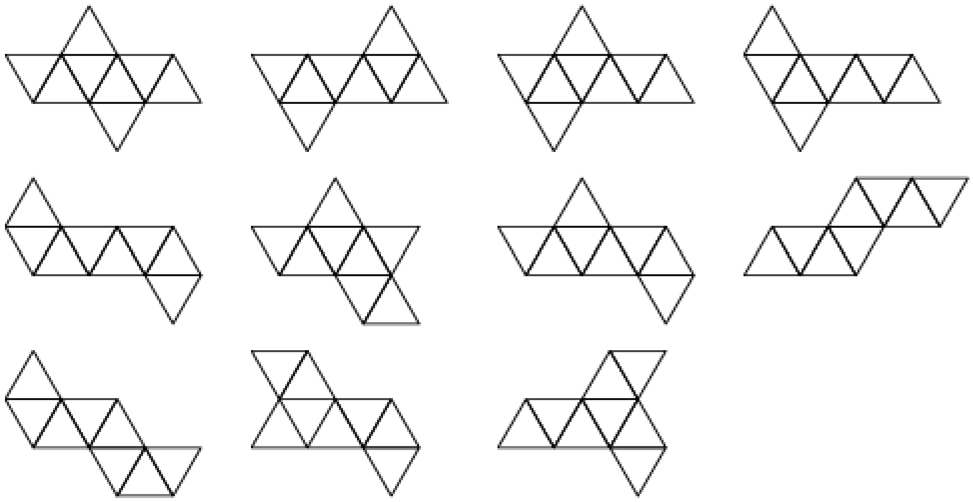


An octahedron is a polyhedron having eight faces. Examples include the augmented triangular prism (Johnson solid J_{49}), boat, gyrobifastigium (Johnson solid J_{26}), heptagonal pyramid, hexagonal prism, (regular) octahedron, square dipyramid, triangular cupola (Johnson solid J_3), tridiminished icosahedron (Johnson solid J_{63}), and truncated tetrahedron.

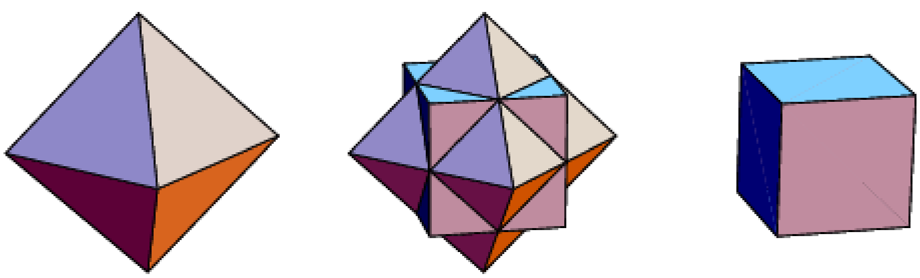
There are 257 convex octahedra, corresponding to the duals of the octahedral graphs. The convex octahedra consisting of regular polygonal faces of equal edge lengths are summarized in the following table. They all have $V - E = 6$, as required by the polyhedral formula.

polyhedron	degree sequence	V	E
truncated tetrahedron	3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3	12	18
heptagonal pyramid	3, 3, 3, 3, 3, 3, 3, 7	8	14
triangular cupola	3, 3, 3, 3, 3, 3, 4, 4, 4	9	15
tridiminished icosahedron	3, 3, 3, 3, 3, 3, 4, 4, 4	9	15
gyrobifastigium	3, 3, 3, 3, 4, 4, 4, 4	8	14
augmented triangular prism	3, 3, 4, 4, 4, 4, 4	7	13
octahedron	4, 4, 4, 4, 4, 4	6	12

The regular octahedron is the Platonic solid P_4 with six polyhedron vertices, 12 polyhedron edges, and eight equivalent equilateral triangular faces, denoted $8\{3\}$. It is also uniform polyhedron U_5 and Wenninger model W_2. It is given by the Schläfli symbol $\{3,4\}$ and Wythoff symbol $4\,|\,2\,3$. The octahedron of unit side length is the antiprism of $n=3$ sides with height $h=\sqrt{6}/3$. The octahedron is also a square dipyramid with equal edge lengths.



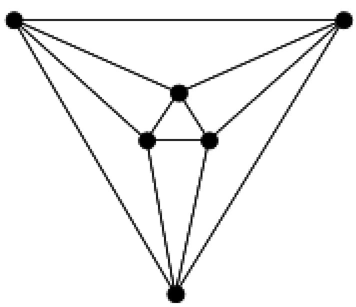
There are 11 distinct nets for the octahedron, the same as for the cube (Buekenhout and Parker 1998). Questions of polyhedron coloring of the octahedron can be addressed using the Pólya enumeration theorem.



The dual polyhedron of an octahedron with unit edge lengths is a cube with edge lengths $1/\sqrt{2}$.

The illustration above shows an origami octahedron constructed from a single sheet of paper (Kasahara and Takahama 1987, pp. 60-61).

Like the cube, it has the O_h octahedral group of symmetries.



The connectivity of the vertices is given by the octahedral graph.

Wikipedia

In geometry, an **octahedron** (plural: octahedra) is a polyhedron with eight faces. A **regular** octahedron is a Platonic solid composed of eight equilateral triangles, four of which meet at each vertex.

An octahedron is the three-dimensional case of the more general concept of a cross polytope.

Dimensions

If the edge length of a regular octahedron is *a*, the radius of a circumscribed sphere (one that touches the octahedron at all vertices) is

$$r_u = \frac{a}{2}\sqrt{2} \approx 0.7071067 \cdot a$$

and the radius of an inscribed sphere (tangent to each of the octahedron's faces) is

$$r_i = \frac{a}{6}\sqrt{6} \approx 0.4082482 \cdot a$$

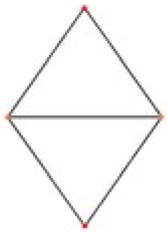
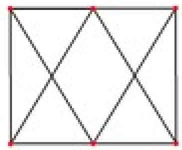
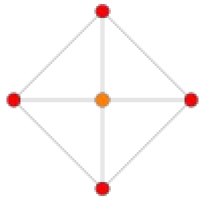
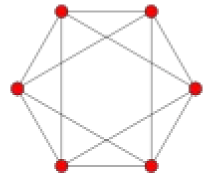
while the midradius, which touches the middle of each edge, is

$$r_m = \frac{a}{2} = 0.5 \cdot a$$

Orthogonal projections

The *octahedron* has four special orthogonal projections, centered, on an edge, vertex, face, and normal to a face. The second and third correspond to the B_2 and A_2 Coxeter planes.

Orthogonal projections

Centered by	Edge	Face Normal	Vertex	Face
Image				
Projective symmetry	[2]	[2]	[4]	[6]

Cartesian coordinates

An octahedron with edge length sqrt(2) can be placed with its center at the origin and its vertices on the coordinate axes; the Cartesian coordinates of the vertices are then

(±1, 0, 0);
(0, ±1, 0);
(0, 0, ±1).

In an $x - y - z$ Cartesian coordinate system, the octahedron with center coordinates $(a, \ b, \ c)$ and radius r is the set of all points $(x, \ y, \ z)$ such that

$$|x-a| + |y-b| + |z-c| = r.$$

Area and volume

The surface area A and the volume V of a regular octahedron of edge length a are:

$$A = 2\sqrt{3}a^2 \approx 3.46410162a^2$$
$$V = \frac{1}{3}\sqrt{2}a^3 \approx 0.471404521a^3$$

Thus the volume is four times that of a regular tetrahedron with the same edge length, while the surface area is twice (because we have 8 vs. 4 triangles).

If an octahedron has been stretched so that it obeys the equation:

$$\left|\frac{x}{x_m}\right| + \left|\frac{y}{y_m}\right| + \left|\frac{z}{z_m}\right| = 1$$

The formula for the surface area and volume expand to become:

$$A = 4\, x_m\, y_m\, z_m \times \sqrt{\frac{1}{x_m^2} + \frac{1}{y_m^2} + \frac{1}{z_m^2}}$$
$$V = \frac{4}{3}\, x_m\, y_m\, z_m$$

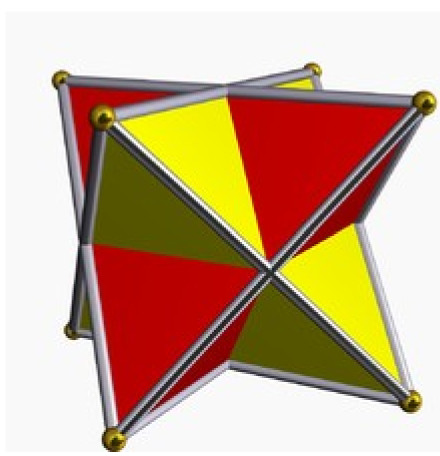
Additionally the inertia tensor of the stretched octahedron is:

$$I = \begin{bmatrix} \frac{1}{10}m(y_m^2 + z_m^2) & 0 & 0 \\ 0 & \frac{1}{10}m(x_m^2 + z_m^2) & 0 \\ 0 & 0 & \frac{1}{10}m(x_m^2 + y_m^2) \end{bmatrix}$$

These reduce to the equations for the regular octahedron when:

$$x_m = y_m = z_m = a\,\frac{\sqrt{2}}{2}$$

Geometric relations



The octahedron represents the central intersection of two tetrahedra

The interior of the compound of two dual tetrahedra is an octahedron, and this compound, called the stella octangula, is its first and only stellation. Correspondingly, a regular octahedron is the result of cutting off from a regular tetrahedron, four regular tetrahedra of half the linear size (i.e. rectifying the tetrahedron). The vertices of the octahedron lie at the midpoints of the edges of the tetrahedron, and in this sense it relates to the tetrahedron in the same way that the cuboctahedron and icosidodecahedron relate to the other Platonic solids. One can also divide the edges of an octahedron in the ratio of the golden mean to define the vertices of an icosahedron. This is done by first placing vectors along the octahedron's edges such that each face is bounded by a cycle, then similarly partitioning each edge into the golden mean along the direction of its vector. There are five octahedra that define any given icosahedron in this fashion, and together they define a *regular compound*.

Octahedra and tetrahedra can be alternated to form a vertex, edge, and face-uniform tessellation of space, called the octet truss by Buckminster Fuller. This is the only such tiling save the regular tessellation of cubes, and is one of the 28 convex uniform honeycombs. Another is a tessellation of octahedra and cuboctahedra.

The octahedron is unique among the Platonic solids in having an even number of faces meeting at each vertex. Consequently, it is the only member of that group to possess mirror planes that do not pass through any of the faces.

Using the standard nomenclature for Johnson solids, an octahedron would be called a *square bipyramid*. Truncation of two opposite vertices results in a square bifrustum.

The octahedron is 4-connected, meaning that it takes the removal of four vertices to disconnect the remaining vertices. It is one of only four 4-connected simplicial well-covered polyhedra, meaning that all of the maximal independent sets of its vertices have the same size. The other three polyhedra with this property are the pentagonal dipyramid, the snub disphenoid, and an irregular polyhedron with 12 vertices and 20 triangular faces.[1]

Uniform colorings and symmetry

There are 3 uniform colorings of the octahedron, named by the triangular face colors going around each vertex: 1212, 1112, 1111.

The octahedron's symmetry group is O_h, of order 48, the three dimensional hyperoctahedral group. This group's subgroups include D_{3d} (order 12), the symmetry group of a triangular antiprism; D_{4h} (order 16), the symmetry group of a square bipyramid; and T_d (order 24), the symmetry group of a rectified tetrahedron. These symmetries can be emphasized by different colorings of the faces.

Irregular octahedra

The following polyhedra are combinatorially equivalent to the regular polyhedron. They all have six vertices, eight triangular faces, and twelve edges that correspond one-for-one with the features of a regular octahedron.

- *Triangular antiprisms*: Two faces are equilateral, lie on parallel planes, and have a common axis of symmetry. The other six triangles are isosceles.
- Tetragonal bipyramids, in which at least one of the equatorial quadrilaterals lies on a plane. The regular octahedron is a special case in which all three quadrilaterals are planar squares.
- Schönhardt polyhedron, a nonconvex polyhedron that cannot be partitioned into tetrahedra without introducing new vertices.

More generally, an octahedron can be any polyhedron with eight faces. The regular octahedron has 6 vertices and 12 edges, the minimum for an octahedron; nonregular octahedra may have as many as 12 vertices and 18 edges.[1] Other nonregular octahedra include the following:

- Hexagonal prism: Two faces are parallel regular hexagons; six squares link corresponding pairs of hexagon edges.
- Heptagonal pyramid: One face is a heptagon (usually regular), and the remaining seven faces are triangles (usually isosceles). It is not possible for all triangular faces to be equilateral.
- Truncated tetrahedron: The four faces from the tetrahedron are truncated to become regular hexagons, and there are four more equilateral triangle faces where each tetrahedron vertex was truncated.
- Tetragonal trapezohedron: The eight faces are congruent kites.

R. Buckminster Fuller and the Tetrahelix

Fuller gave quite explicit and far – seeing description of the Tetrahelix, surmising the existence of two states of matter – the 8 x 8 Satva and the 9 x 9 Raja states, both of which enter into the composition of the Tetrahelix. In this case, Fuller must have deduced the existence of two states of matter from qualities of the Tetrahelix.

Moreover, Fuller goes into great detail about the relationship between the Tetrahelix and the DNA helix. The author shall present an additional paper on this theme, but as shall shortly be seen, Fuller drew these connections many decades ago.

In his Sinergetics, Fuller suggests the definition of the Tetrahelix:

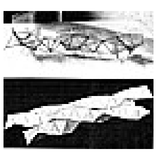


Fig. 933.01

933.00 **Tetrahelix**

933.01 The tetrahelix is a helical array of triple-bonded tetrahedra. (See Illus. 933.01) We have a column of tetrahedra with straight edges, but when face-bonded to one another, and the tetrahedra's edges are interconnected, they altogether form a hyperbolic-parabolic, helical column. The column spirals around to make the helix, and it takes just ten tetrahedra to complete one cycle of the helix.

In this section, Fuller discusses the binary aspects of the Tetrahelix, which implies Yin and Yang qualities:

933.07 When we address two or more positive or two or more negative tetrahelixes together, the positives nestle their angling forms into one another, as the negatives nestle likewise into one another's forms.

The next section indicates the importance of carbon and its relationship to the Tetrahelix and DNA, with an eye to the formation of life:

931.61 The closest-packing concept was developed in respect to spherical aggregates with the convex and concave octahedra and vector equilibria spaces between the spheres. Spherical closest packing overlooks a much closer packed condition of energy structures, which, however, had been comprehended by organic chemistry—that of quadrivalent and fourfold bonding, which corresponds to outright congruence of the octahedra or tetrahedra themselves. When carbon transforms from its soft, pressed-cake, carbon black powder (or charcoal) arrangement to its diamond arrangement, it converts from the so-called closest arrangement of triple bonding to quadrivalence. We call this self-congruence packing, as a single tetrahedron arrangement in contradistinction to closest packing as a neighboring-group arrangement of spheres.

932.01 The four chemical compounds guanine, cytosine, thymine, and adenine, whose first letters are GCTA, and of which DNA always consists in various paired code pattern sequences, such as GC, GC, CG, AT, TA, GC, in which A and T are always paired as are G and C. The pattern controls effected by DNA in all biological structures can be demonstrated by equivalent variations of the four individually unique spherical radii of two unique pairs of spheres which may be centered in any variation of series that will result in the viral steerability of the shaping of the DNA tetrahelix prototypes. (See Sec. 1050.00 et. seq.)

In the following section, Fuller describes the "ribbon" of a wave which determines the connections between the tetrahedral and the helix. While Fuller fails to specify a source for this wave, the author theorizes that the wave originates with a spiral created from the Magic Square at the center of the Clifford Clock and the Clock of Complex Spaces. Just has H.L. Coxeter theorized that a polyhedra emanates from rotations about a single point in space, so does Fuller's "ribbon" emanate from the Magic Square, the most likely candidate for which is the Svas Tika (swastika) Magic Square, which determines a revolving spiral – arm movement.

The 60 – degree pattern appears to fit nicely with the Fano Plane and the Octonions, which the author posits as a key element of the mathematics of the Tetrahelix. The ribbon works with the octahedra, the icosahedra and the tetrahedra.

930.11 Exploring the multi-ramifications of spontaneously regenerative re-angulations and triangulations, we introduce upon a continuous ribbon a 60-degree-patterned, progressively alternating, angular bounce-off inwards from first one side and then the other side of the ribbon, which produces a wave pattern whose length is the interval along any one side between successive bounce-offs which, being at 60 degrees in this case, produces a series of equiangular triangles along the strip.

As seen from one side, the equiangular triangles are alternately oriented as *peak away,* then *base away,* then *peak away* again, etc. This is the patterning of the only equilibrious, never realized, angular field state, in contradistinction to its sine-curve wave, periodic realizations of progressively accumulative, disequilibrious aberrations, whose peaks and valleys may also be patterned between the same length wave intervals along the sides of the ribbon as that of the equilibrious periodicity. (See Illus. 930.11.)

930.20 **Pattern Strips Aggregate Wrapabilities:** The equilibrious state's 60- degree rise-and-fall lines may also become successive cross-ribbon fold-lines, which, when successively partially folded, will produce alternatively a tetrahedral- or an octahedral- or an icosahedral-shaped spool or reel upon which to roll-mount itself repeatedly: the tetrahedral spool having four successive equiangular triangular facets around its equatorial girth, with no additional triangles at its polar extremities; while in the case of the octahedral reel, it wraps closed only six of the eight triangular facets of the octahedron, which six lie around the octahedron's equatorial girth with two additional triangles left unwrapped, one each triangularly surrounding each of its poles; while in the case of the icosahedron, the equiangle-triangulated and folded ribbon wraps up only 10 of the icosahedron's 20 triangles, those

10 being the 10 that lie around the icosahedron's equatorial girth, leaving five triangles uncovered around each of its polar vertexes. (See Illus. 930.20.)

The next sections give details about the ribbon:

930.23 The tetrahedron requires only *one* wrap-up ribbon; the octahedron *two*; and the icosahedron *three,* to cover all their respective numbers of triangular facets.

930.24 If each of the ribbon-strips used to wrap-up, completely and symmetrically, the tetra, octa, and icosa, consists of transparent tape; and those tapes have been divided by a set of equidistantly interspaced lines running parallel to the ribbon's edges; and three of these ribbons wrap the tetrahedron, six wrap the octahedron, and nine the icosahedron; then all the four equiangular triangular facets of the tetrahedron, eight of the octahedron, and 20 of the icosahedron, will be seen to be symmetrically subdivided into smaller equi-angle triangles whose total number will be N^2, the second power of the number of spaces between the ribbon's parallel lines.

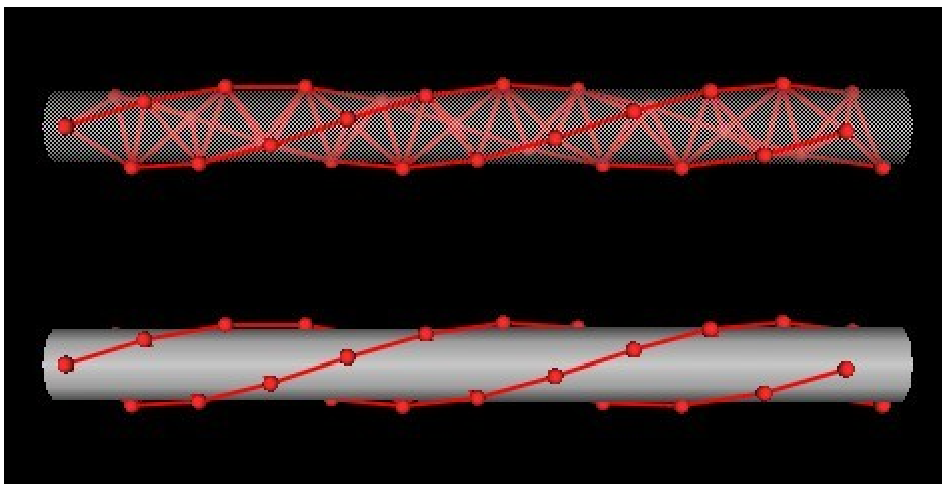
R. Gray

TETRAHELIX DATA

The tetrahelix of Fuller's Synergetics consists of face bond regular tetrahedra. The mathematics for this spiraling structure is quit einteresting. Despite the tetrahelix composition of regular tetrahedra (the "simplest" polyhedron), I have not been able to find a simple way to calculate the information for the tetrahelix.

From the Zheng paper (see References below): "The tetrahedral helix is called the 'Bernal spiral' in association with discussions of liquid structures in the physics literature."

The vertices of the regular tetrahedra of the tetrahelix all lay on the surface of a cylinder. Let us visualize this cylinder lying along the z-axis.



The radius of the cylinder:

$$r = (3 \text{ sqrt}(3) / 10) \text{ EL}$$

where EL is the edge length of the tetrahedra used to build the tetrahelix.

Let us put a vertex (call it V0) of one of the tetrahedra on the x-axis. That is

$$V0 = (r, 0, 0)$$

Then the next vertex of the tetrahelix (V1) will be at the coordinates

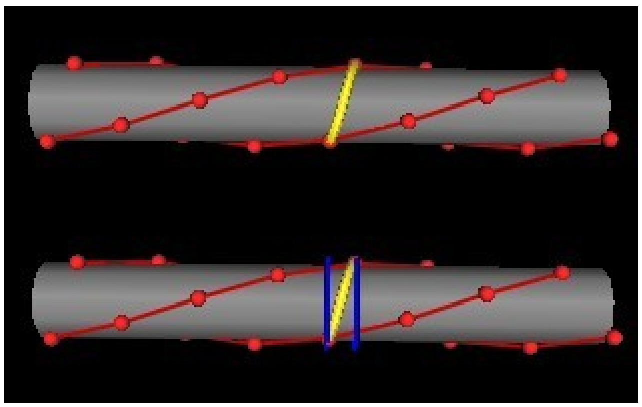
$$V1 = (r \cos(theta), r \sin(theta), h)$$

where theta is the angle around the z-axis and is given by

theta = arccos(-2/3) (approximately 131.8103149 degrees)

and where h is the distance in the z-axis direction and is given by

$$h = (1/\mathrm{sqrt}(10))\ EL$$



In the above figures, the yellow band connects a vertex to the "next" vertex, while the distance h is the distance between the 2 blue bands around the cylinder.

In general, the coordinates for the vertices of a **Counter Clockwise** tetrahelix Vn (n = 0, 1, 2, 3, ...) are given by

$$Vn = (r \cos(n*theta), r \sin(n*theta), n*h)$$

The coordinates for the vertices of a **Clockwise** tetrahelix

Vn (n = 0, 1, 2, 3, ...) are given by

$$Vn = (r \cos(n*theta), - r \sin(n*theta), n*h)$$

Note that cos(theta) = -2/3 and that sin(theta) = sqrt(5)/3. You can calculate exact expressions for the vertex coordinates by using these relations together with the following trig identities:

cos(n*theta) = cos(theta)*cos((n-1)*theta) - sin(theta)*sin((n-1)*theta)

sin(n*theta) = sin(theta)*cos((n-1)*theta) + cos(theta)*sin((n-1)*theta)

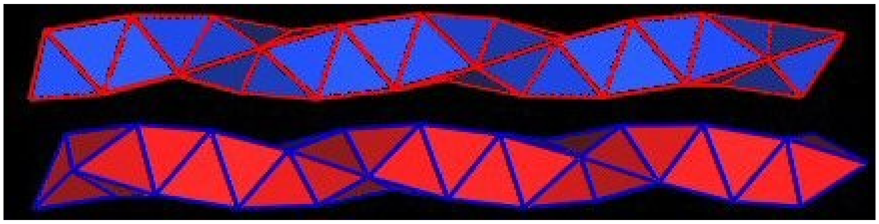
One of the reasons that deriving the above information is difficult is that the axis of symmetry of the cylinder (the axis through the center of the cylinder) does not pass through the center of volume of the tetrahedra. The distance from the z-axis to the tetrahedron center of volume is given by the equation

dist. = (sqrt(2)/10) EL

Therefore, all the Tetrahelix cylinder axis of symmetry pass tangentially by a sphere of radius (sqrt(2)/10)EL centered at the Tetrahedron's center of volume.

Comments

- Tetrahelix come in 2 orientations; a right-handed spiral and a left-handed spiral.



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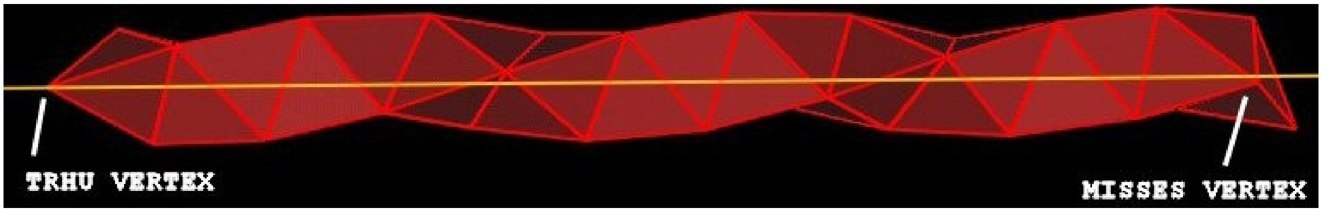
- The vertices of the tetrahelix never line up. That is, no two vertices will ever rest directly above one another, since the theta angle above is an irrational number.
-

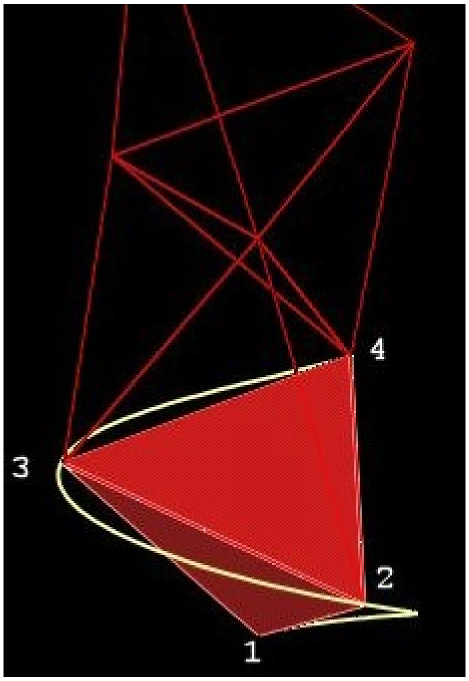


-
- It may prove possible to "nest" 3 additional tetrahelix about the original tetrahelix in a tight bundle, yet this proves impossible since that face binding 5 tetrahedra together about a single edge/axis leaves a gap (which Fuller calls the unzipping angle.) The dihedral angle of the regular tetrahedron is
-

  D-tetra = arccos(1/3) (approximately 70.528779 degrees)

- 5 times this amount is approx 352.643895 degrees, which is approx 7.356105 degrees (the unzipping angle) short of 360 degrees. So the tetrahedra of multiple helixes can not pack together without gaps.

- Twelve (12) possible Tetrahelix pass through a single Tetrahedron: 6 Clockwise and 6 Counter clockwise. As the figure shows, the spiral/coil associated with a Tetrahelix going through a Tetrahedron passes through all 4 vertices of the Tetrahedron sequentially. Since the spiral has the same symmetry axes as the Tetrahelix, we can count the number of possible different spirals to count the number of Tetrahelix.
-



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- We label the Tetrahedron's vertices by numbering them 1, 2, 3, and 4. Then that equals 4x3x2x1 = 24 combinations for the order in which the spiral can pass through the vertices. However, 1/2 of these are simply reversals: (1,2,3,4) has the same spiral symmetry axis as the spiral (4,3,2,1). That leaves 12 spirals/Tetrahelix.

  For more details on the way 12 Tetrahelix axes pass through a single Tetrahedron, see the next web page.

There is a Clockwise and a Counterclockwise spiral associated with every Tetrahelix.

Note that the spacing between adjacent loops in the spiral is different for the Counterclockwise spiral versus the Clockwise spiral.

In both cases, the spirals sequentially pass through all the vertices in a Tetrahedron.

In both cases, the spiral travels a distance along the z-axis by an amount:

h = (1/sqrt(10)) EL

But in one case, the spiral travels around the Tetrahelix by an angular amount of

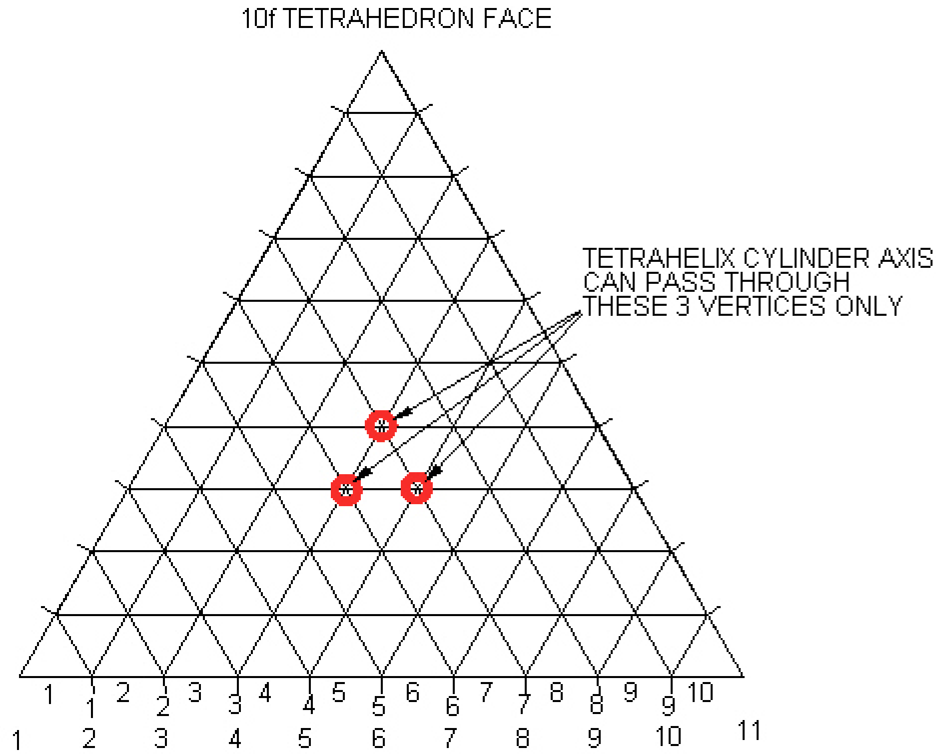
theta = arccos(-2/3) (approximately 131.8103149 degrees)

and in the other case, the spiral travels around the Tetrahelix by an angular amount of

alpha = 360 - arccos(-2/3) (approximately 228.1896851 degrees)

- (My notes on this (dated 1997) suggests that I am not the originator of this. Unfortunately, they do not indicate who suggested this originally.)

  If the tetrahedra used to build the tetrahelix are 10-frequency (have 11 vertices per edge) then the axis of the surrounding tetrahelix cylinder will always pass through the tetrahedra faces at an inner triangular face vertex. The "triangular face coordinate" which the symmetry axis passes through is (7,3).



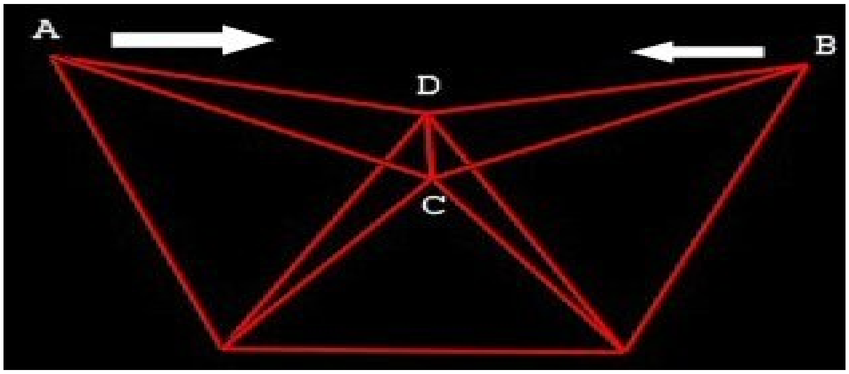
- For a finite length Tetrahelix, all of the Tetrahelix's vertices can be given rational (x, y, z) coordinates. Then by scaling, the coordinates' x, y, z, components can be made to be integers.

- I have found that if you allow the vertices to be flexible and allow some of the tetrahedra edges to expand in length, the tetrahelix can fold up into another (shorter) tetrahelix. When it does, it passes through an Octahedron phase.
-

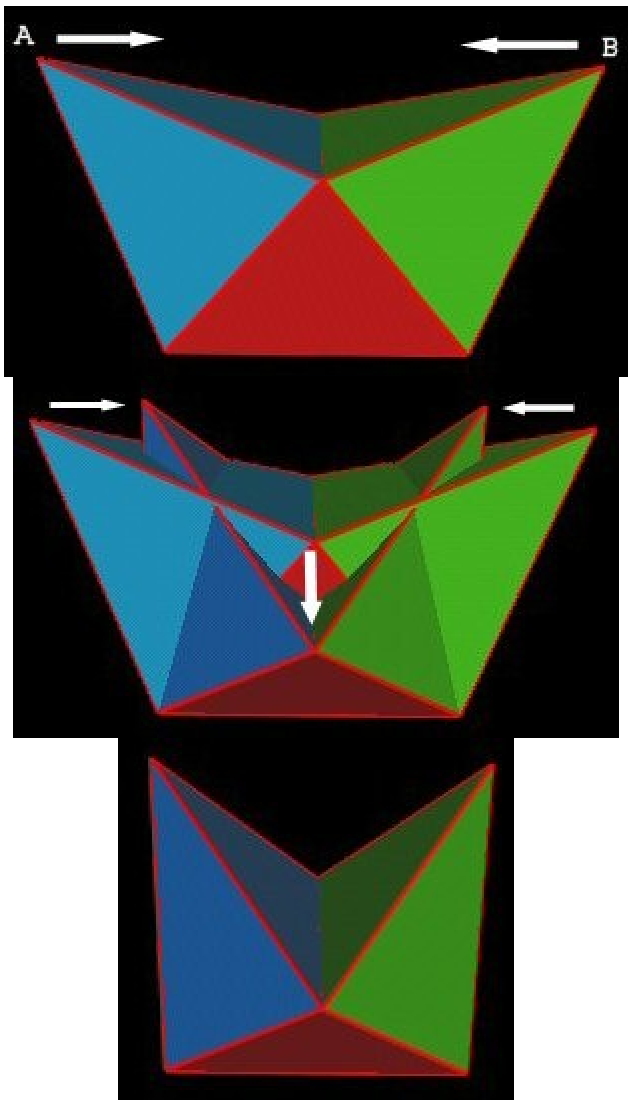
The following sequence of images helpd one visualize the transformation.

First, consider the Tetrahelix as consisting of a number of 3-Tetrahedra units. By folding up each of these units, the Tetrahelix is reduced.

We can fold up a 3-Tetra unit as shown in the following figures.

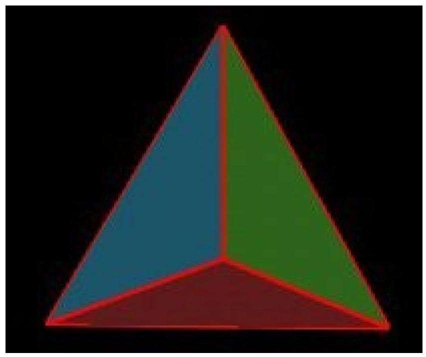


Not that only the C-to-D edge needs to change its length. All other edges of the Tetrahedra remain the same.



At this point, the 3 Tetrahedra are very close to defining an Octahedron. They actually do form an Octahedron as the vertices "A" and "B" are brought closer togther.

When vertices "A" and "B" are brought together, a double Tetrahedron is formed. That is two Tetrahedra face bound togther. (This figure only shows one of the 2 Tetrahedra. The other is hidden behind this one.)



The transformation from the 3 Tetrahedra to the Octahedron Fuller calls the "Richter Transformation". See Color plate 6 in Fuller's book Synergetics. I am not aware that Fuller continued the transformation described here to transform a Tetrahelix.

-

- Joe Matto suggested to me (October, 2004) that 2 Tetrahelix can intersect each other at 90 degrees. If we define "intersect" to mean that the axis of symmetry of 2 Tetrahelix pass through a common Tetrahedron and pass through a common point at 90 degrees, then this statement is not true. The 2 Tetrahelix "intersect" each other at
-

2 arcsin(1/sqrt(10)) = 36.86989765... degrees.

However, if we define "intersect" to mean that the 2 Tetrahelix pass through a common Tetrahedron and thier symmetry axes pass by each other at 90 degrees, then this is true.

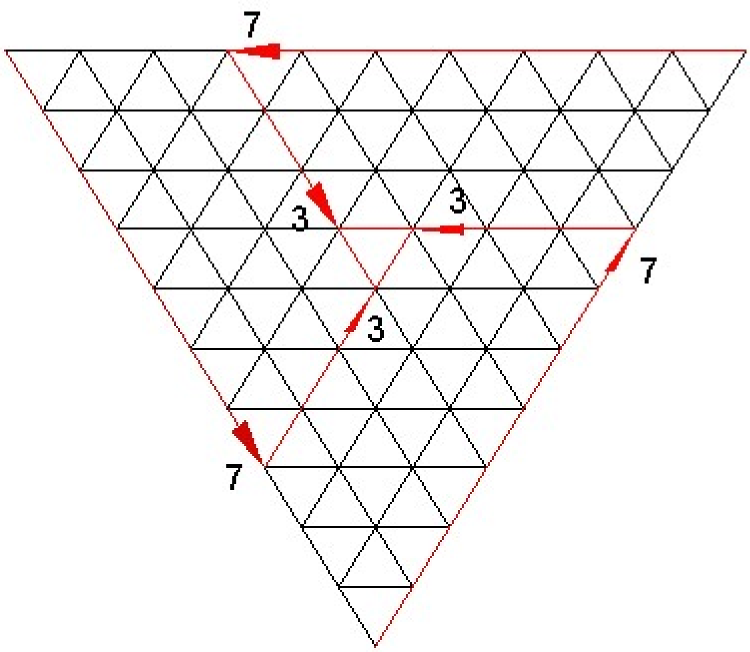
- There are 2 ways to place the end of a Tetrahelix flat on the top of a table such that the Tetrahelix rises above the table. See the calculations here. The angles which the symmetry axis makes with the table top are:

-
  90 - arccos(sqrt(1/15)) = 14.96321744... degrees
- and
  90 - arccos(sqrt(3/5)) = 50.76847952... degrees

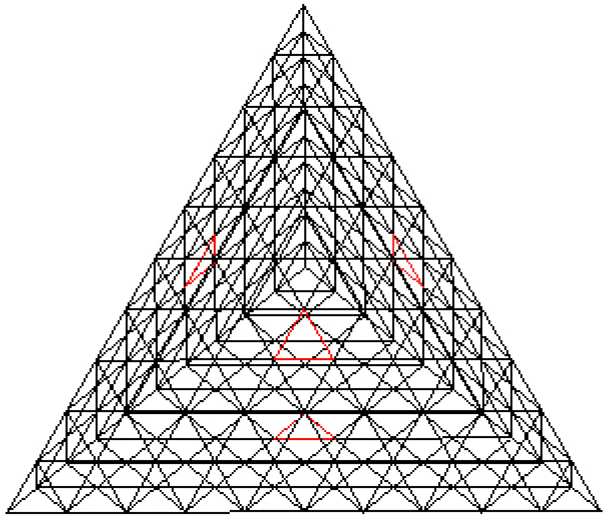
Tetrahelix Axes Passing Thru A Single Tetrahedron

We know that the symmetry axis of a Tetrahelix passing through a Tetrahedron's triangular face must pass through a triangle face coordinate (7, 3). (See this web page for details.)

We highlight the 3 (7,3) triangle face coordinates by drawing a red triangle.

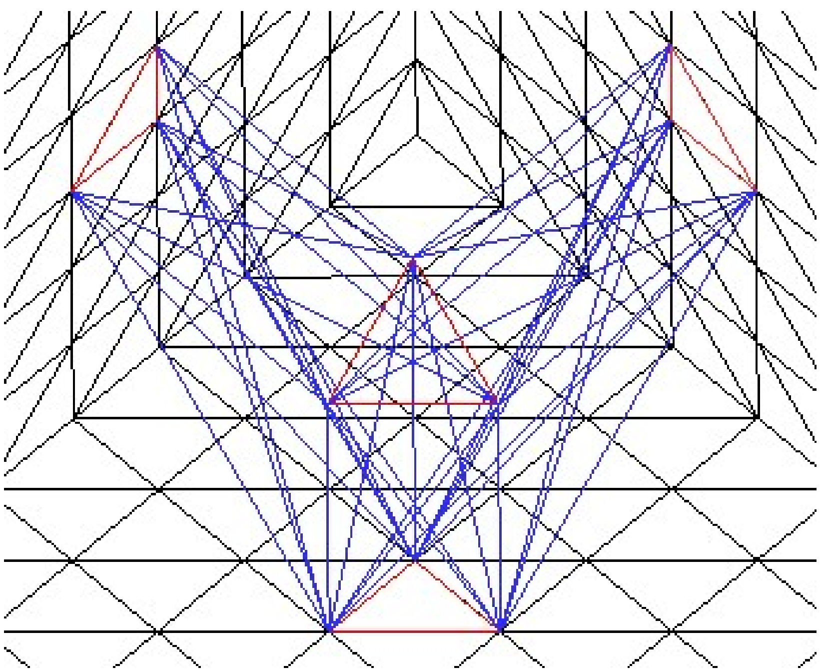


We divide all 4 faces of the Tetrahedron into a 10-frequency grid and draw the (7,3) triangles in red.



We next connect all the vertices of the red triangles to all other red triangle vertices.

Since we know that the Tetrahelix symmetry axis passes into a Tetrahedron and out of a Tetrahedron through only these (7,3) positions, some of these lines must serve as symmetry axes for all possible Tetrahelix passing through this Tetrahedron.



Tetrahelix Axis Passes Thru (7,3) Face Coordinate

We want to prove that the cylinder axis of the Tetrahelix passes through the Tetrahedron's triangle face at triangle face coordinate (7, 3).

The radius of the Tetrahelix cylinder is given by

$$r = \frac{3\sqrt{3}}{10}\,EL$$

where EL is the edge length of the Tetrahedra making up the Tetrahelix.

The Tetrahelix can be positioned so that its (x, y, z) coordinates are given by the equation

where (approximately 131.8103149 degrees) and

$$h = \frac{1}{\sqrt{10}}\,EL$$

Using the equations

it is easy to calculate values for and .

n		
0	1	0
1		$\frac{\sqrt{5}}{3}$
2		$\frac{-4\sqrt{5}}{9}$
3		$\frac{7\sqrt{5}}{27}$

The n-th Terahelix vertex will then have the coordinate

For the “first” 4 vertices, with EL=1, we get

$$V_0 = \left(\frac{3\sqrt{3}}{10}, 0.0, 0.0 \right)$$

$$V_1 = \left(\frac{-6\sqrt{3}}{30}, \frac{\sqrt{15}}{10}, \frac{1}{10} \right)$$

$$V_2 = \left(\frac{-3\sqrt{3}}{90}, \frac{-12\sqrt{15}}{90}, \frac{2}{10} \right)$$

$$V_3 = \left(\frac{66\sqrt{3}}{270}, \frac{21\sqrt{15}}{270}, \frac{3}{10} \right)$$

What (7, 3) means is that we travel 7 (out of 10) units along one edge and then 3 units (out of 10) parallel to another edge of the triangular face.

Define V_a to be the vector from vertex V_0 toward vertex V_1 but which is only 7 (out of 10) units in length. Then define V_b to be the vector from vertex V_1 toward vertex V_2 and which is 3 (out of 10) units in length.

Then the vector to (one of) the face point (7, 3) is given by V0 + Va + Vb. If this vector has no x or y components then it must lay on the z-axis, which is the symmetry axis of the Tetrahelix.

We calculate V_a to be

$$V_a \equiv \frac{7}{10}\left(V_1 - V_0\right) = \frac{7}{10}\left(\frac{-6\sqrt{3}}{30} - \frac{3\sqrt{3}}{10}, \frac{\sqrt{15}}{10}, \frac{1}{10} \right)$$

$$V_a = \left(\frac{-35\sqrt{3}}{100}, \frac{7\sqrt{15}}{100}, \frac{7}{100} \right)$$

And we calculate V_b to be

$$V_b \equiv \frac{3}{10}\left(V_2 - V_1\right) = \frac{3}{10}\left(\frac{-3\sqrt{3}}{90} - \frac{-6\sqrt{3}}{30}, \frac{-12\sqrt{15}}{90} - \frac{\sqrt{15}}{10}, \frac{2}{10} - \frac{1}{10} \right)$$

$$V_b = \left(\frac{45\sqrt{3}}{900}, \frac{-63\sqrt{15}}{900}, \frac{3}{100} \right)$$

Then the vector to the point face coordinate point (7, 3) is given by

$$V_c \equiv V_0 + V_a + V_b = \left(\frac{3\sqrt{3}}{10} + \frac{-35\sqrt{3}}{100} + \frac{45\sqrt{3}}{900}, \frac{7\sqrt{15}}{100} + \frac{-63\sqrt{15}}{900}, \frac{7}{100} + \frac{3}{100} \right)$$

$$V_c = \left(\frac{270\sqrt{3}}{900} + \frac{-315\sqrt{3}}{900} + \frac{45\sqrt{3}}{900}, \frac{63\sqrt{15}}{900} + \frac{-63\sqrt{15}}{900}, \frac{1}{10} \right)$$

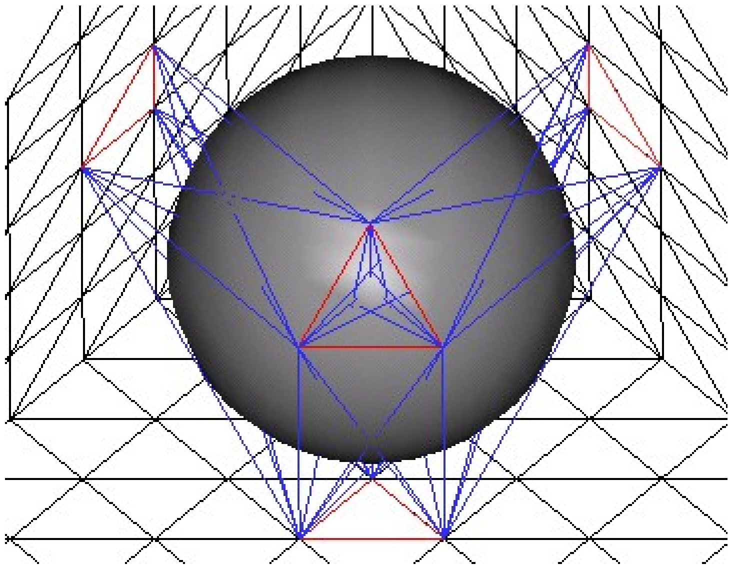
$$V_c = \left(0.0, 0.0, \frac{1}{10} \right)$$

This lays on the z-axis. So the symmetry axis of the Tetrahelix does pass through the triangular face coordinate (7,3).

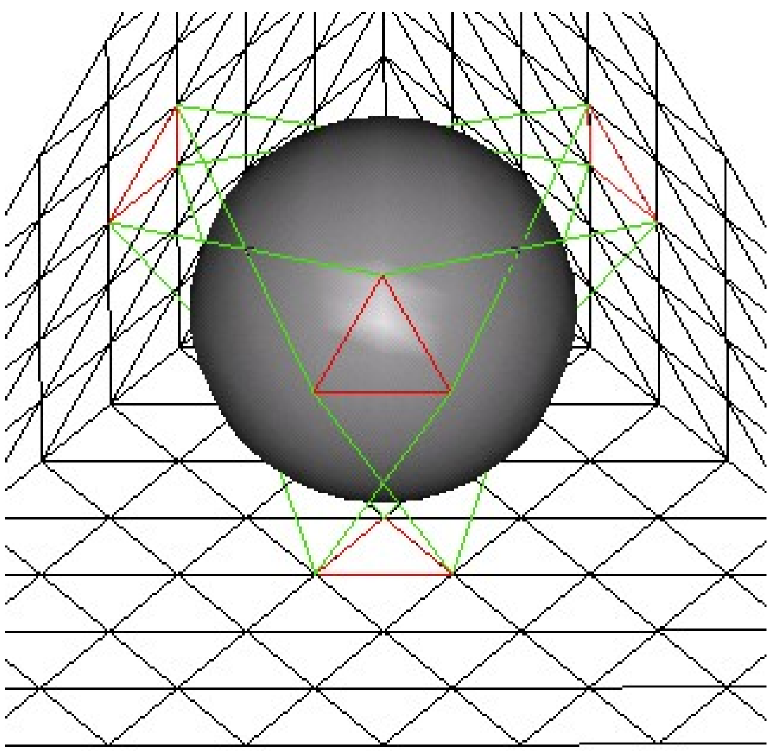
There are 3 such face points depending on which of the 3 vertices of the triangular face is used to begin measuring the 7 (and then 3) units of length.

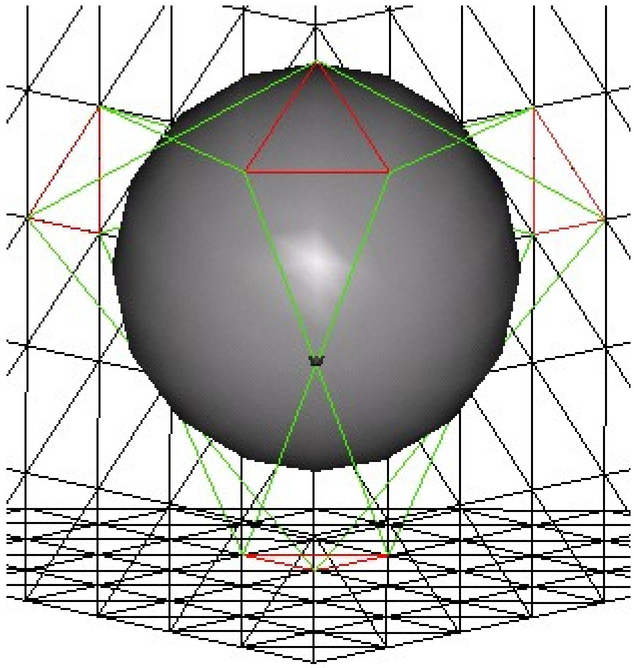
We also know that this symmetry axis passes by the center of volume of the Tetrahedron at a distance of (sqrt(2)/10)EL. (See this web page for details.)

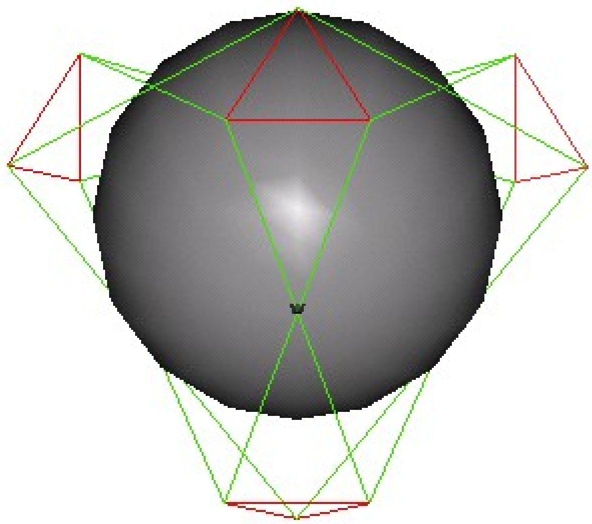
So we place a sphere with this radius value at the center of volume of the Tetrahedron.



The lines which pass the sphere at a tangent point on the sphere will be a symmetry axis for a Tetrahelix. These lines will be shown in green.





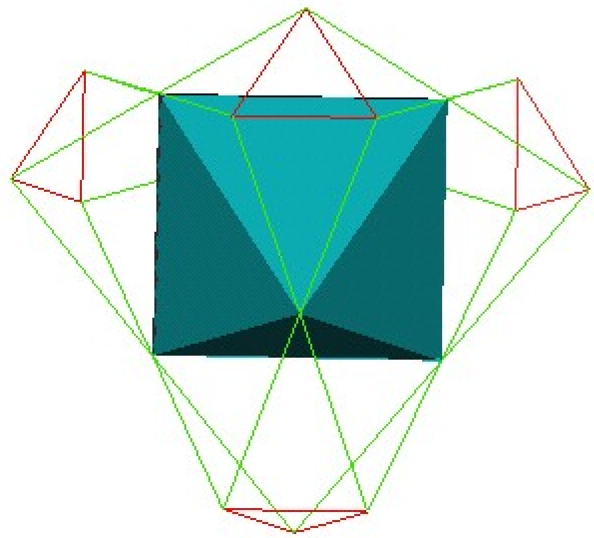


There are 12 green lines, so 12 Tetrahelices pass through a single Tetrahedron. Six of these will have a Clockwise screw sense and 6 will have a Counter Clockwise screw sense.

It would be very interesting to know what breaks the symmetry of screw sense. How is it that one green line gets assigned to a Clockwise screw sense Tetrahelix and another gets assigned to a Counter Clockwise? At this point, one line seems the same as any other line.

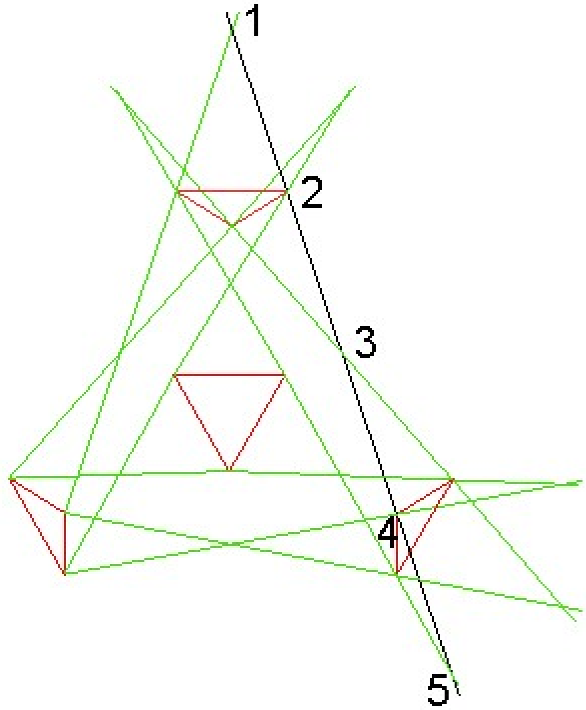
Note that the green lines come in crossing pairs. There are 6 intersection points which define an Octahedron.

The angle of the intersecting green lines gives the angle at which the Tetrahelix intersect each other.



However, it should be noted that the symmetry axis of two Tetrahelix which share the same Tetrahedron do not have to intersect inside the shared Tetrahedron.

As the following figure shows, a Tetrahelix's symmetry axis will intersect 5 other Tetrahelix axis of symmetry and 2 of the intersection points are outside the Tetrahedron.



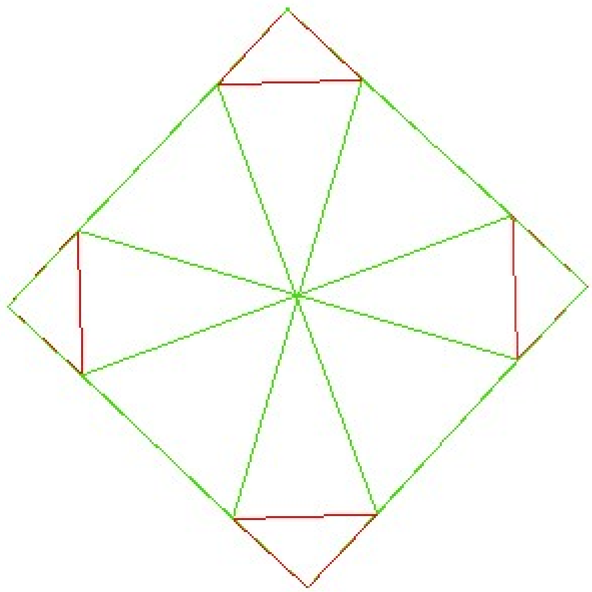
Intersection points 1 and 5 in the above figure are outside the Tetrahedron. Points 2 and 4 are on the face of the Tetrahedron and intersection point 3 is inside the Tetrahedron.

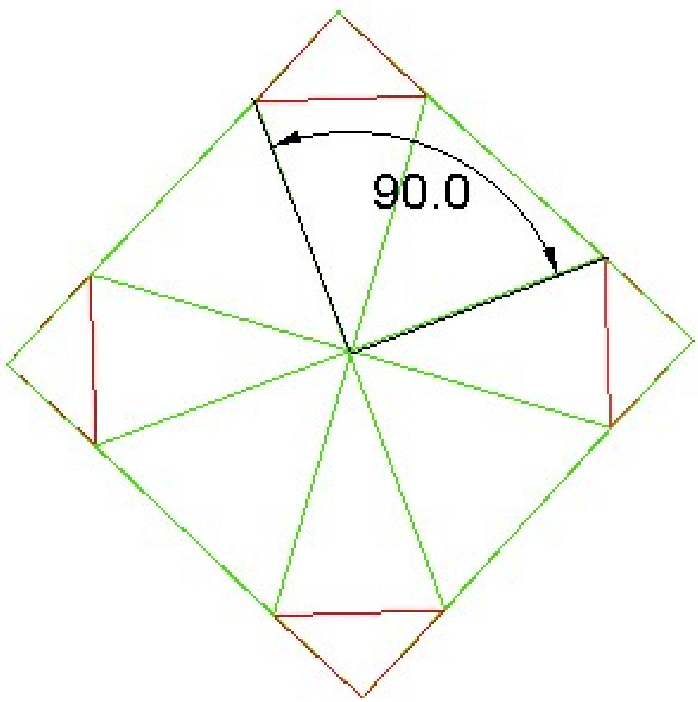
It has been suggested to me by Joe Matto that 2 Tetrahelix can "intersect" each other at 90 degrees. This is not possible if by "intersect" one means that the axis of symmetry of the Tetrahelix intersect at 90 degrees.

However, recall that the opposite edge of a Tetrahedron pass by each other at 90 degrees. That is, the two opposite edges share the same mid-edge point axis and one edge is rotated by 90 degrees about this shared axis with respect to the other edge.

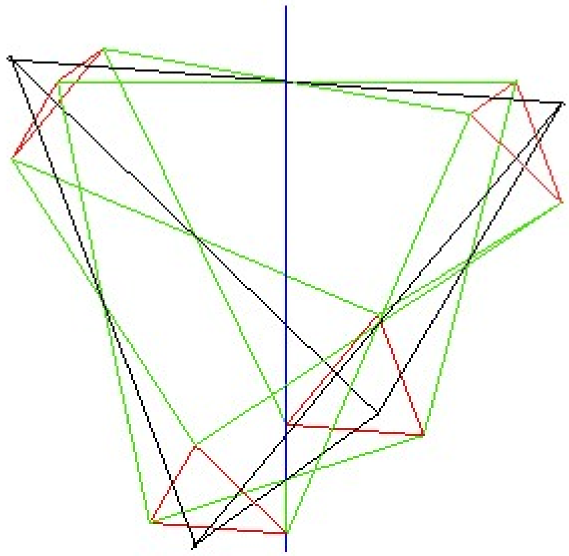
In a similar way, the axis of symmetry of 2 Tetrahelix can pass by each other at 90 degrees. Note that the 2 Tetrahelix are also passing through the same Tetrahedron. So, one could say that the Tetrahelix are intersecting each other at 90 degrees.

Here is a figure showing the symmetry axis lines from another point of view showing that one passes under another and are rotated about a common axis by 90 degrees, just like the Tetrahedron case shown above.





Here is another perspective of this in which I draw in a Tetrahedron (in black) and a mid-edge to mid-edge axis of the Tetrahedron (in blue).



More details and calculations are given on the next page.

Tetrahelix Axis Passes Thru (7,3) Face Coordinate

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$$r = \frac{3\sqrt{3}}{10}\,EL$$

where EL is the edge length of the Tetrahedra making up the Tetrahelix.

The Tetrahelix can be positioned so that its (x, y, z) coordinates are given by the equation

where (approximately 131.8103149 degrees) and

$$h = \frac{1}{\sqrt{10}}\,EL$$

Using the equations

it is easy to calculate values for and .

n		
0	1	0
1		$\frac{\sqrt{5}}{3}$
2		$\frac{-4\sqrt{5}}{9}$
3		$\frac{7\sqrt{5}}{27}$

The n-th Terahelix vertex will then have the coordinate

For the "first" 4 vertices, with EL=1, we get

$$V_0 = \left(\frac{3\sqrt{3}}{10}, 0.0, 0.0 \right)$$

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$$V_3 = \left(\frac{66\sqrt{3}}{270}, \frac{21\sqrt{15}}{270}, \frac{3}{10} \right)$$

What (7, 3) means is that we travel 7 (out of 10) units along one edge and then 3 units (out of 10) parallel to another edge of the triangular face.

Define V_a to be the vector from vertex V_0 toward vertex V_1 but which is only 7 (out of 10) units in length. Then define V_b to be the vector from vertex V_1 toward vertex V_2 and which is 3 (out of 10) units in length.

Then the vector to (one of) the face point (7, 3) is given by V0 + Va + Vb. If this vector has no x or y components then it must lay on the z-axis, which is the symmetry axis of the Tetrahelix.

We calculate V_a to be

$$V_a \equiv \frac{7}{10}\left(V_1 - V_0\right) = \frac{7}{10}\left(\frac{-6\sqrt{3}}{30} - \frac{3\sqrt{3}}{10}, \frac{\sqrt{15}}{10}, \frac{1}{10} \right)$$

$$V_a = \left(\frac{-35\sqrt{3}}{100}, \frac{7\sqrt{15}}{100}, \frac{7}{100} \right)$$

And we calculate V_b to be

$$V_b \equiv \frac{3}{10}\left(V_2 - V_1\right) = \frac{3}{10}\left(\frac{-3\sqrt{3}}{90} - \frac{-6\sqrt{3}}{30}, \frac{-12\sqrt{15}}{90} - \frac{\sqrt{15}}{10}, \frac{2}{10} - \frac{1}{10}\right)$$

$$V_b = \left(\frac{45\sqrt{3}}{900}, \frac{-63\sqrt{15}}{900}, \frac{3}{100}\right)$$

Then the vector to the point face coordinate point (7, 3) is given by

$$V_c \equiv V_0 + V_a + V_b = \left(\frac{3\sqrt{3}}{10} + \frac{-35\sqrt{3}}{100} + \frac{45\sqrt{3}}{900}, \frac{7\sqrt{15}}{100} + \frac{-63\sqrt{15}}{900}, \frac{7}{100} + \frac{3}{100}\right)$$

$$V_c = \left(\frac{270\sqrt{3}}{900} + \frac{-315\sqrt{3}}{900} + \frac{45\sqrt{3}}{900}, \frac{63\sqrt{15}}{900} + \frac{-63\sqrt{15}}{900}, \frac{1}{10}\right)$$

$$V_c = \left(0.0, 0.0, \frac{1}{10}\right)$$

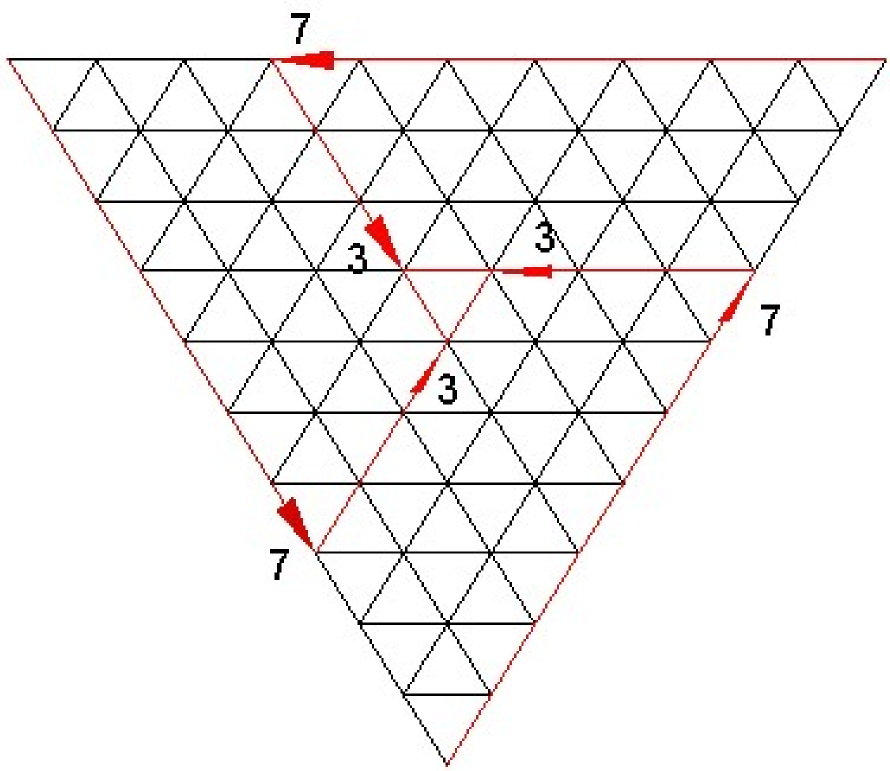
This lays on the z-axis. So the symmetry axis of the Tetrahelix does pass through the triangular face coordinate (7,3).

There are 3 such face points depending on which of the 3 vertices of the triangular face is used to begin measuring the 7 (and then 3) units of length.

12 Tetrahelix-Tetrahedron Intersection Points

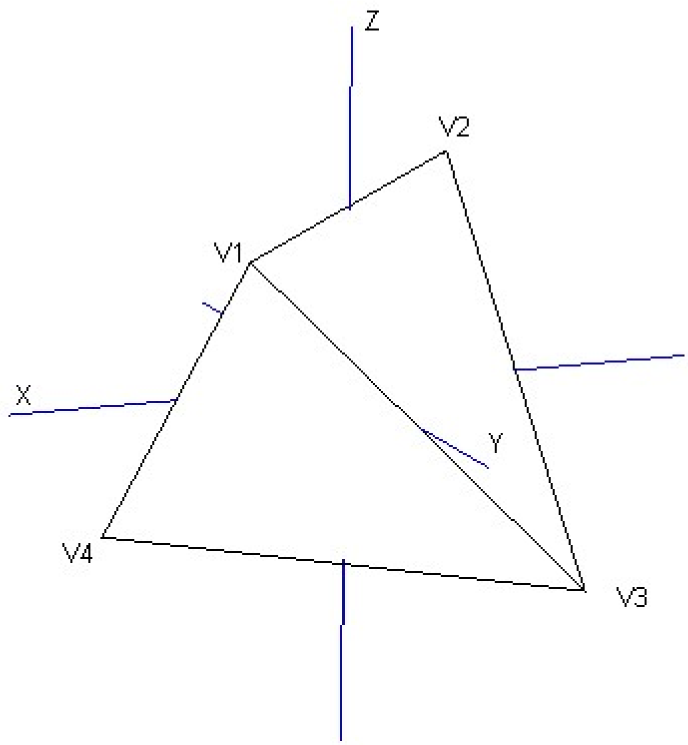
We calculate the (x, y, z) coordinates for the 3x4=12 points (3 per Tetrahedron triangular face) for the triangle face coordinates (7, 3). This will allows us to then calculate other properties of the Tetrahelix.

Recall that the Tetrahelix axis passes through the (7, 3) points of the Tetrahedron's triangular face.



We position the Tetrahedron so that its 4 vertices are at

V1 = (1, 1, 1)
V2 = (-1, -1, 1)
V3 = (-1, 1, -1)
V4 = (1, -1, -1)



The Tetrahedron edge length is then

$$EL = dist(V2 - V1) = sqrt((-1 - 1)^2 + (-1 - 1)^2 + (1 - 1)^2)$$
$$EL = sqrt(4 + 4 + 0) = 2 sqrt(2) = 2.828427125...$$

For the V1-V2-V3 triangular face, we let

Va = (V2 - V1) = (-2, -2, 0)
Vb = (V3 - V2) = (0 , 2, -2)
Vc = (V1 - V3) = (2, 0, 2)

Then

P1 = V1 + (7/10)Va + (3/10)Vb = (1 - 14/10, 1 - 14/10 + 6/10, 1 - 6/10)
P1 = (-4/10, 2/10, 4/10) = (-0.4, 0.2, 0.4)

P2 = V2 + (7/10)Vb + (3/10)Vc = (-1 + 6/10, -1 + 14/10, 1 - 14/10 + 6/10)
P2 = (-4/10, 4/10, 2/10) = (-0.4, 0.4, 0.2)

P3 = V3 + (7/10)Vc + (3/10)Va = (－1 + 14/10 - 6/10, 1 - 6/10, －1 + 14/10)
P3 = (- 2/10, 4/10, 4/10) = (－0.2, 0.4, 0.4)

For the V1-V2-V4 triangular face, we let

Va = (V2 - V1) = (－2, －2, 0)
Vb = (V4 - V2) = (2 , 0, －2)
Vc = (V1 - V4) = (0, 2, 2)

Then

P1 = V1 + (7/10)Va + (3/10)Vb = (1 - 14/10 + 6/10, 1 - 14/10, 1 - 6/10)
P1 = (2/10, －4/10, 4/10) = (0.2, －0.4, 0.4)

P2 = V2 + (7/10)Vb + (3/10)Vc = (－1 + 14/10, －1 + 6/10, 1 - 14/10 + 6/10)
P2 = (4/10, - 4/10, 2/10) = (0.4, －0.4, 0.2)

P3 = V4 + (7/10)Vc + (3/10)Va = (1 - 6/10, －1 + 14/10 - 6/10, －1 + 14/10)
P3 = (4/10, －2/10, 4/10) = (0.4, －0.2, 0.4)

For the V1-V3-V4 triangular face, we let

Va = (V3 - V1) = (－2, 0, －2)
Vb = (V4 - V3) = (2 , －2, 0)
Vc = (V1 - V4) = (0, 2, 2)

P1 = V1 + (7/10)Va + (3/10)Vb = (1 - 14/10 + 6/10, 1 - 6/10, 1 - 14/10)
P1 = (2/10, 4/10, - 4/10) = (0.2, 0.4, －0.4)

P2 = V3 + (7/10)Vb + (3/10)Vc = (－1 + 14/10, 1 - 14/10 + 6/10, －1 + 6/10)
P2 = (4/10, 2/10, －4/10) = (0.4, 0.2, －0.4)

P3 = V4 + (7/10)Vc + (3/10)Va = (1 - 6/10, －1 + 14/10, －1 + 14/10 - 6/10)
P3 = (4/10, 4/10, －2/10) = (0.4, 0.4, －0.2)

For the V2-V3-V4 triangular face, we let

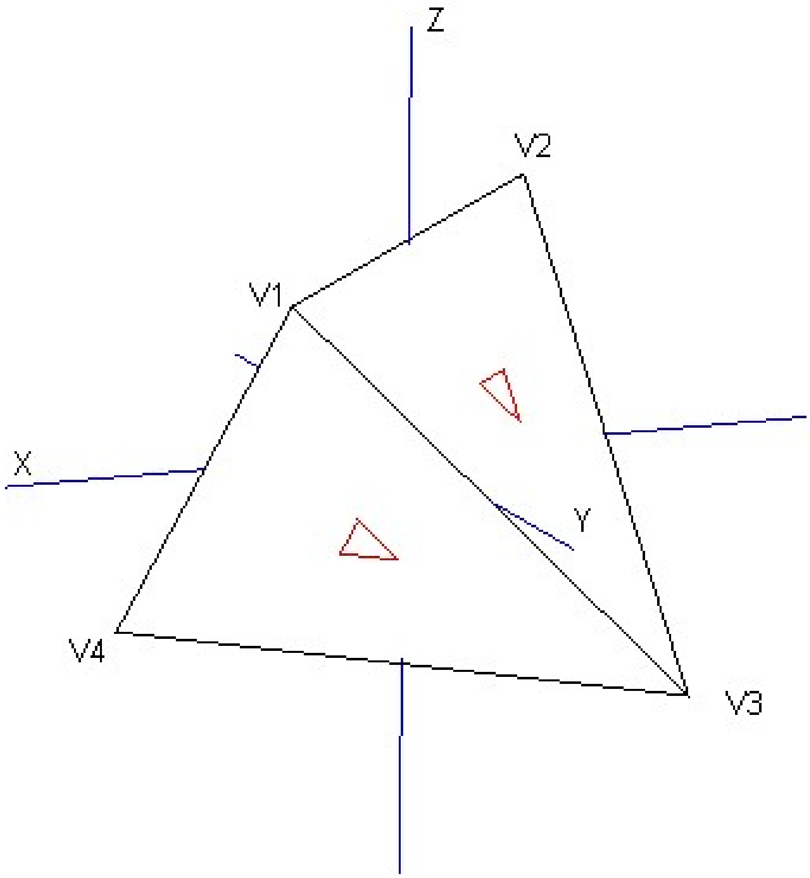
Va = (V3 - V2) = (0, 2, －2)
Vb = (V4 - V3) = (2 , －2, 0)
Vc = (V2 - V4) = (－2, 0, 2)

P1 = V2 + (7/10)Va + (3/10)Vb = (－1 + 6/10, －1 + 14/10 - 6/10, 1 - 14/10)
P1 = (－4/10, －2/10, - 4/10) = (－0.4, －0.2, －0.4)

P2 = V3 + (7/10)Vb + (3/10)Vc = (－1 + 14/10 - 6/10, 1 - 14/10, －1 + 6/10)
P2 = (－2/10, －4/10, －4/10) = (－0.2, －0.4, －0.4)

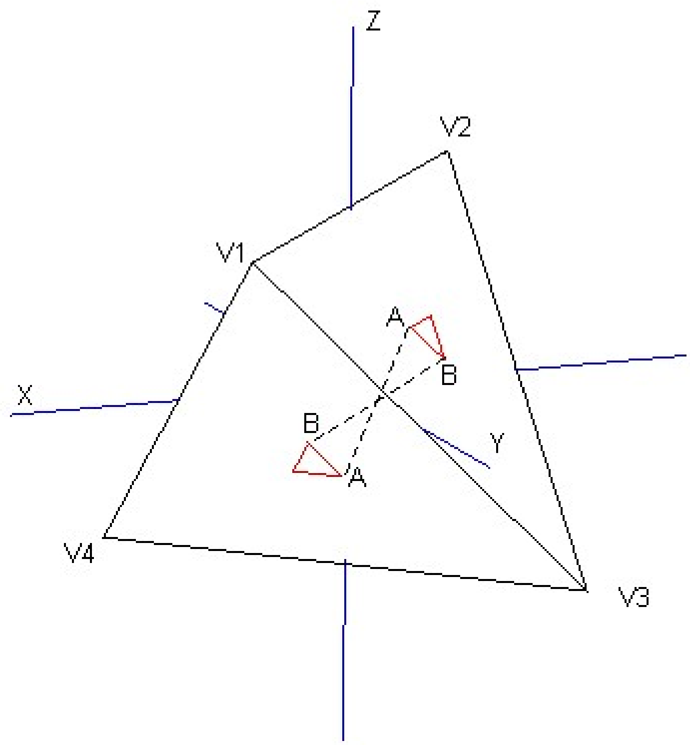
P3 = V4 + (7/10)Vc + (3/10)Va = (1 - 14/10, －1 + 6/10, －1 + 14/10 - 6/10)
P3 = (－4/10, －4/10, －2/10) = (－0.4, －0.4, －0.2)

Now that we know the (7,3) points we can draw the small triangle red triangle on each of the 4 Tetrahedron's faces.



We know that the Tetrahelix axes make crossing lines through these (7, 3) triangle face points.

For example, if we look at just 2 of the red triangles, then two of the Tetrahelix axes pass through the 4 points as follows.



"A" goes to "A" and "B" goes to "B".

The two "A" point coordinates are

$$A(123) = (-0.2, 0.4, 0.4), A(134) = (0.2, 0.4, -0.4)$$

The length of the line segment A-to-A is given by

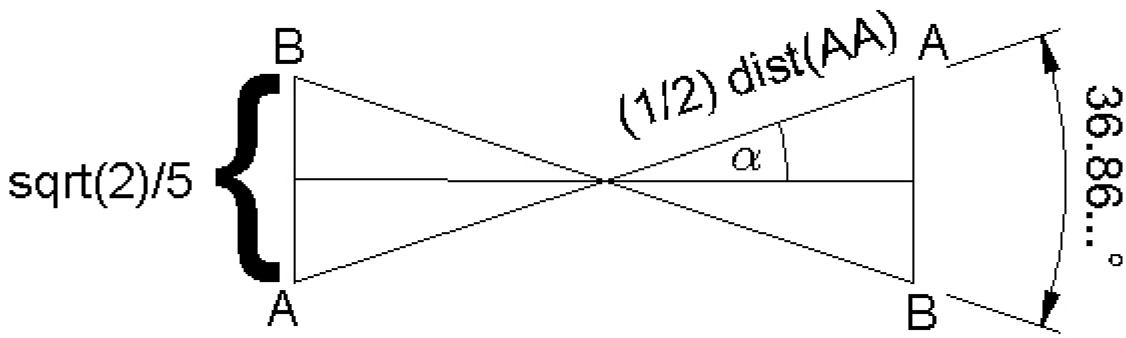
$$\text{dist} = \text{sqrt}((0.2 + 0.2)\text{^}2 + (0.4 - 0.4)\text{^}2 + (-0.4 - 0.4)\text{^}2)$$
$$\text{dist} = \text{sqrt}(0.16 + 0.0 + 0.64) = \text{sqrt}(0.8)$$
$$\text{dist} = \text{sqrt}(8/10) = 2 / \text{sqrt}(5) = 0.894427191...$$

We can calculate the crossing angle as follows:

Since the edge length of the Tetrahedron is EL = 2 sqrt(2) the edge length of the small red triangle is

$$ELs = EL / 10 = \text{sqrt}(2) / 5.$$

We let alpha = (1/2) crossing angle.



Then

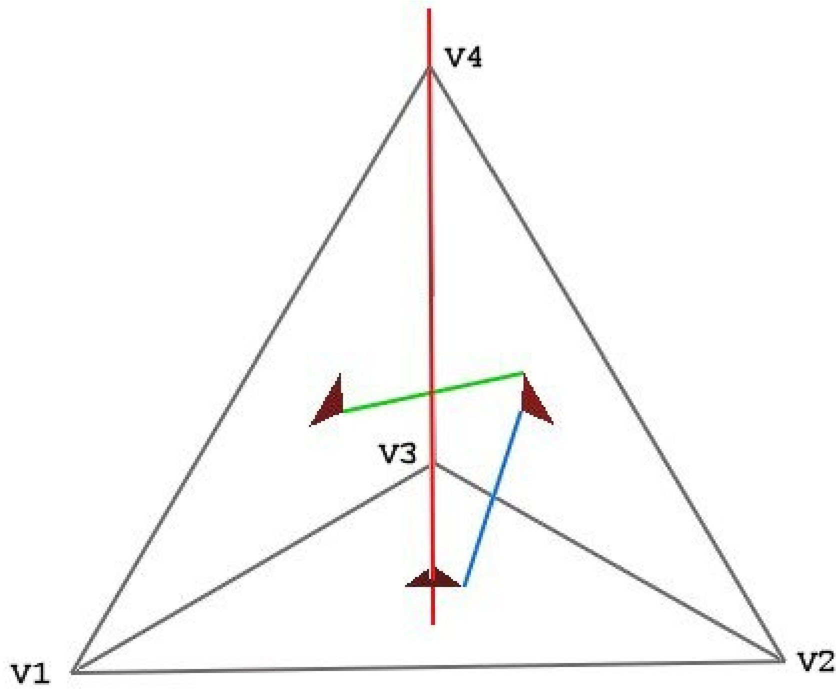
sin(alpha) = (ELs/2) / ((2/sqrt(5)) / 2)
sin(alpha) = (sqrt(2) / 10) / (1 / sqrt(5))
sin(alpha) = sqrt(5) / (5 sqrt(2))
sin(alpha) = 1 / sqrt(10)

Then the crossing angle is

alpha = 2 arcsin(1/sqrt(10)) = 36.86989765... degrees.

We now wish to calculate the angle which a Tetrahelix makes with a table top when the a Tetrahedron face is placed flat on the table.

We first need a vector which is perpendicular to a Tetrahedron face. This can easily be found by summing the vectors to vertices V1, V2, and V3. (The red line in the next figure is perpendicular to the V1,V2,V3 face of the Tetrahedron.)



This is essentially taking the average of the 3 vectors and multiplying by 3. Recall that the average of the vectors is given by

$$Avg(123) = (1/3)(V1 + V2 + V3)$$

So, multiplying by 3 we get

$$Vn(123) = (V1 + V2 + V3) = (-1, 1, 1)$$

This is a vector pointing out through the Face Center of the V1, V2, V3 face.

This vector Vn(123) is normal (perpendicular) the the V1, V2, V3 face.

If we put the Tetrahedron of the table with its V1, V2, V3 face flat on the table, this vector will point down through the table. So, we reverse the direction of this vector so that it will point up from the table top.

$$Vn(123) = (1, -1, -1)$$

For calculation purposes, we make the length of this vector to be 1. In the Figures, the length is not 1 and is drawn as a red line.

$$Vn(123) = (1/\mathrm{sqrt}(3))(1, -1, -1) = (1/\mathrm{sqrt}(3), -1/\mathrm{sqrt}(3), -1/\mathrm{sqrt}(3))$$

We will use the vector dot product to calculate the angles.

$$A.B = |A|\ |B| \cos(\mathrm{theta})$$
$$\mathrm{theta} = \arccos(A.B\ /\ (|A||B|))$$

The vector connecting the 2 "A" points above

$$A(123) = (–0.2, 0.4, 0.4)$$
$$A(134) = (0.2, 0.4, –0.4)$$

is given by

$$VAA = A(134) - A(123) = (0.4, 0.0, -0.8) = (2/5, 0, -4/5)$$

(see the blue line in the figures.)

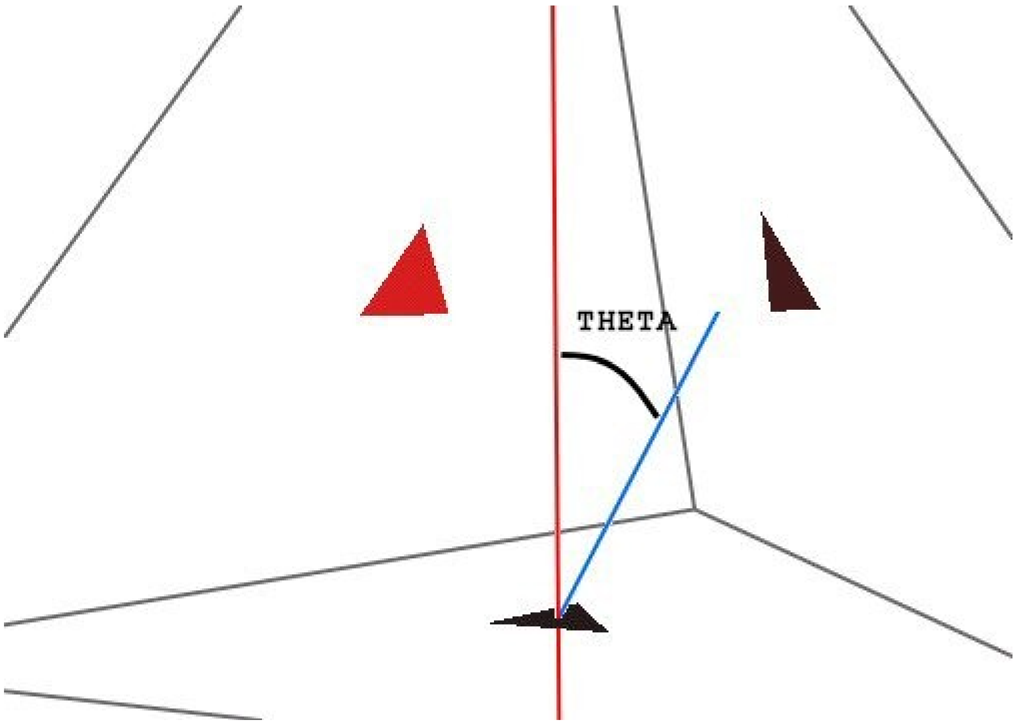
Its magnitude is

$$\mathrm{mag}(VAA) = \mathrm{sqrt}(0.4\text{^}2 + (-0.8)\text{^}2) = \mathrm{sqrt}(0.16 + 0.64)$$
$$\mathrm{mag}(VAA) = \mathrm{sqrt}(0.8) = \mathrm{sqrt}(8/10) = 2\ /\ \mathrm{sqrt}(5)$$

The unit vector in the VAA direction is then

$$UAA = (\mathrm{sqrt}(5)/2)(2/5, 0, -4/5)$$
$$UAA = (1/\mathrm{sqrt}(5), 0, -2/\mathrm{sqrt}(5))$$

Note that vectors can be moved to any position as long as we don't change their direction nor their magnitude. So I move it (blue line) in the Figure so that it is at the face center. The red line (perpendicular line to the Tetrahedron face) also passes through the face center point.



The dot product is

$$(1/\mathrm{sqrt}(3), -1/\mathrm{sqrt}(3), -1/\mathrm{sqrt}(3)).(1/\mathrm{sqrt}(5), 0, -2/\mathrm{sqrt}(5))$$
$$(1/\mathrm{sqrt}(15) + 2/\mathrm{sqrt}(15)) = 3/\mathrm{sqrt}(15) = \mathrm{sqrt}(3/5)$$

So theta is

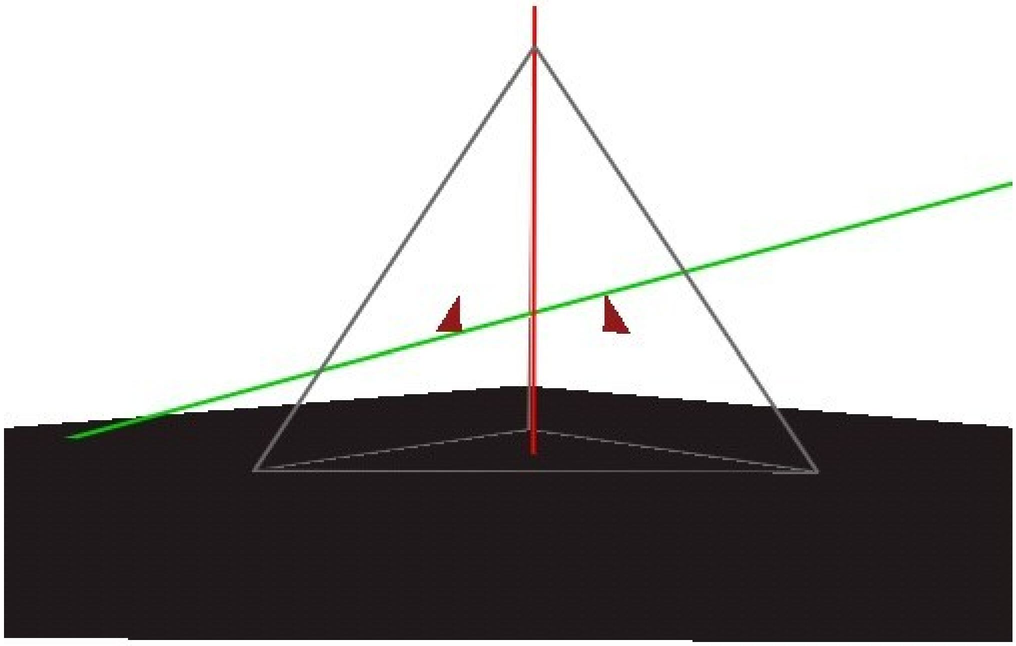
$$\text{theta} = \arccos(\mathrm{sqrt}(3/5)) = 39.23152048\ldots \text{ degrees}$$

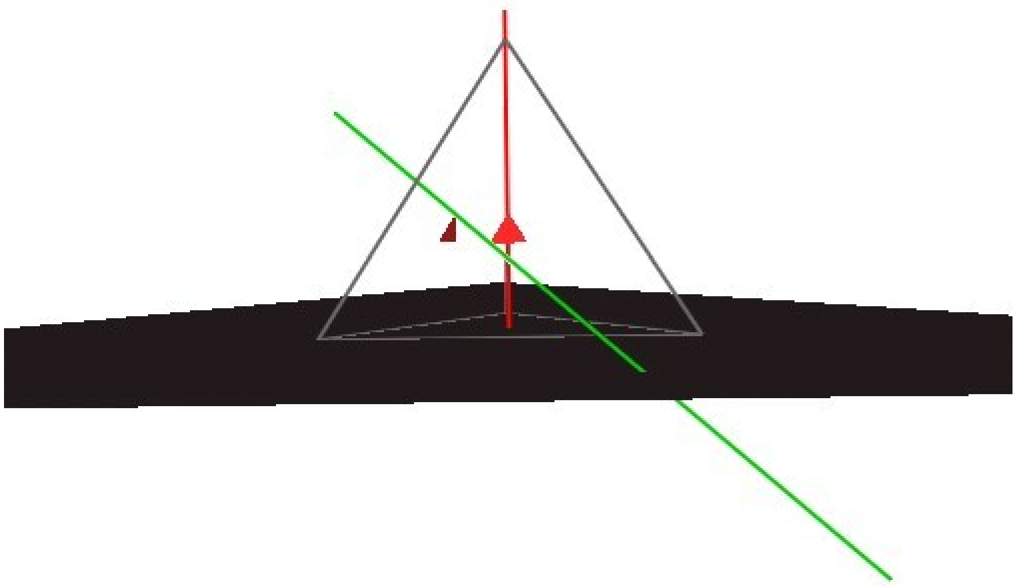
Which means the Tetrahelix makes an angle of

$$90 - \arccos(\mathrm{sqrt}(3/5)) = 50.76847952\ldots \text{ degrees}$$

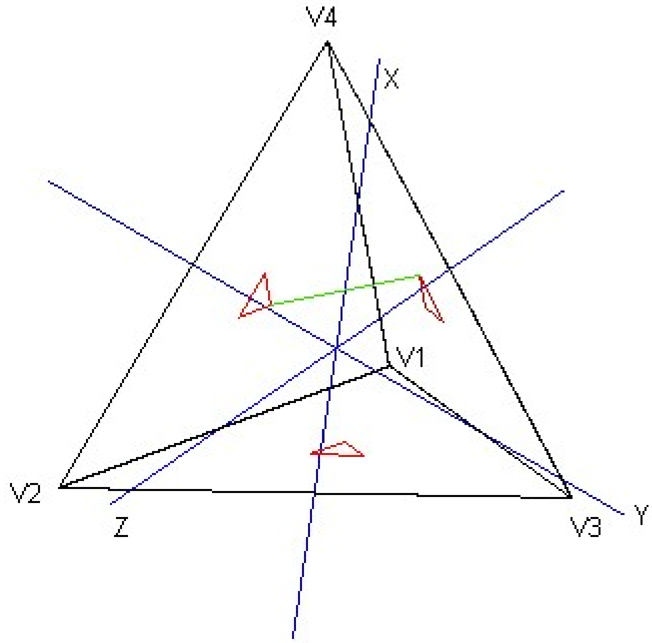
with the table's surface.

There is another angle at which a Tetrahelix can make with the table when the Tetrahedron is placed flat on the table. See the green line in the above figure. This is another axis of symmetry for some Tetrahelix. It passes through the table top some distance away from the Tetrahedron as the following Figures show.





There are several line segments we could chose for the calculation. The following 2 Figures shows the line segment in green that I will use. Yes, its very hard to see the orientation.



The two points selected are:

P3 = (0.4, -0.2, 0.4) from face (1.2.4), and
P2 = (0.4, 0.2, -0.4) from face (1.3.4).

The vector along this line segment is then

$$V = (0.4, 0.2, -0.4) – (0.4, –0.2, 0.4)$$
$$V = (0.0, 0.4, -0.8) = (0, 2/5, -4/5)$$

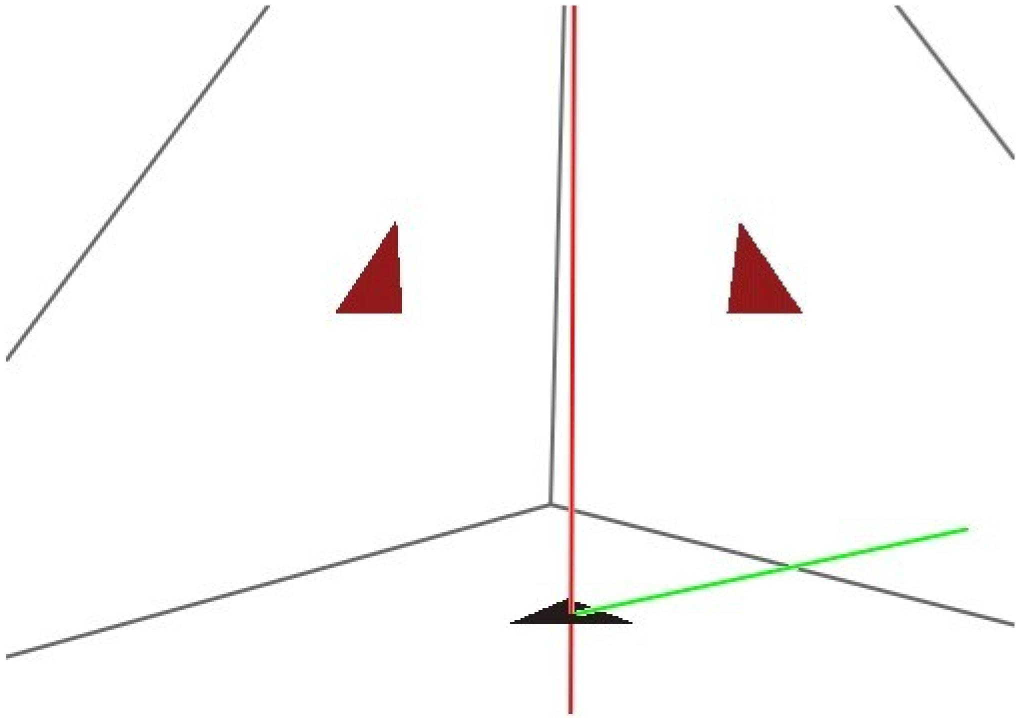
As before, the magnitude of this vector is

$$mag(V) = sqrt(0.4^2 + (-0.8)^2) = sqrt(0.16 + 0.64)$$
$$mag(V) = sqrt(0.8) = sqrt(8/10) = 2 / sqrt(5)$$

The unit vector in the direction of th egreen line segment is then

$$UV = (sqrt(5)/2)(0, 2/5, -4/5)$$
$$UV = (0, 1/sqrt(5), -2/sqrt(5))$$

We can place this vector anywhere we want. So we place it at the center of the 1.2.3 face.

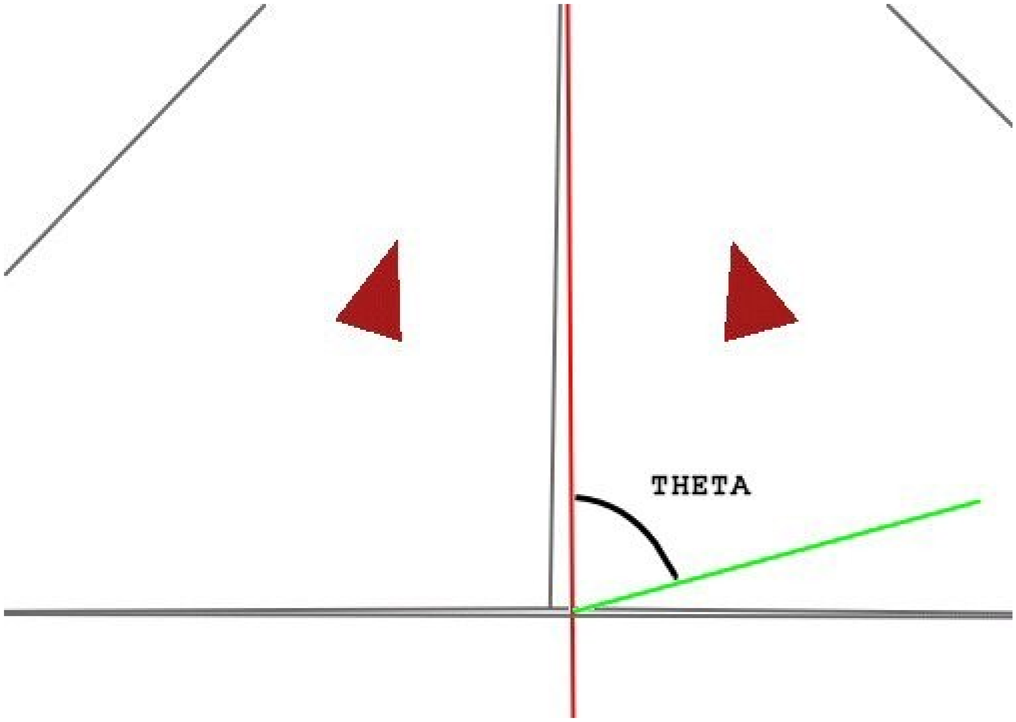


The dot product with the face unit normal is given by

D = (1/sqrt(3), -1/sqrt(3), -1/sqrt(3)).(0, 1/sqrt(5), -2/sqrt(5))
D = 0 - 1/sqrt(15) + 2/sqrt(15) = 1/sqrt(15)

So, for this case, theta is

theta = arccos(sqrt(1/15)) = 75.0367825... degrees



Which means this Tetrahelix makes an angle of

$$90 - \arccos(\mathrm{sqrt}(1/15)) = 14.96321744... \text{ degrees}$$

with the table.

References

Here is a list of references. However, I did not use any of these references for my calculations.

- H. S. M. Coxeter, *Introduction to Geometry*, Wiley: New York, 1969, p. 412.

- Chong Zheng, Roald Hoffman, David R. Nelson, "A Helical Face-Sharing Tetrahedron Chain with Irrational Twist, Stella Quadrangula, and Related Matters", J. Am. Chem. Soc., 112, pp. 3784-3791, 1990.

I have not seen the following references. These references are given in the Zheng paper.

- A. H. Boerdjik, Phillips Res. Rep., 7, p. 303, 1952.

- J. D. Bernal, Proc. R. Soc. London Ser. A, 280, p. 299, 1964.

- P. J. Steinhardt, D. R. Nelson, M. Ronchetti, Phys. Rev. Lett., 47, p. 1297, 1981.

- P. J. Steinhardt, D. R. Nelson, M. Ronchetti, Phys. Rev. B, 28, p. 784, 1983.

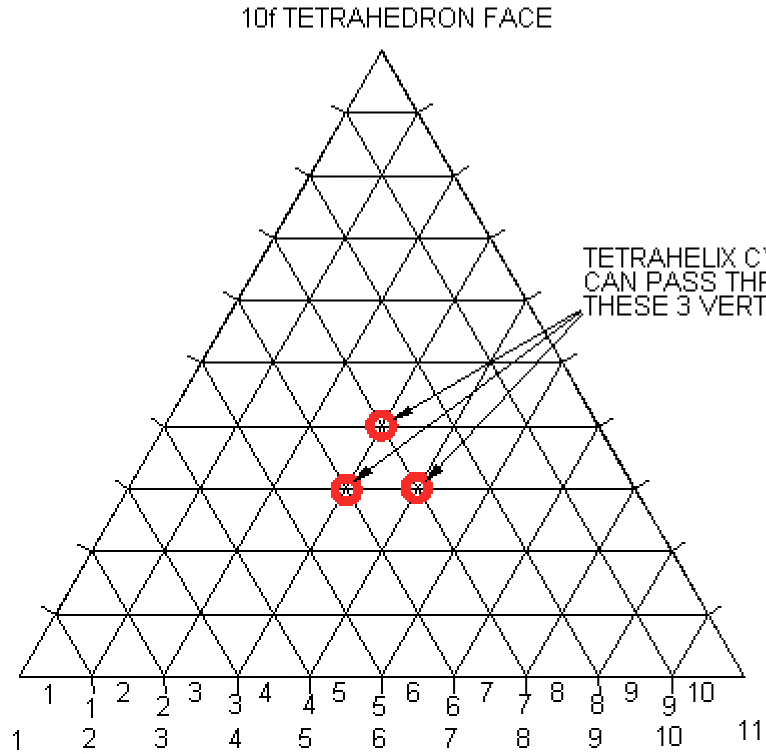
- D. R. Nelson, Phys. Rev. B, 28, p. 5515, 1983.

Conclusion

Discussion of R. Grey Section

Grey certainly develops and articulates the model left by R. Buckminster Fuller. The most interesting aspect of his work is the following:

- The "triangular face coordinate" which the symmetry axis passes through is (7,3).



Viewing the BC – Helix from this angle suggests a number of related mathematical structures: the Binomial Pyramid, attributed to Zhang Hui, to the Hindus as Mt. Meru and to Blaise Pascal. Frank “Tony” Smith has done considerable work on his massive website to illustrate similar numerical relationships among Clifford and other higher algebras.

Since ours is a combinatorial universe, could we construct a triangle like this after the Binomial Pyramid? The shape is suggestive of many

TABLE I. Properties of the regular (Platonic) polyhedra

Polyhedron	Face type	Vertex degrees	Number of edges	Number of faces	Number of vertices
Tetrahedron {3,3}	Triangle	3	6	4	4
Octahedron {3,4}	Triangle	4	12	8	6
Cube {4,3}	Square	3	12	6	8
Icosahedron {3,5}	Triangle	5	30	20	12
Dodecahedron {5,3}	Pentagon	3	30	12	20

possibilities, including the Fano Plane and the Octonions, which obviously play a part in the BC – Helix.

Tony Smith and the late Robert de Marrais have suggested the relationship of Sedenions to various lattices, specifically the Leech Lattice, the E8 Lattice and the Barnes – Wall Lattice. The triangular shape of the latter suggests a relationship to the BC – Helix, while the involvement of DNA with the helix implies a relationship to the Leech Lattice. Indeed, the author will later present a paper which explores the relationship of these lattices to the BC – Helix and the 64 hexagrams of the I Ching, as well as the 64 amino acid combinations of DNA.

Given the involvement of the E8 Lattice, one might include the Freudenthal – Tits Magic Square of Exceptional Lie Algebras, which is related to triality.

A \ B	R	C	H	O
R	A_1	A_2	C_3	F_4
C	A_2	$A_2 \times A_2$	A_5	E_6
H	C_3	A_5	D_6	E_7
O	F_4	E_6	E_7	E

	B	R	C	H	O
A	*der*(A/B)	0	0	$\mathfrak{sp}_1$	$\mathfrak{g}_2$
R	0	$\mathfrak{so}_3$	$\mathfrak{su}_3$	$\mathfrak{sp}_3$	$\mathfrak{f}_4$
C	0	$\mathfrak{su}_3$	$\mathfrak{su}_3 \oplus \mathfrak{su}_3$	$\mathfrak{su}_6$	$\mathfrak{e}_6$
H	$\mathfrak{sp}_1$	$\mathfrak{sp}_3$	$\mathfrak{su}_6$	$\mathfrak{so}_{12}$	$\mathfrak{e}_7$
O	$\mathfrak{g}_2$	$\mathfrak{f}_4$	$\mathfrak{e}_6$	$\mathfrak{e}_7$	$\mathfrak{e}_8$

Note that by construction, the row of the table with A=**R** gives **der**($J_3(B)$), and similarly vice versa.

Barnes Wall Lattice

Another related lattice is that of the Barnes – Wall Lattice, shown below: notice the position of E8 here, as well as D4. If we may draw parallels between Exceptional Lie Algebra and the BW Lattice, it appears that higher forms of organization may exist beyond E8.

200 *CHAPTER 14. INTRODUCTION TO LATTICE AND TRELLIS CODES*

The $|u|u+v|$ construction suggests the following tableau of BW lattices. Here $D_4 = BW_4$, $E_8 = BW_8$, and $\Lambda_n = BW_n$ for $n = 2^{m+1} \geq 16$. Also, we use $R^2 = 2I_{2^m}$.

$$
\begin{array}{cccccccc}
\mathbb{Z}^2 & & & & & & & \\
 & D_4 & & & & & & \\
R\mathbb{Z}^2 & & E_8 & & & & & \\
 & RD_4 & & \Lambda_{16} & & & & \\
2\mathbb{Z}^2 & & RE_8 & & \Lambda_{32} & & & \\
 & 2D_4 & & R\Lambda_{16} & & \Lambda_{64} & & \\
2R\mathbb{Z}^2 & & 2E_8 & & R\Lambda_{32} & & \Lambda_{128} & \\
 & 2RD_4 & & 2\Lambda_{16} & & R\Lambda_{64} & & \Lambda_{256} \\
4\mathbb{Z}^2 & & 2RE_8 & & 2\Lambda_{32} & & R\Lambda_{128} & & \Lambda_{512}
\end{array}
$$

Figure 4. *Tableau of Barnes-Wall lattices.*

In this tableau each BW lattice lies halfway between the two lattices of half the dimension that are used to construct it in the $|u|u+v|$ construction, from which we can immediately deduce its normalized volume.

The Magic Triangle

While apparently square in shape, the Magic Square can be fashioned into a triangle, as has been done by P. Cvitanovic.

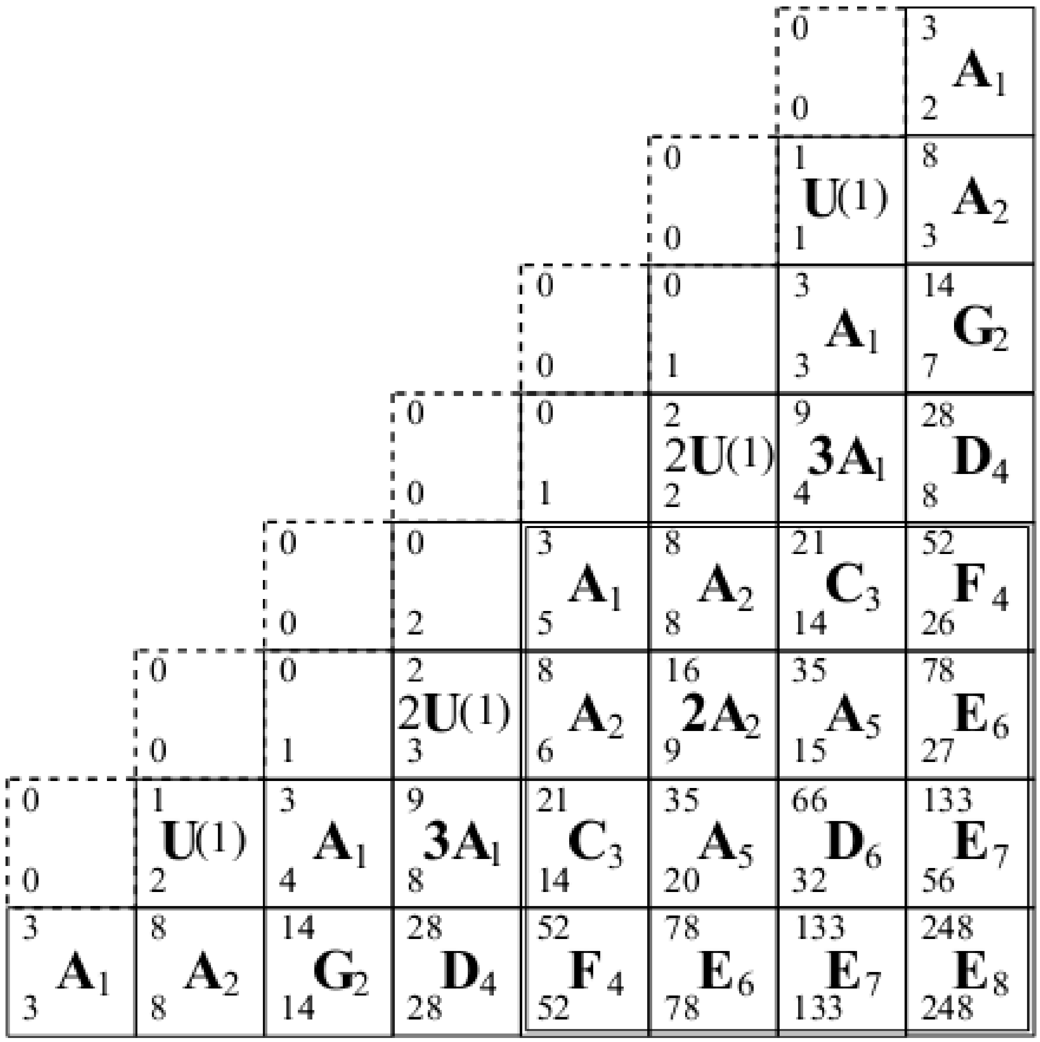


Figure 1.1 The "Magic Triangle" for Lie algebras. The "Magic Square" is framed by the double line. For a discussion, consult chapter 21.

Hypothesis

Would it prove possible to combine the Exceptional Lie Algebras of the three Magic Triangles into one face of the BC – Helix? What might be the result? E8 on each vertex with the remainder filling in.

In fact, it appears that there exist two versions of the BC Helix, one containing proportionately more of the 8 x 8 equilibrious Satva state, and related to the more stable structures of DNA, as discussed in detail by R. Buckminster Fuller and R. Gray as the Tetrahelix.

The second version appears more akin to what has been oddly termed the Pearce Cluster, since Peter Pearce disavows this title for his own creation, preferring "Oblate Icosahedral system."

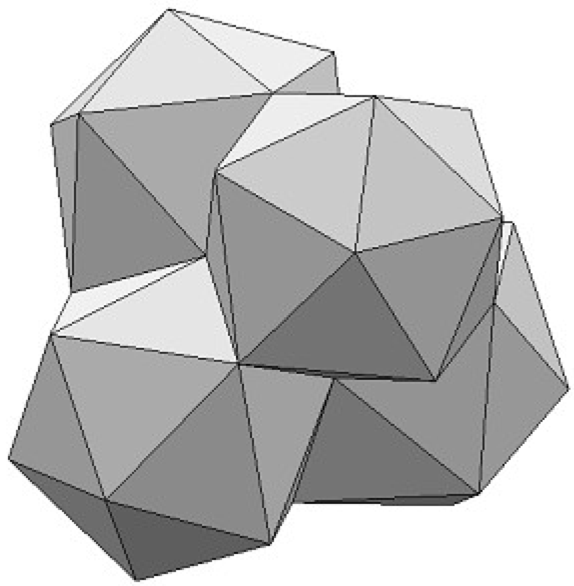


Fig. 6. The "Pearce" cluster; four icosahedra in face contact.

The author theorizes that the Pearce Cluster or the Oblated Icosahedral System forms when matter is programmed toward the more dynamic state of 9 x 9 Raja type. That is to say that matter forms has has been described in the Qi Men Dun Jia Model until it reaches the BC – Helix, which apparently includes both the 8 x 8 and the 9 x 9 state of matter, before choosing one type over the other.

Thus, for example, if matter were developing towards a DNA strand, then this would be stable Satva matter of the 8 x 8 type, and the BC – Helix would develop into this formation. On the other hand, if there were a bit micro bit of dynamic matter that was programmed for a different use, then one would anticipate the formation of the Pearce

Cluster / Oblate Icosahedra structure.

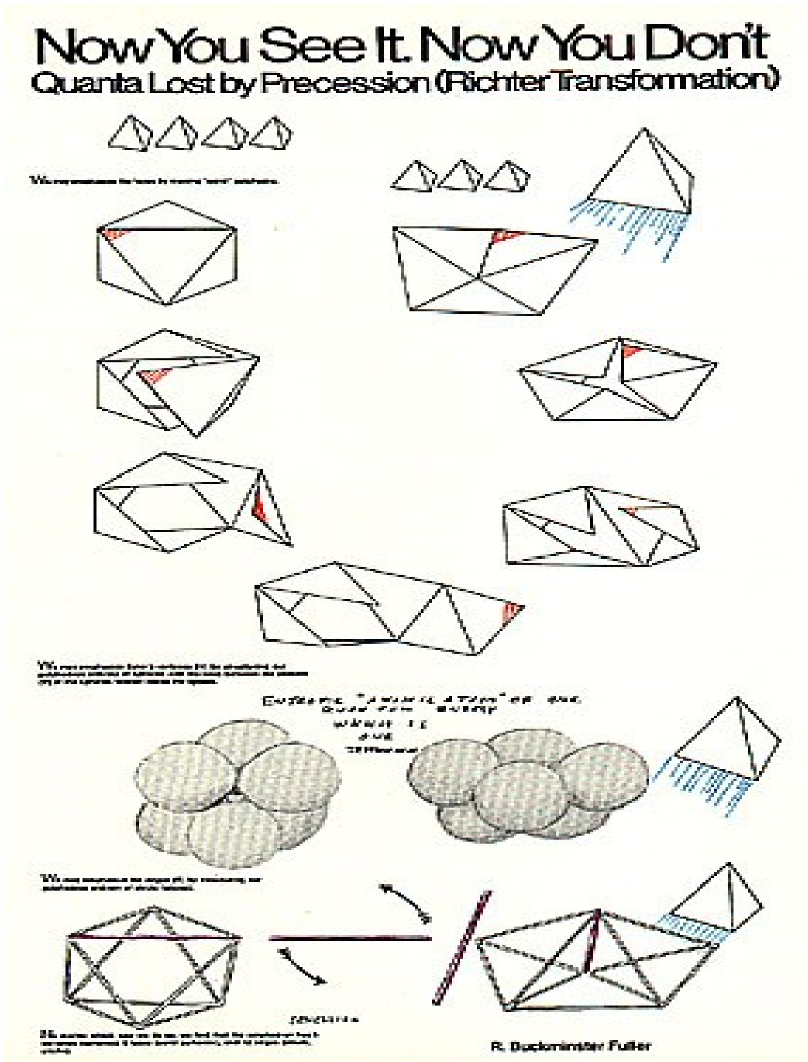
In a future paper the author hopes to show how these lattices, Magic Squares and Magic Triangles extend towards the Golay Code and DNA amino acid coding, as suggested by R. Buckminster Fuller.

Eccentricities of the BC – Helix

The BC – Helix or the Tetrahelix exhibit certain functional eccentricities that perhaps relate closely to one another as the helix carries out its functions. That is to say, that instead of simply dismissing these as odd aspects of the polytope, we may rather assemble these eccentricities to see whether and how they may fit together, for the purpose of serving some function that remains as yet unknown.

As mentioned above, the Octonions have been dismissed as useless for physics since they lose the associative ability, while sedenions lose the division property. At the same time, the BC – Helix reduces from the dodecahedral shape to the octahedral, as discussed by R. Gray and R. Buckminster Fuller in the Richter Transformation. It may prove possible that the loss of functionality of the higher algebras may be connected to the reduction of the BC – Helix to an octahedral.

The Richter Transformation



"Quanta Lost by Precession" boasts the subtitle, teasingly suggesting the appropriate role of the Octonions and Sedenions in the BC – Helix, since most mathematicians and physicists complain about the loss of functionality in the Octonions and Sedenions, the Octonions losing associativity and the Sedenions losing divisibility. Roger Penrose calls the Octonions the "lost cause" of physics, while Frank "Tony" Smith wrote that the best use of the Sedenions is to put the Octonions into complete perspective.

If the Richter Transformation signals the transition from a 3 Tetrahedron to an Octahedron, then we might well expect a loss of functionality in the algebras which comprise the geometric structures. That is to say, that if indeed “Quanta is Lost by Precession,” then the reason may well be that nature requires this loss in order to carry out the transformation from one geometric structure to another, from one state of matter to another, ie, from 9 x 9 Raja matter to 8 x 8 Satva matter.

Therefore, there may well exist a logical reason why the Octonions and Sedenions lose their properties, just as the Richter Transformation results in a loss of quanta. A butterfly must shed its cocoon at some point. Here Tony Smith describes the loss of divisibility in the Sedenions:

```
The 0-grade 1-dimensional scalar space of Cl(0,15)
represents the sedenion real axis.

There is an 8 to 1 correspondence between the
1x128 minimal ideal SSPINOR on which SL acts by Clifford
and
the 1x16 sedenion column vectors on which
SL acts by matrix-vector action.

This leads
to failure of the division algebra property of sedenions,
because the map SL from SSPINOR to S is 8 to 1
and invertible.
```

The eccentricities of BC – Helix continue, when we learn that the spiral which forms the helix pass through the vertices in sequence. We find that the axis fails to pass through the center of the object but is slightly off – center; and therefore the sections never actually line up smoothly. Wikipedia states that the BC – Helix is the only stacking Platonic Solid which is not rotationally repetitive, due to the helical pitch per cell.

We learn that there are clockwise and anti – clockwise versions of the helix, and that the clockwise has a length of 131.8103149, while the anti – clockwise version has a length of 228.1896851.

We learn from R. Buckminster Fuller that the BC – Helix attempts but forever fails to attain an equilibrious periodicity, probably related to the 8 x 8 Satva state of matter, while mostly striving in the more dynamic state, which perhaps Fuller understood without knowing that he was discussing the 9 x 9 Raja sate – the sign of a true genius.

In section 930.11, Fuller tells us that the helix forms in the direction of peak to base to peak again, thus underscoring this dynamic process.

Finally, there is the problem of the Hopf Fibration. After reading Tony Smith's piece called, “Why Not Sedenions,” it occurs that the Barnes Wall Lattices, along with the Leech Lattice and E8 Lattice, appear in the even dimensions, which suggests the stable 8 x 8 Satva structure, while the Hopf Fibration may only appear in odd dimensions. As noted above, the BC – Helix appears to fluctuate during its development between the two states of matter, and the problem of dimension, or combinatorial counts, may relate to which type finally emerges in the structure.

The partially completed chart below illustrates this process:

In odd dimensions we find Hopf Fibration while in even dimensions we find Barnes – Wall Lattices, including Leech and E8. What is the 14 Dimension Lattice that corresponds to G2?

Dim		Lattice	Type
1			
2		Z2	Barnes - Wall
3	Hopf Fibration		
4		D4	
5			
6			
7	Hopf Fibration		
8		E8	Barnes - Wall
9			
10			
11			
12			
13			
14	G2 Exceptional Lie Algebra		
15	Hopf Fibration		
16		Laminated	Barnes - Wall

The Barnes – Wall lattices can be constructed from the Leech Lattice A24.

The BC – Helix appears in two types, clockwise and counterclockwise, and this may relate to the problem as well. Does the clockwise type correspond to the 8 x 8 version, and vice versa?

This problem proves worthy of additional study and the author is planning a future paper on the subject of the Hopf Fibration and lattices.

Applications

The BC – Helix has a wide range of applications across various disciplines, but especially in crystallography and minerals. We list a few of them here.

Abstract:

Helices and dense packing of spherical objects are two closely related problems. For instance, the Boerdijk-Coxeter helix, which is obtained as a linear packing of regular tetrahedra, is a very efficient solution to some close-packing problems. The shapes of biological helices result from various kinds of interaction forces, including steric repulsion. Thus, the search for a maximum density can lead to structures related to the Boerdijk-Coxeter helix. Examples are presented for the -helix structure in proteins and for the structure of the protein collagen, but there are other examples of helical packings at different scales in biology. Models based on packing efficiency related to the Boerdijk-Coxeter helix, explain, mainly from topological arguments, why the number of amino acids per turn is close to 3.6 in -helices and 2.7 in collagen.

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(MIT Press, 1990).

Twisted Octonions

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he European Physical Journal B - Condensed Matter and Complex Systems
November 1999, Volume 12, Issue 2, pp 309-318

Boerdijk-Coxeter helix and biological helices

- J.F. Sadoc,
- N. Rivier

[PDF]**The c-brass structure and the Boerdijk–Coxeter helix**

Adobe PDF
The c-brass structure and the **Boerdijk–Coxeter** helix E.A. Lord *, S. Ranganathan Department of Metallurgy, Indian Institute of Science, Bangalore 560012, India

Appendix I

R. Buckminster Fuller on the Tetrahelix

930.00 **Tetrahelix: Unzipping Angle**

930.10 **Continuous Pattern Strip: "Come and Go"**

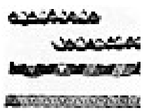
Fig. 930.11

930.11 Exploring the multiramifications of spontaneously regenerative reangulations and triangulations, we introduce upon a continuous ribbon a 60-degree-patterned, progressively alternating, angular bounce-off inwards from first one side and then the other side of the ribbon, which produces a wave pattern whose length is the interval along any one side between successive bounce-offs which, being at 60 degrees in this case, produces a series of equiangular triangles along the strip. As seen from one side, the equiangular triangles are alternately oriented as *peak away,* then *base away,* then *peak away* again, etc. This is the patterning of the only equilibrious, never realized, angular field state, in contradistinction to its sine-curve wave, periodic realizations of progressively accumulative, disequilibrious aberrations, whose peaks and valleys may also be patterned between the same length wave intervals along the sides of the ribbon as that of the equilibrious periodicity. (See Illus. 930.11.)

930.20 **Pattern Strips Aggregate Wrapabilities:**The equilibrious state's 60- degree rise-and-fall lines may also become successive cross-ribbon fold-lines, which, when successively partially folded, will produce alternatively a tetrahedral- or an octahedral- or an icosahedral-shaped spool or reel upon which to roll-mount itself repeatedly: the tetrahedral spool having four successive equiangular triangular facets around its equatorial girth, with no additional triangles at its polar extremities; while in the case of the octahedral reel, it wraps closed only six of the eight triangular facets of the octahedron, which six lie around the octahedron's equatorial girth with two additional triangles left unwrapped, one each triangularly surrounding each of its poles; while in the case of the icosahedron, the equiangle-triangulated and folded ribbon wraps up only 10 of the icosahedron's 20 triangles, those 10 being the 10 that lie around the icosahedron's equatorial girth, leaving five triangles uncovered around each of its polar vertexes. (See Illus. 930.20.)

930.21 The two uncovered triangles of the octahedron may be covered by wrapping only one more triangularly folded ribbon whose axis of wraparound is one of the XYZ symmetrical axes of the octahedron.

930.22 Complete wrap-up of the two sets of five triangles occurring around each of the two polar zones of the icosahedron, after its equatorial zone triangles are completely enclosed by one ribbon-wrapping, can be accomplished by employing only two more such alternating, triangulated ribbon-wrappings .

930.23 The tetrahedron requires only *one* wrap-up ribbon; the octahedron *two*; and the icosahedron *three,* to cover all their respective numbers of triangular facets. Though all their faces are covered, there are, however, alternate and asymmetrically arrayed, open and closed edges of the tetra, octa, and icosa, to close all of which in an even-number of layers of ribbon coverage per each facet and per each edge of the three-and-only prime structural systems of Universe, requires *three,* triangulated, ribbon-strip wrappings for the tetrahedron; six for the octahedron; and *nine* for the icosahedron.

930.24 If each of the ribbon-strips used to wrap-up, completely and symmetrically, the tetra, octa, and icosa, consists of transparent tape; and those tapes have been divided by a set of equidistantly interspaced lines running parallel to the ribbon's edges; and three of these ribbons wrap the tetrahedron, six wrap the octahedron, and nine the icosahedron; then all the four equiangular triangular facets of the tetrahedron, eight of the octahedron, and 20 of the icosahedron, will be seen to be symmetrically subdivided into smaller equiangle triangles whose total number will be N^2, the second power of the number of spaces between the ribbon's parallel lines.

930.25 All of the vertexes of the intercrossings of the three-, six-, nine-ribbons' internal parallel lines and edges identify the centers of spheres closest-packed into tetrahedra, octahedra, and icosahedra of a frequency corresponding to the number of parallel intervals of the ribbons. These numbers (as we know from Sec. 223.21) are:

$2F^2 + 2$ for the tetrahedron;

$4F^2 + 2$ for the octahedron; and

$10F^2 + 2$ for the icosahedron (or vector equilibrium).

930.26 Thus we learn sum-totally how a ribbon (band) wave, a waveband, can self-interfere periodically to produce in-shuntingly all the three prime structures of Universe and a complex isotropic vector matrix of successively shuttle-woven tetrahedra and octahedra. It also illustrates how energy may be wave-shuntingly self-knotted or self- interfered with (see Sec. 506), and their energies impounded in local, high-frequency systems which we misidentify as only-seemingly-static matter.

931.00 Chemical Bonds

931.10 **Omnicongruence:** When two or more systems are joined vertex to vertex, edge to edge, or in omnicongruence-in single, double, triple, or quadruple bonding, then the topological accounting must take cognizance of the congruent vectorial build in growth. (See Illus. 931.10.)

931.20 **Single Bond:** In a single-bonded or univalent aggregate, all the tetrahedra are joined to one another by only one vertex. The connection is like an electromagnetic universal joint or like a structural engineering pin joint; it can rotate in any direction around the joint. The mutability of behavior of single bonds elucidates the compressible and load-distributing behavior of gases.

931.30 **Double Bond:** If two vertexes of the tetrahedra touch one another, it is called double-bonding. The systems are joined like an engineering *hinge;* it can rotate only perpendicularly about an axis. Double-bonding characterizes the load-distributing but noncompressible behavior of liquids. This is edge-bonding.

931.40 **Triple Bond:** When three vertexes come together, it is called a *fixed* bond, a three-point landing. It is like an engineering fixed joint; it is *rigid.* Triple-bonding elucidates both the formational and continuing behaviors of crystalline substances. This also is face-bonding.

931.50 **Quadruple Bond:**When four vertexes are congruent, we have quadruple-bonded *densification.* The relationship is quadrivalent. Quadri-bond and mid-edge coordinate tetrahedron systems demonstrate the super-strengths of substances such as diamonds and metals. This is the way carbon suddenly becomes very dense, as in a diamond. This is multiple self-congruence.

931.51 The behavioral hierarchy of bondings is integrated four-dimensionally with the synergies of mass-interattractions and precession.

931.60 **Quadrivalence of Energy Structures Closer-Than-Sphere Packing:**In 1885, van't Hoff showed that all organic chemical structuring is tetrahedrally configured and interaccounted in vertexial linkage. A constellation of tetrahedra linked together entirely by such single-bonded universal jointing uses lots of space, which is the openmost condition of flexibility and mutability characterizing the behavior of gases. The medium- packed condition of a double-bonded, hinged arrangement is still flexible, but sum-totally as an aggregate, allspace-filling complex is noncompressible–as are liquids. The closest- packing, triple-bonded, fixed-end arrangement corresponds with rigid-structure molecular compounds.

931.61 The closest-packing concept was developed in respect to spherical aggregates with the convex and concave octahedra and vector equilibria spaces between the spheres. Spherical closest packing overlooks a much closer packed condition of energy structures, which, however, had been comprehended by organic chemistry–that of quadrivalent and fourfold bonding, which corresponds to outright congruence of the octahedra or tetrahedra themselves. When carbon transforms from its soft, pressed-cake, carbon black powder (or charcoal) arrangement to its diamond arrangement, it converts from the so-called closest arrangement of triple bonding to quadrivalence. We call this self-congruence packing, as a single tetrahedron arrangement in contradistinction to closest packing as a neighboring-group arrangement of spheres.

931.62 Linus Pauling's X-ray diffraction analyses revealed that all metals are tetrahedrally organized in configurations interlinking the gravitational centers of the compounded atoms. It is characteristic of metals that an alloy is stronger when the different metals' unique, atomic, constellation symmetries have congruent centers of gravity, providing mid-edge, mid-face, and other coordinate, interspatial accommodation of the elements' various symmetric systems.

931.63 In omnitetrahedral structuring, a triple-bonded linear, tetrahedral array may coincide, probably significantly, with the DNA helix. The four unique quanta corners of the tetrahedron may explain DNA's unzipping dichotomy as well as–T-A; G-C–patterning control of all reproductions of all biological species.

932.00 **Viral Steerability**

932.01 The four chemical compounds guanine, cytosine, thymine, and adenine, whose first letters are GCTA, and of which DNA always consists in various paired code pattern sequences, such as GC, GC, CG, AT, TA, GC, in which A and T are always paired as are G and C. The pattern controls effected by DNA in all biological structures can be demonstrated by equivalent variations of the four individually unique spherical radii of two unique pairs of spheres which may be centered in any variation of series that will result in the viral steerability of the shaping of the DNA tetrahelix prototypes. (See Sec. 1050.00 et. seq.)

932.02 One of the main characteristics of DNA is that we have in its helix a structural patterning instruction, all four-dimensional patterning being controlled only by frequency and angle modulatability. The coding of the four principal chemical compounds, GCTA, contains all the instructions for the designing of all the patterns known to biological life. These four letters govern the coding of the life structures. With new life, there is a parent-child code controls unzipping. There is a dichotomy and the new life breaks off from the old with a perfect imprint and control, wherewith in turn to produce and design others.

933.00 Tetrahelix

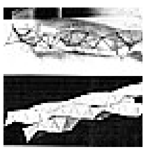

Fig. 933.01

933.01 The tetrahelix is a helical array of triple-bonded tetrahedra. (See Illus. 933.01) We have a column of tetrahedra with straight edges, but when face-bonded to one another, and the tetrahedra's edges are interconnected, they altogether form a hyperbolic-parabolic, helical column. The column spirals around to make the helix, and it takes just ten tetrahedra to complete one cycle of the helix.

933.02 This tetrahelix column can be equiangle-triangular, triple-ribbon-wave produced as in the methodology of Secs. 930.10 and 930.20 by taking a ribbon three-panels wide instead of one-panel wide as in Sec. 930.10. With this triple panel folded along both of its interior lines running parallel to the three-band-wide ribbon's outer edges, and with each of the three bands interiorly scribed and folded on the lines of the equiangle-triangular wave pattern, it will be found that what might at first seem to promise to be a straight, prismatic, three-edged, triangular-based column–upon matching the next-nearest above, wave interval, outer edges of the three panels together (and taping them together)–will form the same tetrahelix column as that which is produced by taking separate equiedged tetrahedra and face-bonding them together. There is no distinguishable difference, as shown in the illustration.

933.03 The tetrahelix column may be made positive (like the right-hand-threaded screw) or negative (like the left-hand-threaded screw) by matching the next-nearest-below wave interval of the triple-band, triangular wave's outer edges together, or by starting the triple-bonding of separate tetrahedra by bonding in the only alternate manner provided by the two possible triangular faces of the first tetrahedron furthest away from the starting edge; for such columns always start and end with a tetrahedron's edge and not with its face.

933.04 Such tetrahelical columns may be made with regular or irregular tetrahedral components because the sum of the angles of a tetrahedron's face will always be 720 degrees, whether regular or asymmetric. If we employed asymmetric tetrahedra they would have six different edge lengths, as would be the case if we had four different diametric balls–G, C, T, A–and we paired them tangentially, G with C, and T with A, and we then nested them together (as in Sec. 623.12), and by continuing the columns in any different combinations of these pairs we would be able to modulate the rate of angular changes to design approximately any form.

933.05 This synergetics' tetrahelix is capable of demonstrating the molecular-compounding characteristic of the Watson-Crick model of the DNA, that of the deoxyribonucleic acid. When Drs. Watson, Wilkins, and Crick made their famous model of the DNA, they made a chemist's reconstruct from the information they were receiving, but not as a microscopic photograph taken through a camera. It was simply a schematic reconstruction of the data they were receiving regarding the relevant chemical associating and the disassociating. They found that a helix was developing.

933.06 They found there were 36 rotational degrees of arc accomplished by each increment of the helix and the 36 degrees aggregated as 10 arc increments in every complete helical cycle of 360 degrees. Although there has been no identification of the tetrahelix column of synergetics with the Watson-Crick model, the numbers of the increments are the same. Other molecular biologists also have found a correspondence of the tetrahelix with the structure used by some of the humans' muscle fibers.

933.07 When we address two or more positive or two or more negative tetrahelixes together, the positives nestle their angling forms into one another, as the negatives nestle likewise into one another's forms.

933.08 **Closest Packing of Different-sized Balls:**It could be that the CCTA tetrahelix derives from the closest packing of different-sized balls. The Mites and Sytes (see Sec. 953) could be the tetrahedra of the GCTA because they are both positive-negative and allspace filling.

Appendix III

A006003 n * (n^2 + 1) / 2. 73
(Formerly M3849)

0, 1, 5, 15, 34, 65, 111, 175, 260, 369, 505, 671, 870, 1105, 1379, 1695, 2056, 2465, 2925, 3439, 4010, 4641, 5335, 6095, 6924, 7825, 8801, 9855, 10990, 12209, 13515, 14911, 16400, 17985, 19669, 21455, 23346, 25345, 27455, 29679, 32020, 34481(list; graph; refs; listen; history; text; internal format)

OFFSET 0,3

COMMENTS Comment from Felice Russo: Write the natural numbers in groups: 1; 2,3; 4,5,6; 7,8,9,10; ... and add the groups. In other words, "sum of the next n natural numbers".

Number of rhombi in an n X n rhombus, if 'crossformed' rhombi are allowed - Matti De Craene (Matti.DeCraene(AT)rug.ac.be), May 14 2000

Also the sum of the integers between T(n-1)+1 and T(n), the n-th triangular number (A000217). Sum of n-th row of A000027 regarded as a triangular array.

Unlike the cubes which have a similar definition, it is possible for 2 elements of this sequence to sum to a third. E.g. a(36)+a(37)=23346+25345=48691=a(46). Might be called 2nd order triangular numbers, thus defining 3rd order triangular numbers (A027441) as n(n^3+1)/2, etc... - Jon Perry, Jan 14 2004

Also as a(n)=(1/6)*(3*n^3+3*n), n>0: structured trigonal diamond numbers (vertex structure 4) (Cf. A000330 = alternate vertex; A000447 = structured diamonds; A100145 for more on structured numbers). - James A. Record (james.record(AT)gmail.com), Nov. 7, 2004.

The sequence M(n) of magic constants for n X n magic squares (numbered 1 through n^2) from n=3 begins M(n)=15, 34, 65, 111, 175, 260, ... - Lekraj

Beedassy, Apr 16 2005. [Comment corrected by Colin Hall, Sep 11 2009]

The sequence Q(n) of magic constants for the n-queens problem in chess begins 0, 0, 0, 0, 34, 65, 111, 175, 260, ... - Paul Muljadi, Aug 23, 2005.

Alternate terms of A057587. - Jeremy Gardiner, Apr 10 2005

Also partial differences of A063488(n) = (2*n-1)*(n^2-n+2)/2. a(n) = A063488(n) - A063488(n-1) for n>1. - Alexander Adamchuk, Jun 03 2006

In an n x n grid of numbers from 1 to n^2, select -- in any manner -- one number from each row and column. Sum the selected numbers. The sum is independent of the choices and is equal to the n-th term of this sequence. - F.-J. Papp (fjpapp(AT)umich.edu), Jun 06 2006

Sequence allows us to find X values of the equation:(X-Y)^3-(X+Y)=0. To find Y values: b(n)=(n^3-n)/2. - Mohamed Bouhamida (bhmd95(AT)yahoo.fr), May 16 2006

For the equation: m*(X-Y)^k-(X+Y)=0 with X>=Y,k>=2 and m is an odd number the X values are given by the sequence defined by: a(n)=(m*n^k+n)/2. The Y values are given by the sequence defined by: b(n)=(m*n^k-n)/2. - Mohamed Bouhamida (bhmd95(AT)yahoo.fr), May 16 2006

If X is an n-set and Y a fixed 3-subset of X then a(n-3) is equal to the number of 4-subsets of X intersecting Y. - Milan Janjic, Jul 30 2007

(m*(2n)^k+n, m*(2n)^k-n) solves the Diophantine equation: 2m*(X-Y)^k-(X+Y)=0 with X>=Y,k>=2 where m is a natural integer. - Mohamed Bouhamida (bhmd95(AT)yahoo.fr), Oct 02 2007

Also c^(1/2) in a^(1/2) + b^(1/2) = c^(1/2) such that a^2 + b = c. - Cino Hilliard (hillcino368(AT)hotmail.com), Feb 09 2008

Number of units of a(n) belongs to a periodic sequence: 0, 1, 5, 5, 4, 5, 1, 5, 0, 9, 5, 1, 0, 5, 9, 5, 6, 5, 5, 9. [From Mohamed Bouhamida (bhmd95(AT)yahoo.fr), Sep 04 2009]

The n-th row sums of Floyd's triangle are 1, 5, 15, 34, 65, 111, 175, 260, ... [From Paul Muljadi, Jan 25 2010]

a(n) = n*A000217(n) - sum(A001477(i), i=0..n-1). [Bruno Berselli, Apr 25 2010]

a(n) is the number of triples (w,x,y) having all terms in {0,...n} such that at least one of these inequalities fails: x+y<w, y+w<x, w+x<y. [Clark Kimberling, Jun 14 2012]

Sum of n-th row of the triangle in A209297. - Reinhard Zumkeller, Jan 19 2013

a(n) = A000217(n) + n*A000217(n-1). [Bruno Berselli, Jun 07 2013]

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S. M. Losanitsch, Die Isomerie-Arten bei den Homologen der Paraffin-Reihe, Chem. Ber. 30 (1897), 1917-1926.

T. P. Martin, Shells of atoms, Phys. Reports, 273 (1996), 199-241, eq. (11).

F.-J. Papp, Colloquium Talk, Department of Mathematics, University of Michigan-Dearborn, 2006 March 6

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J. D. Bell, A translation of Leonhard Euler's "De Quadratis Magicis", E795

Milan Janjic, Two Enumerative Functions

Eric Weisstein's World of Mathematics, Magic Constant.

Wikipedia, Floyd's triangle [From Paul Muljadi, Jan 25 2010]

Index entries for sequences related to linear recurrences with constant coefficients

Index entries for sequences related to magic squares

FORMULA binomial(n, 3)+binomial(n-1, 3)+binomial(n-2, 3).
G.f.: x*(1+x+x^2)/(x-1)^4. - Floor van Lamoen (fvlamoen(AT)hotmail.com), Feb 11 2002.
Partial sums of A005448, centered triangular numbers: 3n(n-1)/2 + 1. - Jonathan Vos Post, Mar 16 2006
Binomial transform of [1, 4, 6, 3, 0, 0, 0,...] = (1, 5, 15, 34, 65,...). - Gary W. Adamson, Aug 10 2007
a(-n) = -a(n). - Michael Somos, Dec 24 2011
a(n) = sum_{k = 1..n} A(k-1, k-1-n) where A(i, j) = i^2 + i*j + j^2 + i + j + 1. - Michael Somos, Jan 02 2012
a(n) = 4*a(n-1) - 6*a(n-2) + 4*a(n-3) - a(n-4), with a(0)=0, a(1)=1, a(2)=5, a(3)=15. Harvey P. Dale, May 16 2012
a(n) = 3*a(n-1) - 3*a(n-2) + a(n-3) + 3. - Ant King, Jun 13 2012

EXAMPLE x + 5*x^2 + 15*x^3 + 34*x^4 + 65*x^5 + 111*x^6 + 175*x^7 + 260*x^8 + ...

MAPLE with (combinat):seq((fibonacci(4, n)+n^3)/4, n=0..41); - Zerinvary Lajos (zerinvarylajos(AT)yahoo.com), May 25 2008

MATHEMATICA Table[n(n^2 + 1)/2, {n, 0, 45}]
LinearRecurrence[{4, -6, 4, -1}, {0, 1, 5, 15}, 50] Harvey P. Dale, May 16 2012

PROG (PARI) { v=vector(100, i, i*(i^2+1)/2); x=vector(1275); c=0; for (i=1, 50, for (j=i, 50, x[c++]=v[j]-v[i])); for (k=1, 1275, for (l=1, 100, if (x[k]==v[l], print(x[k]); break))) } (Perry)
(PARI) {a(n) = n * (n^2 + 1) / 2} /* Michael Somos, Dec 24 2011 */
(Haskell)
a006003 n = n * (n ^ 2 + 1) `div` 2
a006003_list = scanl (+) 0 a005448_list
-- Reinhard Zumkeller, Jun 20 2013

CROSSREFS Cf. A000330, A000537, A066886, A057587, A027480.
Cf. A000578 (cubes).
Cf. A007742, A005449.
(1/12)*t*(n^3-n)+n for t = 2, 4, 6, ... gives A004006, A006527, A006003, A005900, A004068, A000578, A004126, A000447, A004188, A004466, A004467, A007588, A062025, A063521, A063522, A063523.
Antidiagonal sums of array in A000027.
Cf. A005448.
Cf. A063488 - Sum of two consecutive terms.
Cf. A118465.
Cf. A226449. [Bruno Berselli, Jun 09 2013]
Cf. A034262.
Cf. A080992.
Sequence in context: A147264 A147150 A162513 * A111385 A026101 A084288
Adjacent sequences: A006000 A006001 A006002 * A006004 A006005 A006006

KEYWORD nonn,easy,nice

AUTHOR N. J. A. Sloane, Simon Plouffe

EXTENSIONS Better description from Albert Rich (Albert_Rich(AT)msn.com) 3/97.
More terms from Robert G. Wilson v, Apr 15 2002
This is a second attempt at correction, first submission is hereby withdrawn. Corrected comment by Lekraj Beedassy on magic squares. n=2 does not exist, not strictly correct to set M(2)=0 Colin Hall, Sep 11 2009

STATUS approved

Appendix IV

Forgetful functor

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In mathematics, in the area of category theory, a **forgetful functor** (also known as a **stripping functor**) 'forgets' or drops some or all of the input's structure or properties 'before' mapping to the output. For an algebraic structure of a given signature, this may be expressed by curtailing the signature: the new signature is an edited form of the old one. If the signature is left as an empty list, the functor is simply to take the **underlying set** of a structure. Because many structures in mathematics consist of a set with an additional added structure, a forgetful functor that maps to the underlying set is the most common case.

Introduction

As examples, there are several forgetful functors from the category of commutative rings. A (unital) ring, described in the language of universal algebra, is an ordered tuple $(R, +, *, a, 0, 1)$ satisfying certain axioms, where "+" and "*" are binary functions on the set R, a is a unary operation corresponding to additive inverse, and 0 and 1 are nullary operations giving the identities of the two binary operations. Deleting the 1 gives a forgetful functor to the category of rings without unit; it simply "forgets" the unit. Deleting "*" and 1 yields a functor to the category of abelian groups, which assigns to each ring R the underlying additive abelian group of R. To each morphism of rings is assigned the same function considered merely as a morphism of addition between the underlying groups. Deleting all the operations gives the functor to the underlying set R.

It is beneficial to distinguish between forgetful functors that "forget structure" versus those that "forget properties". For example, in the above example of commutative rings, in addition to those functors that delete some of the operations, there are functors that forget some of the axioms. There is a functor from the category **CRing** to **Ring** that forgets the axiom of commutativity, but keeps all the operations. Occasionally the object may include extra sets not defined strictly in terms of the underlying set (in this case, which part to consider the underlying set is a matter of taste, though this is rarely ambiguous in practice). For these objects, there are forgetful functors that forget the extra sets that are more general.

Most common objects studied in mathematics are constructed as underlying sets along with extra sets of structure on those sets (operations on the underlying set, privileged subsets of the underlying set, etc.) which may satisfy some axioms. For these objects, a commonly considered forgetful functor is as follows. Let $\mathcal{C}$ be any category based on sets, e.g. groups - sets of elements - or topological spaces - sets of 'points'. As usual, write $\mathrm{Ob}(\mathcal{C})$ for the objects of $\mathcal{C}$ and write $\mathrm{Fl}(\mathcal{C})$ for the morphisms of the same. Consider the rule:

$$A \text{ in } \mathrm{Ob}(\mathcal{C}), \quad A \mapsto |A| = \text{the underlying set of } A,$$
$$u \text{ in } \mathrm{Fl}(\mathcal{C}), \quad u \mapsto |u| = \text{the morphism, } u\text{, as a map of sets.}$$

The functor $|\ |$ is then the forgetful functor from $\mathcal{C}$ to $\mathbf{Set}$, the category of sets.

Forgetful functors are almost always faithful. Concrete categories have forgetful functors to the category of sets—indeed they may be *defined* as those categories that admit a faithful functor to that category.

Forgetful functors that only forget axioms are always fully faithful; every morphism that respects the structure between objects that satisfy the axioms automatically also respects the axioms. Forgetful functors that forget structures need not be full; some morphisms don't respect the structure. These functors are still faithful though; distinct morphisms that do respect the structure are still distinct when the structure is forgotten. Functors that forget the extra sets need not be faithful; distinct morphisms respecting the structure of those extra sets may be indistinguishable on the underlying set.

In the language of formal logic, a functor of the first kind removes axioms. The second kind removes predicates. The third kind remove types.

An example of the first kind is the forgetful functor **Ab** → **Grp**. One of the second kind is the forgetful functor **Ab** → **Set**. A functor of the third kind is the functor **Mod** → **Ab**, where **Mod** is the fibred category of all modules over arbitrary rings. To see this, just choose a ring homomorphism between the underlying rings that does not change the ring action. Under the forgetful functor, this morphism yields the identity. Note that an object in **Mod** is a tuple, which includes a ring and an abelian group, so which to forget is a matter of taste.

Left Adjoint: Free

Forgetful functors tend to have left adjoints, which are 'free' constructions. For example:

- free module: the forgetful functor from $\mathbf{Mod}(R)$(the category of R-module) to $\mathbf{Set}$has left adjoint Free_R, with $X \mapsto \mathrm{Free}_R(X)$, the free R-module with basis X.
- free group
- free lattice
- tensor algebra
- free category, adjoint to the forgetful functor from categories to quivers

For a more extensive list, see (Mac Lane 1997).

As this is a fundamental example of adjoints, we spell it out: adjointness means that given a set X and an object (say, an R-module) M, maps *of sets* $X \to M$ correspond to maps of modules $\mathrm{Free}_R(X) \to M$: every map of sets yields a map of modules, and every map of modules comes from a map of sets.

In the case of vector spaces, this is summarized as: "A map between vector spaces is determined by where it sends a basis, and a basis can be mapped to anything."

Symbolically:

$$\mathrm{Hom}_{\mathbf{Mod}_R}(\mathrm{Free}_R(X), M) = \mathrm{Hom}_{\mathbf{Set}}(X, \mathrm{Forget}(M)).$$

The unit of the free–forget adjunction is the "inclusion of a basis": $X \to \mathrm{Free}_R(X)$.

Fld, the category of fields, furnishes an example of a forgetful functor with no adjoint. There is no field satisfying a free universal property for a given set.