# Numerical Integration of the Negative Energy Density in the Natario Warp Drive Spacetime using 3 different Natario Shape Functions

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**Abstract**

Warp Drives are solutions of the Einstein Field Equations that allows superluminal travel within the framework of General Relativity. There are at the present moment two known solutions: The Alcubierre warp drive discovered in 1994 and the Natario warp drive discovered in 2001. However as stated by both Alcubierre and Natario themselves the warp drive violates all the known energy conditions because the stress energy momentum tensor is negative implying in a negative energy density. While from a classical point of view the negative energy is forbidden the Quantum Field Theory allows the existence of very small amounts of it being the Casimir effect a good example as stated by Alcubierre himself.The major drawback concerning negative energies for the warp drive is the huge amount of negative energy able to sustain the warp bubble.Ford and Pfenning computed the amount of negative energy needed to maintain an Alcubierre warp drive and they arrived at the result of 10 times the mass of the entire Universe for a stable warp drive configuration rendering the warp drive impossible.We introduce here 3 new shape functions that defines the Natario warp drive spacetime and we choose one of these functions as an excellent candidate to low the negative energy density requirements to affordable levels.We demonstrate in this work that both Alcubierre and Natario warp drives have two warped regions and not only one.One of these warped regions is associated with geometry and the other is associated with negative energy requirements.We also discuss Horizons and Doppler Blueshifts that affects the Alcubierre spacetime but not the Natario counterpart.

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# 1 Introduction

The Warp Drive as a solution of the Einstein Field Equations of General Relativity that allows superluminal travel appeared first in 1994 due to the work of Alcubierre.([1]) The warp drive as conceived by Alcubierre worked with an expansion of the spacetime behind an object and contraction of the spacetime in front.The departure point is being moved away from the object and the destination point is being moved closer to the object.The object do not moves at all[1].It remains at the rest inside the so called warp bubble but an external observer would see the object passing by him at superluminal speeds(pg 8 in [1])(pg 1 in [2]).

Later on in 2001 another warp drive appeared due to the work of Natario.([2]).This do not expands or contracts spacetime but deals with the spacetime as a "strain" tensor of Fluid Mechanics(pg 5 in [2]). Imagine the object being a fish inside an aquarium and the aquarium is floating in the surface of a river but carried out by the river stream.The warp bubble in this case is the aquarium whose walls do not expand or contract. An observer in the margin of the river would see the aquarium passing by him at a large speed but inside the aquarium the fish is at the rest with respect to his local neighborhoods.

However the major drawback that affects the warp drive is the quest of large negative energy requirements enough to sustain the warp bubble.While from a classical point of view negative energy densities are forbidden the Quantum Field Theory allows the existence of very small quantities of such energies but unfortunately the warp drive requires immense amounts of it.Ford and Pfenning computed the negative energy density needed to maintain a warp bubble and they arrived at the conclusion that in order to sustain a stable configuration able to perform interstellar travel the amount of negative energy density is of about 10 times the mass of the Universe and they concluded that the warp drive is impossible.(see pg 10 in [3] and pg 78 in [5]).

Another drawback that affects the warp drive is the quest of the interstellar navigation:Interstellar space is not empty and from a real point of view a ship at superluminal speeds would impact asteroids,comets,interstellar space dust and photons of Cosmic Background Radiation(COBE).

According to Clark,Hiscock and Larson a single collision between a ship and a COBE photon would release an amount of energy equal to the photosphere of a star like the Sun.(see pg 11 in [9]).And how many photons of COBE we have per cubic centimeter of space??

These highly energetic collisions would pose a very serious threat to the astronauts as pointed out by McMonigal,Lewis and O'Byrne (see pg 10 in [10]).

Another problem:these highly energetic collisions would raise the temperature of the warp bubble reaching the Hawking temperature as pointed out by Barcelo,Finazzi and Liberati.(see pg 6 in in [11]).At pg 9 they postulate that all future spaceships cannot bypass 99 percent of the light speed.

In section 4 we will see that these problems of interstellar navigation affects the Alcubierre warp drive but not the Natario one.

[1] do not violates Relativity

The last drawback raised against the warp drive is the fact that inside the warp bubble an astronaut cannot send signals with the speed of the light to control the front of the bubble because an Horizon(causally disconnected portion of spacetime)is established between the astronaut and the warp bubble.We discuss this in section 4 and in section 5 we discuss a possible way to overcome the Horizon problem using only General Relativity.

Recently Harold White discovered that by a manipulation of the parameter @ in the original shape function that defines the Alcubierre spacetime the amounts of negative energy density needed to maintain the warp drive can be lowered to more reasonable levels.(see pg 4 fig 2 in [8])(see pg 8 in [17]).

In this work we introduce 3 shape functions that defines the Natario spacetime and we choose one of these functions as the best candidate to lower the negative energy density requirements to arbitrary low levels.

We adopted the International System of Units where $G = 6,67 \times 10^{-11}$ $\frac{Newton \times meters^2}{kilograms^2}$ and $c = 3 \times 10^8$ $\frac{meters}{seconds}$ and not the Geometrized System of units in which $c = G = 1$.

We consider here a Natario warp drive with a radius $R = 100$ meters a thickness parameter with the values $@ = 50000$,$@ = 75000$ and $@ = 100000$ moving with a speed 200 times faster than light implying in a $vs = 2 \times 10^2 \times 3 \times 10^8 = 6 \times 10^{10}$ and a $vs^2 = 3,6 \times 10^{21}$

We also adopt a warp factor as a dimensionless parameter in our 3 different Natario shape functions with a value $WF = 200$

This work is a companion of our works [4] and [13] and we recommend a previous study of them because this work is intended to be a continuation of [4] and [13] but by far much more advanced.

This work is organized as follows:

- Section 2)-Outlines the problems of the immense magnitude in negative energy density when a ship travels with a speed of 200 times faster then light.

- Section 3)-The most important section in this work.Introduces the 3 new shape functions that defines the Natario warp drive spacetime outlining the existence of two warped regions one associated with geometry and the other associated with energy.One of these functions is the best candidate to lower the energy density requirements in the Natario warp drive to affordable levels.

- Section 4)-Outlines the major advantages of the Natario warp drive spacetime when compared to its Alcubierre counterpart.The Natario warp drive can survive to the Horizons and Doppler Blueshift problem.It can also survive against the objections raised by Clark,Hiscock,Larson, McMonigal,Lewis,O'Byrne,Barcelo,Finazzi and Liberati

- Section 5)-Outlines the possibility of how to overcome the Horizon problem from an original point of view of General Relativity.

## 2 The Problem of the Negative Energy in the Natario Warp Drive Spacetime-The Unphysical Nature of Warp Drive

The negative energy density for the Natario warp drive is given by(see pg 5 in [2])

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2\sin^2\theta\right] \tag{1}$$

Converting from the Geometrized System of Units to the International System we should expect for the following expression(see eqs 21 and 23 pg 6 in [4]):

$$\rho = -\frac{c^2}{G}\frac{vs^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{rs}{2}n''(rs)\right)^2\sin^2\theta\right]. \tag{2}$$

Rewriting the Natario negative energy density in cartezian coordinates we should expect for[2]:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2 + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{3}$$

In the equatorial plane:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{4}$$

Note that in the above expressions the warp drive speed $vs$ appears raised to a power of 2. Considering our Natario warp drive moving with $vs = 200$ which means to say 200 times light speed in order to make a round trip from Earth to a nearby star at 20 light-years away in a reasonable amount of time(in months not in years) we would get in the expression of the negative energy the factor $c^2 = (3 \times 10^8)^2 = 9 \times 10^{16}$ being divided by $6,67 \times 10^{-11}$ giving $1,35 \times 10^{27}$ and this is multiplied by $(6 \times 10^{10})^2 = 36 \times 10^{20}$ coming from the term $vs = 200$ giving $1,35 \times 10^{27} \times 36 \times 10^{20} = 1,35 \times 10^{27} \times 3,6 \times 10^{21} = 4,86 \times 10^{48}$ !!!

A number with 48 zeros!!!Our Earth have a mass[3] of about $6 \times 10^{24} kg$

This term is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!!!or better:The amount of negative energy density needed to sustain a warp bubble at a speed of 200 times faster than light requires the magnitude of the masses of 1.000.000.000.000.000.000.000.000 planet Earths for both Alcubierre and Natario cases!!!!

And multiplying the mass of Earth by $c^2$ in order to get the total positive energy "stored" in the Earth according to the Einstein equation $E = mc^2$ we would find the value of $54 \times 10^{40} = 5,4 \times 10^{41} Joules$.

Earth have a positive energy of $10^{41} Joules$ and dividing this by the volume of the Earth(radius $R_{Earth} =$ 6300 km approximately) we would find the positive energy density of the Earth.Taking the cube of the Earth radius $(6300000m = 6,3 \times 10^6)^3 = 2,5 \times 10^{20}$ and dividing $5,4 \times 10^{41}$ by $(4/3)\pi R_{Earth}^3$ we would find the value of $4,77 \times 10^{20} \frac{Joules}{m^3}$. So Earth have a positive energy density of $4,77 \times 10^{20} \frac{Joules}{m^3}$ and we are talking about negative energy densities with a factor of $10^{48}$ for the warp drive while the quantum theory allows only microscopical amounts of negative energy density.

[2] see Appendix A

[3] see Wikipedia:The free Encyclopedia

So we would need to generate in order to maintain a warp drive with 200 times light speed the negative energy density equivalent to the positive energy density of $10^{28}$ Earths!!!!

A number with 28 zeros!!!.Unfortunately we must agree with the major part of the scientific community that says:"Warp Drive is impossible and unphysical!!"(eg Ford-Pfenning)(see pg 10 in [3] and pg 78 in [5]).

However looking better to the expression of the negative energy density in the equatorial plane of the Natario warp drive:

$$\rho = T_{\mu\nu} u^{\mu} u^{\nu} = -\frac{c^2}{G} \frac{v_s^2}{8\pi} \left[ 3(n'(rs))^2 \right] \tag{5}$$

We can see that a very low derivative and hence its square can perhaps obliterate the huge factor of $10^{48}$ ameliorating the negative energy requirements to sustain the warp drive.By manipulating the term @ in the original Alcubierre shape function Harold White lowered these requirements for the Alcubierre warp drive.(see pg 4 fig 2 in [8])(see pg 8 in [17])

In the next section we will introduce 3 new shape functions that defines the Natario warp drive spacetime and we will choose one that allows the reduction of the negative energy requirements from 10 times the mass of the Universe that would render the warp drive as impossible and unphysical to arbitrary low values.

## 3 Numerical Integration of the Negative Energy Density in the Natario Warp Drive Spacetime using 3 different Natario Shape Functions

According to Natario(pg 5 in [2]) any function that gives 0 inside the bubble and $\frac{1}{2}$ outside the bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region is a valid shape function for the Natario warp drive.

The 3 Natario warp drive continuous shape functions ($A$,$B$ and $C$) introduced here and its derivatives squares can be defined by:

- Function $A$

$$n(rs) = [\frac{1}{2}][1 - f(rs)]^{WF} \tag{6}$$

$$n'(rs)^2 = [\frac{1}{4}]WF^2[1 - f(rs)]^{2(WF-1)}f'(rs)^2 \tag{7}$$

- Function $B$

$$n(rs) = [\frac{1}{2}][1 - f(rs)^{WF}] \tag{8}$$

$$n'(rs)^2 = [\frac{1}{4}]WF^2[f(rs)^{2(WF-1)}]f'(rs)^2 \tag{9}$$

- Function $C$

$$n(rs) = [\frac{1}{2}][1 - f(rs)^{WF}]^{WF} \tag{10}$$

$$n'(rs)^2 = [\frac{1}{4}]WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}]f'(rs)^2 \tag{11}$$

Each one of these shape functions gives the result of $n(rs) = 0$ inside the warp bubble and $n(rs) = \frac{1}{2}$ outside the warp bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region(see pg 5 in [2]).Then each one of these functions is a valid shape function for the Natario warp drive spacetime.

Function $A$ was already introduced in [4].(see eq 38 pg 9 and eqs 39 and 40 pg 10 in [4]).It also appears in section 4 pg 10 in [13].

Note that the Alcubierre shape function is being used to define its 3 Natario shape function counterparts.Below is presented the Alcubierre shape function and its derivative square.(see eqs 6 and 7 pg 4 in [1] or top of pg 4 in [2])[4].

$$f(rs) = \frac{1}{2}[1 - tanh[@(rs - R)] \tag{12}$$

$$f'(rs)^2 = \frac{1}{4}[\frac{@^2}{cosh^4[@(rs - R)]}] \tag{13}$$

.

$$rs = \sqrt{(x - xs)^2 + y^2 + z^2} \tag{14}$$

According with Alcubierre any function $f(rs)$[5] that gives 1 inside the bubble and 0 outside the bubble while being $1 > f(rs) > 0$ in the Alcubierre warped region is a valid shape function for the Alcubierre warp drive.(see eqs 6 and 7 pg 4 in [1] or top of pg 4 in [2]).

In the Alcubierre shape function $xs$ is the center of the warp bubble where the ship resides. $R$ is the radius of the warp bubble and @ is the Alcubierre dimensionless parameter[6] related to the thickness.According to Alcubierre these can have arbitrary values.We outline here the fact that according to pg 4 in [1] the parameter @ can have arbitrary values.This is very important for the White analysis as we will see later.

The shape function $f(rs)$ have a value of 1 inside the warp bubble and zero outside the warp bubble while being $0 < f(rs) < 1$ in the warp bubble walls.$rs$ is the path of the so-called Eulerian observer that starts at the center of the bubble $xs$ and ends up outside the warp bubble.In our case we consider the equatorial plane and we have for $rs$ the following expression[7].

$$rs = \sqrt{(x - xs)^2} \tag{15}$$

$$rs = x - xs \tag{16}$$

The term $WF$ in the 3 Natario shape functions is dimensionless too:it is the warp factor that will squeeze the region where the derivatives of the Natario shape functions are different than 0.

For each one of the 3 Natario shape functions introduced here it is easy to figure out when $f(rs) = 1$(interior of the Alcubierre bubble) then $n(rs) = 0$(interior of the Natario bubble) and when $f(rs) = 0$(exterior of the Alcubierre bubble)then $n(rs) = \frac{1}{2}$(exterior of the Natario bubble).

We consider here a Natario warp drive with a radius $R = 100$ meters a thickness parameter with the values $@ = 50000$,$@ = 75000$ and $@ = 100000$ and a warp factor with a value $WF = 200$.

We provide numerical plots of these values for all the 3 Natario shape functions.

[4] $tanh[@(rs+R)] = 1$,$tanh(@R) = 1$ for values of the Alcubierre thickness parameter $@ = 50000$,$@ = 75000$ and $@ = 100000$

[5] $f(rs)$ and its derivative square are dimensionless.see section 2 in [13]

[6] in this work we use numerical plots of a Natario warp bubble with a radius $R = 100$ meters and dimensionless parameter @ with the values $@ = 50000$,$@ = 75000$ and $@ = 100000$ and not $@ = 50000$ meters,and not $@ = 75000$ meters and not $@ = 100000$ meters

[7] see Appendix A

- Numerical Plot when @ = 50000 for the Functions $A$ and $B$

Function $A$

| $rs$ | $f(rs)$ | $n(rs)$ | $f'(rs)^2$ | $n'(rs)^2$ |
|---|---|---|---|---|
| $9,99970000000E+001$ | 1 | 0 | $2,650396620740E-251$ | 0 |
| $9,99980000000E+001$ | 1 | 0 | $1,915169647489E-164$ | 0 |
| $9,99990000000E+001$ | 1 | 0 | $1,383896564748E-077$ | 0 |
| $1,00000000000E+002$ | 0,5 | $3,111507638931E-061$ | $6,250000000000E+008$ | $9,681479787123E-108$ |
| $1,00001000000E+002$ | 0 | 0,5 | $1,383896486082E-077$ | $1,383896486082E-073$ |
| $1,00002000000E+002$ | 0 | 0,5 | $1,915169538624E-164$ | $1,915169538624E-160$ |
| $1,00003000000E+002$ | 0 | 0,5 | $2,650396470082E-251$ | $2,650396470082E-247$ |

Function $B$

| $rs$ | $f(rs)$ | $n(rs)$ | $f'(rs)^2$ | $n'(rs)^2$ |
|---|---|---|---|---|
| $9,99970000000E+001$ | 1 | 0 | $2,650396620740E-251$ | $2,650396620740E-247$ |
| $9,99980000000E+001$ | 1 | 0 | $1,915169647489E-164$ | $1,915169647489E-160$ |
| $9,99990000000E+001$ | 1 | 0 | $1,383896564748E-077$ | $1,383896564748E-073$ |
| $1,00000000000E+002$ | 0,5 | $3,111507638931E-061$ | $6,250000000000E+008$ | $9,681479787123E-108$ |
| $1,00001000000E+002$ | 0 | 0,5 | $1,383896486082E-077$ | 0 |
| $1,00002000000E+002$ | 0 | 0,5 | $1,915169538624E-164$ | 0 |
| $1,00003000000E+002$ | 0 | 0,5 | $2,650396470082E-251$ | 0 |

Above are the numerical plots for a Natario warp drive spacetime with a $R = 100$ meters bubble radius when the Eulerian observer $rs$ initially at the center of the bubble($rs = 0$) approaches the end of the bubble ($rs \simeq R$)[8].Inside the bubble($rs < R$) $f(rs) = 1$ and $n(rs) = 0$ and outside the bubble($rs > R$) $f(rs) = 0$ and $n(rs) = 0,5$ according to both Alcubierre and Natario shape functions requirements.

Note that when $rs = 9,99970000000E+001$ still inside the bubble $f(rs) = 1$ but the derivative square is not zero and when $rs = 1,00003000000E+002$ already outside the bubble $f(rs) = 0$ but the derivative square is also not zero.

When $rs < 9,99970000000E+001$ or $rs > 1,00003000000E+002$ the derivatives of $f(rs) = 0$.Since the negative energy density in both Alcubierre and Natario warp drive spacetime is directly proportional to the derivative square of the shape functions we are interested in the region where the derivative squares are not zero.The expressions for the negative energy density in both Alcubierre and Natario warp drive spacetimes are given by:(see eq 8 pg 6 in [3] and pg 5 in [2])[9][10][11]:

$$\langle T^{\mu\nu} u_\mu u_\nu \rangle = \langle T^{00} \rangle = \frac{1}{8\pi} G^{00} = -\frac{1}{8\pi} \frac{v_s^2(t)[y^2 + z^2]}{4r_s^2(t)} \left( \frac{df(r_s)}{dr_s} \right)^2, \tag{17}$$

$$\rho = T_{\mu\nu} u^\mu u^\nu = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[ 3(n'(rs))^2 (\frac{x}{rs})^2 + \left( n'(rs) + \frac{r}{2} n''(rs) \right)^2 (\frac{y}{rs})^2 \right] \tag{18}$$

[8]the surface of the bubble is delimited by its radius.so the bubble terminates when $rs = R$

[9]$f(rs)$ is the Alcubierre shape function.Equations written in the Geometrized System of Units $c = G = 1$

[10]$n(rs)$ is the Natario shape function.Equation written the Geometrized System of Units $c = G = 1$

[11]Equation written in Cartezian Coordinates.See Appendix $A$

Note that when the derivative square of the Alcubierre shape function is zero(flat spacetime) ($f'(rs)^2 = 0$) then the derivative squares of the 3 Natario shape functions are also zero too.

Examining the plots of both Functions $A$ and $B$ we can see that both warp drives wether in Alcubierre or Natario case have two warped regions:

- 1)-The warped region where $1 > f(rs) > 0$ or $0 < n(rs) < \frac{1}{2}$ according with both Alcubierre and Natario requirements.-This warped region is known as the Geometrized warped region.
- 2)-The warped region where the derivative squares of both Alcubierre and Natario shape functions are not zero resulting in a non-null negative energy density and in a non-flat spacetime.-This warped region is known as the Energized warped region

Note that for both Functions $A$ and $B$ the Geometrized warped region is distributed in a thin layer over the neighborhoods of the bubble radius $rs \simeq R$ in both Alcubierre and Natario cases and it is the same in both Functions $A$ and $B$ but the Energized warped region extends itself over regions inside and outside the bubble ($rs < R$) and ($rs > R$) in the Alcubierre case only being also equal in both Functions $A$ and $B$.So in the Alcubierre case we have negative energy density inside and outside the bubble where $f(rs) = 1$ or $f(rs) = 0$ but the derivative squares ($f'(rs)^2 \neq 0$) and not only in the region where $1 > f(rs) > 0$.

Note that the Energized warped region in the Natario case is not equal in both Functions:

- 1)-In the case of Function $A$ the Energized warped region starts at the bubble radius ($rs = R$) and extends itself to the regions outside the bubble but in the neighborhoods of the bubble radius ($rs >= R$,$rs \cong R$).Faraway from the bubble radius but outside the bubble the derivative of the Natario shape function is zero and we recover again flat spacetime.So in the case of Function $A$ we have negative energy density outside the bubble.
- 2)-In the case of Function $B$ the Energized warped region starts inside the bubble but in the neighborhoods of the bubble radius ($rs <= R$,$rs \cong R$) and terminates exactly in the bubble radius($rs = R$). Faraway from the bubble radius but still inside the bubble the derivative of the Natario shape function is zero and we recover again flat spacetime.So in the case of Function $B$ we have negative energy density inside the bubble.

Why the Energized warped region in Alcubierre case for both Functions $A$ and $B$ is the same but the Energized warped region in the Natario case is diametrically opposed between the Functions $A$ and $B$ ??

Examining again the square of the derivatives of the shape function in both Functions $A$ and $B$:

$$n'(rs)^2_A = [\frac{1}{4}]WF^2[1 - f(rs)]^{2(WF-1)}f'(rs)^2 \tag{19}$$

$$n'(rs)^2_B = [\frac{1}{4}]WF^2[f(rs)^{2(WF-1)}]f'(rs)^2 \tag{20}$$

Inside the bubble $f(rs) = 1$ and $[1 - f(rs)]^{2(WF-1)} = 0$ resulting in a $n'(rs)^2_A = 0$.This is the reason why Function $A$ dont have derivatives inside the bubble.Outside the bubble $f(rs) = 0$ and $[f(rs)^{2(WF-1)}] = 0$ resulting in a $n'(rs)^2_B = 0$.This is the reason why Function $B$ dont have derivatives outside the bubble.

- Numerical Plot when $@ = 75000$ for the Functions $A$ and $B$

Function $A$

| $rs$ | $f(rs)$ | $n(rs)$ | $f'(rs)^2$ | $n'(rs)^2$ |
|---|---|---|---|---|
| $9,99980000000E+001$ | 1 | 0 | $5,963392481410E-251$ | 0 |
| $9,99990000000E+001$ | 1 | 0 | $1,158345097767E-120$ | 0 |
| $1,00000000000E+002$ | 0,5 | $3,111507638931E-061$ | $1,406250000000E+009$ | $2,178332952103E-107$ |
| $1,00001000000E+002$ | 0 | 0,5 | $1,158344999000E-120$ | $1,158344999000E-116$ |
| $1,00002000000E+002$ | 0 | 0,5 | $5,963391972940E-251$ | $5,963391972940E-247$ |

Function $B$

| $rs$ | $f(rs)$ | $n(rs)$ | $f'(rs)^2$ | $n'(rs)^2$ |
|---|---|---|---|---|
| $9,99980000000E+001$ | 1 | 0 | $5,963392481410E-251$ | $5,963392481410E-247$ |
| $9,99990000000E+001$ | 1 | 0 | $1,158345097767E-120$ | $1,158345097767E-116$ |
| $1,00000000000E+002$ | 0,5 | $3,111507638931E-061$ | $1,406250000000E+009$ | $2,178332952103E-107$ |
| $1,00001000000E+002$ | 0 | 0,5 | $1,158344999000E-120$ | 0 |
| $1,00002000000E+002$ | 0 | 0,5 | $5,963391972940E-251$ | 0 |

- Numerical Plot when $@ = 100000$ for the Functions $A$ and $B$

Function $A$

| $rs$ | $f(rs)$ | $n(rs)$ | $f'(rs)^2$ | $n'(rs)^2$ |
|---|---|---|---|---|
| $9,99990000000E+001$ | 1 | 0 | $7,660678807684E-164$ | 0 |
| $1,00000000000E+002$ | 0,5 | $3,111507638931E-061$ | $2,500000000000E+009$ | $3,872591914849E-107$ |
| $1,00001000000E+002$ | 0 | 0,5 | $7,660677936765E-164$ | $7,660677936765E-160$ |

Function $B$

| $rs$ | $f(rs)$ | $n(rs)$ | $f'(rs)^2$ | $n'(rs)^2$ |
|---|---|---|---|---|
| $9,99990000000E+001$ | 1 | 0 | $7,660678807684E-164$ | $7,660678807684E-160$ |
| $1,00000000000E+002$ | 0,5 | $3,111507638931E-061$ | $2,500000000000E+009$ | $3,872591914849E-107$ |
| $1,00001000000E+002$ | 0 | 0,5 | $7,660677936765E-164$ | 0 |

Above are the plots of the Functions $A$ and $B$ for two Natario warp drives each one with a radius $R = 100$ meters with thickness parameters $@ = 75000$ and $@ = 100000$.[12]

Note that as higher the thickness parameter @ becomes as thicker or thinner the limits or the boundaries of the Energized warped region(the region where the derivatives of $f(rs)$ are not zero)becomes too.

Note that the warp drives with thickness parameter $@ = 100000$ have an Energized warped region with a width (a thickness) smaller than its $@ = 50000$ and $@ = 75000$ counterparts.

A warp drive with a thickness parameter $@ = 1000000000000000000000$ would have the limits of the Energized warped region infinitely close to each other resulting in an infinitely small thickness.

[12]and not $@ = 75000$ meters and not $@ = 100000$ meters.the parameter @ is dimensionless

- Numerical plot for Function $C$ with $@ = 50000$

| $rs$ | $f(rs)$ | $n(rs)$ | $f'(rs)^2$ | $n'(rs)^2$ |
|---|---|---|---|---|
| $9,99970000000E+001$ | 1 | 0 | $2,650396620740E-251$ | 0 |
| $9,99980000000E+001$ | 1 | 0 | $1,915169647489E-164$ | 0 |
| $9,99990000000E+001$ | 1 | 0 | $1,383896564748E-077$ | 0 |
| $1,00000000000E+002$ | 0,5 | 0,5 | $6,250000000000E+008$ | $3,872591914849E-103$ |
| $1,00001000000E+002$ | 0 | 0,5 | $1,383896486082E-077$ | 0 |
| $1,00002000000E+002$ | 0 | 0,5 | $1,915169538624E-164$ | 0 |
| $1,00003000000E+002$ | 0 | 0,5 | $2,650396470082E-251$ | 0 |

- Numerical plot for Function $C$ with $@ = 75000$

| $rs$ | $f(rs)$ | $n(rs)$ | $f'(rs)^2$ | $n'(rs)^2$ |
|---|---|---|---|---|
| $9,99980000000E+001$ | 1 | 0 | $5,963392481410E-251$ | 0 |
| $9,99990000000E+001$ | 1 | 0 | $1,158345097767E-120$ | 0 |
| $1,00000000000E+002$ | 0,5 | 0,5 | $1,406250000000E+009$ | $8,713331808411E-103$ |
| $1,00001000000E+002$ | 0 | 0,5 | $1,158344999000E-120$ | 0 |
| $1,00002000000E+002$ | 0 | 0,5 | $5,963391972940E-251$ | 0 |

- Numerical plot for Function $C$ with $@ = 100000$

| $rs$ | $f(rs)$ | $n(rs)$ | $f'(rs)^2$ | $n'(rs)^2$ |
|---|---|---|---|---|
| $9,99990000000E+001$ | 1 | 0 | $7,660678807684E-164$ | 0 |
| $1,00000000000E+002$ | 0,5 | 0,5 | $2,500000000000E+009$ | $1,549036765940E-102$ |
| $1,00001000000E+002$ | 0 | 0,5 | $7,660677936765E-164$ | 0 |

Above are the plots of the Function $C$ for 3 Natario warp drives with thicknesses of $@ = 50000$ $@ = 75000$ and $@ = 100000$

Note that like in the previous cases of the Functions $A$ and $B$ the Energized warped region in Function $C$ extends beyond the limits of the Geometrized warped region in the Alcubierre case.Also as higher the parameter @ becomes as thicker or thinner the Energized warped region in the Alcubierre case becomes too.

But for this particular case of the Function $C$ the Energized warped region boundary limits in the Natario warp drive coincides with the boundary limits of the Geometrized warped region.

This means to say that the negative energy density in the Natario warp drive using the Function $C$ is contained or restrained in the boundary limits where the Natario shape function passes from 0 to $\frac{1}{2}$ or better:$0 < n(rs) < \frac{1}{2}$ and in this case we dont have negative energy density inside and outside the bubble but only in the warp bubble walls.[13]

Then the thickness of the Energized warped region in the Natario warp drive using Function $C$ is much smaller than in its counterparts that uses the Functions $A$ or $B$.

[13]the walls of the bubble are the regions where $0 < n(rs) < \frac{1}{2}$

Note that in Function $C$ the Energized warped region is distributed in a very thin layer over the neighborhoods of the bubble radius starting in a region very close to the bubble radius
$(rs <= R, rs \cong R, R - rs \cong 0)$ and terminating also in a region very close to the bubble radius
$(rs >= R, rs \cong R, rs - R \cong 0)$

Calling the point where the Energized warped region begins as point $a$ and the point where the Energized warped region ends as point $b$ then the point $b$ lies almost infinitely closed to the point $a$ resulting in an Energized warped region of almost infinite small thickness.

$(rs <= R, rs \cong R, R - rs \cong 0, R - rs = a, a \cong 0)$
$(rs >= R, rs \cong R, rs - R \cong 0, rs - R = b, b \cong 0)$
$(a \cong 0, b \cong 0, b >= a, b \cong a, b - a \cong 0)$

Why this behavior that contains both the Geometrized and Energized warped regions in the same boundary limits $a$ and $b$ occurs only with Function $C$ and not also with the Functions $A$ and $B$ too??

Starting with the square of the derivative of the shape function for the Function $C$:

$$n'(rs)^2_C = [\frac{1}{4}]WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}]f'(rs)^2 \quad (21)$$

Inside the bubble $f(rs) = 1$ and $[1 - f(rs)^{WF}]^{2(WF-1)} = 0$ resulting in a $n'(rs)^2_C = 0$.This is the reason why the Function $C$ dont have derivatives inside the bubble.

Outside the bubble $f(rs) = 0$ and $[f(rs)^{2(WF-1)}] = 0$ resulting also in a $n'(rs)^2_C = 0$.This is the reason why the Function $C$ dont have derivatives outside the bubble.

In the Geometrized warped region for the Alcubierre warp drive $1 > f(rs) > 0$.In this region the derivatives of the Function $C$ do not vanish because if $f(rs) < 1$ then $f(rs)^{WF} << 1$ resulting in an $[1 - f(rs)^{WF}]^{2(WF-1)} << 1$ but $[1 - f(rs)^{WF}]^{2(WF-1)} > 0$.Also if $f(rs) < 1$ then $[f(rs)^{2(WF-1)}] << 1$ too but $[f(rs)^{2(WF-1)}] > 0$

Note that if $[1 - f(rs)^{WF}]^{2(WF-1)} << 1$ and $[f(rs)^{2(WF-1)}] << 1$ their product
$[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}] <<<< 1$

Note that inside the Alcubierre Geometrized warped region $1 > f(rs) > 0$ when $f(rs)$ approaches 1 $n'(rs)^2_C$ approaches 0 due to the factor $[1 - f(rs)^{WF}]^{2(WF-1)}$ and when $f(rs)$ approaches 0 $n'(rs)^2_C$ approaches 0 again due to the factor $[f(rs)^{2(WF-1)}]$ .

The maximum value for $n'(rs)^2_C \cong 10^{-102}$ occurs in the middle of the Alcubierre Geometrized warped region where $f(rs) = 0,5$.Note that this value for the square of the derivative of the Natario shape function can obliterate the factor $10^{48}$ resulting in an extremely low level of negative energy density.

Due to the fact that the values of $n'(rs)^2_C$ grows from 0 to $10^{-102}$ and decreases to 0 again we choose Function $C$ as the best candidate to low the negative energy density in the Natario warp drive spacetime.

Back again to the negative energy density in the Natario warp drive[14]:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{22}$$

The total energy needed to sustain the Natario warp bubble is obtained by integrating the negative energy density $\rho$ over the volume of the Natario warped region(The region where the derivatives of the Natario shape function are not null)(points $a$ and $b$ the beginning and the end of the Natario Energized warped region).
Since we are in the equatorial plane then only the term in $rs$ accounts for and the total energy integral can be given by:

$$E = \int(\rho)drs = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\int(3(n'(rs))^2)drs = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\int((n'(rs))^2)drs \tag{23}$$

Above we placed the constant terms $c$,$G$ and $vs^2$ outside the integral. But the integral now becomes:

$$\int((n'(rs))^2)drs = \int([\frac{1}{4}]WF^4[1-f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}]f'(rs)^2)drs \tag{24}$$

$$\int((n'(rs))^2)drs = \frac{1}{4}WF^4\int([1-f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}]f'(rs)^2)drs \tag{25}$$

Since $WF$ ia also a constant.Then the total energy integral for the Natario warp drive is given by:

$$E = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\int((n'(rs))^2)drs = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\frac{1}{4}WF^4\int([1-f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}]f'(rs)^2)drs \tag{26}$$

Unfortunately integrals of this form do not have known primitives and also the integration methods to compute the integral are not known.In order to compute the total energy needed to sustain the Natario warp bubble we must employ the numerical integration by the Trapezoidal Rule.(see pg 4 in [18])[15]

$$\int_a^b f(x)dx = (b-a)\frac{f(b)+f(a)}{2} \cong 0 \dashrightarrow b-a \cong 0 \dashrightarrow b \cong a \dashrightarrow f(b) \cong f(a) \tag{27}$$

The total energy integral needed to sustain the Natario warp bubble now becomes:

$$E = \int_a^b \rho drs = \int_a^b -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3n'(rs))^2\right]drs = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\int_a^b\left[n'(rs)^2\right]drs \tag{28}$$

$$\int_a^b\left[(n'(rs)^2\right]drs = (b-a)\frac{n'(b)^2+n'(a)^2}{2} \tag{29}$$

$$E = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}(b-a)\frac{n'(b)^2+n'(a)^2}{2} \simeq 0 \dashrightarrow b \simeq a \dashrightarrow b-a \simeq 0 \tag{30}$$

The beginning of the region where the derivatives of the shape function ceases to be zero is the beginning of the Natario Energized warped (point $a$).The end of the region where the derivatives of the shape function still are not zero is the end of the Natario Energized warped region (point $b$)

[14]written in the International System of units
[15]see Wikipedia the Free Encyclopedia

If the difference between the points $a$ and $b$ is close to zero due to the fact that the points $a$ and $b$ are infinitely close to each other then this is enough to obliterate the value of $10^{48}$ or any other value.$(b \simeq a)(b - a \simeq 0)$

However there exists an error margin in the integration by the Trapezoidal Rule.The correct procedure is to decompose the area to be integrated in $n$ slices and integrate numerically still by the Trapezoidal Rule separately each slice and in the end we sum the result of all the slices integrated.This method is known as the Composite Trapezoidal Rule.(see pg 15 in [18])

As higher the number $n$ of slices is as more precise the result of the integral by the Composite Trapezoidal Rule is.(see pgs 14 and 15 in [18]).Remember that a warp bubble wether in the Alcubierre or Natario cases with a radius of 100 meters moving at 200 times light speed have the total amount of negative energy equal to the product of $10^{48}$ by the integral of the square derivatives of the shape function in the region between the point $b$(end of the Energized warped region) and the point $a$(beginning of the Energized warped region).If we want to integrate from the point $a$ to point $b$ using the Composite Trapezoidal Rule reducing the error margin we must divide the region between $a$ and $b$ in $n$ slices and integrate separately each slice also by the Trapezoidal Rule and in the end we sum the result of all the integrations.As higher the number $n$ of slices as accurate the integration becomes.Following pg 14 and 15 in [18] if we want to divide the region between $a$ and $b$ in $n$ slices each slice have a width given by:

$$h = \frac{b-a}{n} \tag{31}$$

And the final integration is the sum of the integration of all the slices by the Trapezoidal Rule given by:

$$\int_a^b n'(rs)^2 drs = \int_a^{a+h} n'(rs)^2 drs + \int_{a+h}^{a+2h} n'(rs)^2 drs + ....... + \int_{a+(n-2)h}^{a+(n-1)h} n'(rs)^2 drs + \int_{a+(n-1)h}^{b} n'(rs)^2 drs \tag{32}$$

Note that each slice above have a width $h$.

Writing the integral using sums we get the following expressions(pg 15 in [18]):

$$\int_a^b n'(rs)^2 drs = \frac{1}{2n}(b-a)[n'(b)^2 + n'(a)^2 + \sum_{i=1}^{n-1} n'(a+ih)^2] \cong 0 \dashrightarrow b-a \cong 0 \dashrightarrow b \cong a \dashrightarrow n >> 1 \tag{33}$$

$$\int_a^b n'(rs)^2 drs = \frac{b-a}{2n}[n'(b)^2 + n'(a)^2 + \sum_{i=1}^{n-1} n'(a+ih)^2] \cong 0 \dashrightarrow b-a \cong 0 \dashrightarrow b \cong a \dashrightarrow n >> 1 \tag{34}$$

Inserting these expressions in the integral of the negative energy density we get:

$$E = \int_a^b \rho drs = \int_a^b -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3n'(rs))^2\right] drs = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\int_a^b \left[n'(rs)^2\right] drs \tag{35}$$

$$E = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\frac{1}{2n}(b-a)[n'(b)^2 + n'(a)^2 + \sum_{i=1}^{n-1} n'(a+ih)^2] \simeq 0 \dashrightarrow n >> 1 \dashrightarrow b \simeq a \dashrightarrow b-a \simeq 0 \tag{36}$$

$$E = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\frac{b-a}{2n}[n'(b)^2 + n'(a)^2 + \sum_{i=1}^{n-1} n'(a+ih)^2] \simeq 0 \dashrightarrow n >> 1 \dashrightarrow b \simeq a \dashrightarrow b-a \simeq 0 \quad (37)$$

Look again to the equations of the total negative energy needed to sustain a Natario warp bubble:

$$E = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\frac{1}{2n}(b-a)[n'(b)^2 + n'(a)^2 + \sum_{i=1}^{n-1} n'(a+ih)^2] \simeq 0 \dashrightarrow n >> 1 \dashrightarrow b \simeq a \dashrightarrow b-a \simeq 0 \quad (38)$$

$$E = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\frac{b-a}{2n}[n'(b)^2 + n'(a)^2 + \sum_{i=1}^{n-1} n'(a+ih)^2] \simeq 0 \dashrightarrow n >> 1 \dashrightarrow b \simeq a \dashrightarrow b-a \simeq 0 \quad (39)$$

Now note an interesting thing:

- 1)-The number of slices raises the accuracy of the integration method and we have the factor $\frac{c^2}{G} \times \frac{vs^2}{8\pi}$ generating the huge factor $10^{48}$ for a bubble speed $vs = 200$ times light speed constraining the negative energy densities in the Natario warp bubble.
- 2)-How about to make the numerical integration by the Trapezoidal Rule using $10^{48}$ slices??.How about to make $n = 10^{48}$ ???

If $n = 10^{48}$ then $n$ in the denominator of the fraction $\frac{b-a}{2n}$ will completely obliterate the factor $\frac{c^2}{G} \times \frac{vs^2}{8\pi}$ in the equations of the total energy integral lowering the negative energy density requirements for the Natario warp bubble.

If we want a rigorous integration of the Natario negative energy density warped region we must divide this region in $10^{48}$ slices each one with a width $h$ of:

$$h = \frac{b-a}{n} = \frac{b-a}{10^{48}} \quad (40)$$

Inserting these values in the total energy integral equations we should expect for:

$$E = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\frac{b-a}{2 \times 10^{48}}[n'(b)^2 + n'(a)^2 + \sum_{i=1}^{n-1} n'(a+ih)^2] \simeq 0 \dashrightarrow n >> 1 \dashrightarrow b \simeq a \dashrightarrow b-a \simeq 0 \quad (41)$$

We choose to divide the Natario negative energy warped region in $10^{48}$ slices due to the factor $\frac{c^2}{G}\frac{v_s^2}{8\pi}$ which is $10^{48}$ for a bubble speed $vs = 200$ times faster than light .In the equation above keeping the bubble radius $R = 100$ meters and the thickness parameter $@ = 5000$ and the warp factor $WF = 200$ all constants and since $c$ and $G$ are constants if we use a different bubble velocity $vs$ higher than 200 times faster than light giving a factor $\frac{c^2}{G}\frac{v_s^2}{8\pi} > 10^{48}$ then the number of slices $n$ needed to integrate accurately by the Trapezoidal Rule the energy density in the Natario warp bubble must be equal to this new factor in order to reduce the total energy integral.

## 4 Horizon and Infinite Doppler Blueshifts in both Alcubierre and Natario Warp Drive Spacetimes

According to pg 6 in [2] warp drives suffers from the pathology of the Horizons and according to pg 8 in [2] warp drive suffer from the pathology of the infinite Doppler Blueshifts that happens when a photon sent by an Eulerian observer to the front of the warp bubble reaches the Horizon.This would render the warp drive impossible to be physically feasible.

For a complete mathematical demonstration of the Horizon and Doppler Blueshift Problems see pg 20 section 6 in [7](basic) and pg 4 section 2 in [6](advanced).The Horizon occurs in both spacetimes.This means to say that the Eulerian observer cannot signal the front of the warp bubble wether in Alcubierre or Natario warp drive because the photon sent to signal will stop in the Horizon..The solution for the Horizon problem must be postponed until the arrival of a Quantum Gravity theory that encompasses both General Relativity and Non-Local Quantum Entanglements of Quantum Mechanics however in the next section we will present a possible solution for this problem that only encompasses General Relativity.

The infinite Doppler Blueshift happens in the Alcubierre warp drive but not in the Natario one.This means to say that Alcubierre warp drive is physically impossible to be achieved but the Natario warp drive is perfectly physically possible to be achieved.

Consider again the negative energy density distribution in the Alcubierre warp drive spacetime(see eq 8 pg 6 in [3])[16][17]:

$$\langle T^{\mu\nu}u_\mu u_\nu\rangle = \langle T^{00}\rangle = \frac{1}{8\pi}G^{00} = -\frac{1}{8\pi}\frac{v_s^2(t)[y^2+z^2]}{4r_s^2(t)}\left(\frac{df(r_s)}{dr_s}\right)^2, \tag{42}$$

And considering again the negative energy density in the Natario warp drive spacetime(see pg 5 in [2])[18][19]:

$$\rho = T_{\mu\nu}u^\mu u^\nu = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2 + \left(n'(rs)+\frac{r}{2}n''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{43}$$

In pg 6 in [2] a warp drive with a x-axis only is considered.In this case for the Alcubierre warp drive $[y^2+z^2]=0$

$$\langle T^{\mu\nu}u_\mu u_\nu\rangle = \langle T^{00}\rangle = \frac{1}{8\pi}G^{00} = -\frac{1}{8\pi}\frac{v_s^2(t)[y^2+z^2]}{4r_s^2(t)}\left(\frac{df(r_s)}{dr_s}\right)^2, = 0 \tag{44}$$

And the negative energy density is zero but the Natario energy density is not zero and given by:.

$$\rho = T_{\mu\nu}u^\mu u^\nu = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2\right] \tag{45}$$

[16] $f(rs)$ is the Alcubierre shape function.Equation written in the Geometrized System of Units $c=G=1$
[17] Equation written in Cartezian Coordinates
[18] $n(rs)$ is the Natario shape function.Equation written the Geometrized System of Units $c=G=1$
[19] Equation written in Cartezian Coordinates.See Appendix $A$

Note that in front of the ship in the Alcubierre case the spacetime is empty but in the Natario case there exists negative energy density in the front of the ship.

According with Natario in pg 7 before section 5.2 in [14] negative energy density means a negative mass density and a negative mass generates a repulsive gravitational field. This repulsive gravitational field in front of the ship in the Natario warp drive spacetime protects the ship from impacts with the interstellar matter.The objections raised by Clark,Hiscock,Larson,McMonigal,Lewis,O'Byrne,Barcelo,Finazzi and Liberati in the introduction of this work are not valid for the Natario warp drive spacetime.

Since the Alcubierre warp drive dont have negative energy in front of the ship but only empty spacetime it doest not have protection against the interstellar medium making valid the objections raised by Clark,Hiscock,Larson,McMonigal,Lewis,O'Byrne,Barcelo,Finazzi and Liberati in the introduction of this work.

The Alcubierre shape function $f(rs)$ is defined as being 1 inside the warp bubble and 0 outside the warp bubble while being $1 > f(rs) > 0$ in the Alcubierre warped region according to eq 7 pg 4 in [1] or top of pg 4 in [2].

Expanding the quadratic term in eq 8 pg 4 in [1] and solving eq 8 for a null-like interval $ds^2 = 0$ we will have the following equation for the motion of the photon sent to the front(see pg 3 in [12] and pg 22 eqs 146 and 147 in [7])[20]:

$$\frac{dx}{dt} = vsf(rs) - 1 \tag{46}$$

Inside the Alcubierre warp bubble $f(rs) = 1$ and $vsf(rs) = vs$.Outside the warp bubble $f(rs) = 0$ and $vsf(rs) = 0$.

Somewhere inside the Alcubierre warped region when $f(rs)$ starts to decrease from 1 to 0 making the term $vsf(rs)$ decreases from $vs$ to 0 and assuming a continuous behavior then in a given point $vsf(rs) = 1$ and $\frac{dx}{dt} = 0$.The photon stops,A Horizon is established and in the Horizon the Doppler Blueshift occurs rendering the Alcubierre warp drive impossible.This due to the fact that there are no negative energy density in the front of the Alcubierre warp drive in the x-axis to deflect the photon.

Now taking the components of the Natario vector defined in the top of pg 5 in [2] and inserting these components in the first equation of pg 2 in [2] and solving for the same null-like interval $ds^2 = 0$ considering only radial motion we will get the following equation for the motion of the photon sent to the front(see eqs 16 and 17 pg 5 in [6])[21]:

$$\frac{dx}{dt} = 2vsn(rs) - 1 \tag{47}$$

The Natario shape function $n(rs)$ is defined as being 0 inside the warp bubble and $\frac{1}{2}$ outside the warp bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region according to pg 5 in [2].

[20] The coordinate frame for the Alcubierre warp drive as in [1] is the remote observer outside the ship

[21] The coordinate frame for the Natario warp drive as in [2] is the ship frame observer in the center of the warp bubble $xs = 0$

Inside the Natario warp bubble $n(rs) = 0$ and $2vsn(rs) = 0$.Outside the warp bubble $n(rs) = \frac{1}{2}$ and $2vsn(rs) = vs$. Somewhere inside the Natario warped region $n(rs)$ starts to increase from 0 to $\frac{1}{2}$ making the term $2vsn(rs)$ increase from 0 to $vs$ and assuming a continuous behavior then in a given point we would have a $2vsn(rs) = 1$ and a $\frac{dx}{dt} = 0$ The photon would stops.A Horizon would be established.

However when the photon reaches the beginning of the Natario warped region it suffers a deflection by the negative energy density in front of the Natario warp drive because this negative energy is not null.So in the case of the Natario warp drive the photon never reaches the Horizon and the Natario warp drive never suffer from the pathology of the infinite Doppler Blueshift due to a different distribution of energy density when compared to its Alcubierre counterpart.This negative energy with repulsive gravitational behavior deflects the photon from inside avoiding it to reach the Horizon and protects the Natario warp drive from the dangers of collisions with the interstellar medium at superluminal speeds.

Adapted from the negative energy in Wikipedia:The free Encyclopedia:

"if we have a small object with equal inertial and passive gravitational masses falling in the gravitational field of an object with negative active gravitational mass (a small mass dropped above a negative-mass planet, say), then the acceleration of the small object is proportional to the negative active gravitational mass creating a negative gravitational field and the small object would actually accelerate away from the negative-mass object rather than towards it."

The Natario warp drive as a solution of the Einstein Field Equations of General Relativity that allows faster than light motion is the first valid candidate for interstellar space travel.

## 5 A Causally Connected Superluminal Natario Warp Drive Spacetime using Micro Warp Bubbles

In 2002 Gauthier,Gravel and Melanson appeared with the idea of the micro warp bubbles.([15],[16])

According to them,microscopical particle-sized warp bubbles may have formed spontaneously immediately after the Big Bang and these warp bubbles could be used to transmit information at superluminal speeds.These micro warp bubbles may exist today.(see abs of [16])

A micro warp bubble with a radius of $10^{-10}$ meters could be used to transport an elementary particle like the electron whose Compton wavelength is $2.43 \times 10^{-12}$ meters at a superluminal speed.These micro warp bubbles may have formed when the Universe had an age between the Planck time and the time we assume that Inflation started.(see pg 306 of [15])

Following the ideas of Gauthier,Gravel and Melanson ([15],[16]) a micro warp bubble can send information or particles at superluminal speeds.(abs of [16],pg 306 in [15]).Since the infinite Doppler Blueshift affect the Alcubierre warp drive but not the Natario one and a superluminal micro warp bubble can only exists without Infinite Doppler Blueshifts[22] we consider in this section only the Natario warp drive spacetime.

The idea of Gauthier,Gravel and Melanson ([15],[16]) to send information at superluminal speeds using micro warp bubbles is very interesting and as a matter of fact shows to us how to solve the Horizon problem. Imagine that we are inside a large superluminal warp bubble and we want to send information to the front.Photons sent from inside the bubble to the front would stop in the Horizon but we also know that incoming photons from outside would reach the bubble.[23] The external observer outside the bubble have all the bubble causally connected while the internal observer is causally connected to the point before the Horizon.Then the external observer can create the bubble while the internal observer cannot. This was also outlined by Everett-Roman in pg 3 in [12]. Unless we find a way to overcome the Horizon problem. We inside the large warp bubble could create and send one of these micro warp bubbles to the front of the large warp bubble but with a superluminal speed $vs2$ larger than the large bubble speed $X = 2vsn(rs)$.Then $vs2 >> X$ or $vs2 >> 2vsn(rs)$ and this would allow ourselves to keep all the warp bubble causally connected from inside overcoming the Horizon problem without the need of the "tachyonic" matter.

- 1)- Superluminal micro warp bubble sent towards the front of the large superluminal warp bubble $vs2 = \frac{dx}{dt} > X - 1 > vs - 1 \dashrightarrow X = 2vsn(rs)$

From above it easy to see that a micro warp bubble with a superluminal speed $vs2$ maintains a large superluminal warp bubble with speed $vs$ causally connected from inside if $vs2 > vs$

[22] assuming a continuous growth of the warp bubble speed $vs$ from zero to a superluminal speed at a given time the speed will be equal to $c$ and the Infinite Doppler Blueshift crashes the bubble.The Alcubierre warp drive can only exists for $vs < c$ so it cannot sustain a micro warp bubble able to shelter particles or information at superluminal speeds

[23] true for the Alcubierre warp drive but not for the Natario one because the negative energy density in the front with repulsive gravitational behavior would deflect all the photons sent from inside and outside the bubble effectively shielding the Horizon from the photon avoiding the catastrophical Infinite Doppler Blueshift

From the point of view of the astronaut inside the large warp bubble he is the internal observer with respect to the large warp bubble but he is the external observer from the point of view of the micro warp bubble so he keeps all the light-cone of the micro warp bubble causally connected to him so he can use it to send superluminal signals to the large warp bubble from inside.(Everett-Roman in pg 3 in [12]).

Gauthier,Gravel and Melanson developed the concept of the micro warp bubble but the idea is at least 5 years younger.The first time micro warp bubbles were mentioned appeared in the work of Ford-Pfenning pg 10 and 11 in [3].

According with Natario in pg 7 before section 5.2 in [14] negative energy density means a negative mass density and a negative mass generates a repulsive gravitational field that deflect photons or positive mass-density particles from the interstellar medium or particles sent to the bubble walls by the astronaut inside the bubble.

However while the negative mass deflects the positive mass or photons[24] a negative mass always attracts another negative mass so the astronaut cannot send positive particles or photons to the large warp bubble but by sending micro warp bubbles these also possesses negative masses that will be attracted by the negative mass of the large warp bubble effectively being a useful way to send signals.

[24] by negative gravitational bending of light

## 6 Conclusion

In this work we introduced 3 new shape functions for the Natario warp drive spacetime and we demonstrated that one of these functions is very efficient lowering the negative energy densities in the Natario warp drive to affordable levels.

Also the objections raised by Clark,Hiscock,Larson,McMonigal,Lewis,O'Byrne,Barcelo,Finazzi and Liberati in the introduction of this work and valid for the Alcubierre warp drive making it physically impossible independently from arbitrary lower levels of negative energy do not affect the Natario warp drive which is perfectly possible to be achieved.This was the main reason behind our interest in the integration of the derivatives of a particular form of the shape function for the Natario warp drive spacetime.

The Natario warp drive once created can survive against all the obstacles pointed as physical impossibilities that rules out the warp drive as a dynamical spacetime.

Lastly and in order to terminate this work:There exists another problem not covered here:the fact that we still dont know how to generate the negative energy density and negative mass and above everything else we dont know how to generate the shape function that distorts the spacetime geometry creating the warp drive effect.So unfortunately all the discussions about warp drives are still under the domain of the mathematical conjectures.

However we are confident to affirm that the Natario warp drive will survive the passage of the Century $XXI$ and will arrive to the Future.The Natario warp drive as the first Human candidate for faster than light interstellar space travel will arrive to the the Century $XXIV$ on-board the future starships up there in the middle of the stars helping the human race to give his first steps in the exploration of our Galaxy

Live Long And Prosper

# 7 Appendix A:The Natario Warp Drive Negative Energy Density in Cartezian Coordinates

The negative energy density according to Natario is given by(see pg 5 in [2])[25]:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2\sin^2\theta\right] \tag{48}$$

In the bottom of pg 4 in [2] Natario defined the x-axis as the polar axis.In the top of page 5 we can see that $x = rs\cos(\theta)$ implying in $\cos(\theta) = \frac{x}{rs}$ and in $\sin(\theta) = \frac{y}{rs}$

Rewriting the Natario negative energy density in cartezian coordinates we should expect for:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2 + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{49}$$

Considering motion in the equatorial plane of the Natario warp bubble (x-axis only) then $[y^2 + z^2] = 0$ and $rs^2 = [(x - xs)^2]$ and making $xs = 0$ the center of the bubble as the origin of the coordinate frame for the motion of the Eulerian observer then $rs^2 = x^2$ because in the equatorial plane $y = z = 0$.

Rewriting the Natario negative energy density in cartezian coordinates in the equatorial plane we should expect for:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{50}$$

[25] $n(rs)$ is the Natario shape function.Equation written in the Geometrized System of Units $c = G = 1$

## 8 Epilogue

- "The only way of discovering the limits of the possible is to venture a little way past them into the impossible."-Arthur C.Clarke[26]
- "The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction. There is no logical path to these laws; only intuition, resting on sympathetic understanding of experience, can reach them"-Albert Einstein[27][28]

## 9 Remarks

References [8] "Warp Field Mechanics 101" and [17] "Warp Field Mechanics 102" by Harold "Sonny" White of NASA Lyndon B.Johnson Space Center Houston Texas are available at NASA Technical Reports Server (NTRS)[29] however we can provide a copy in PDF Acrobat reader of these references for those interested.

Reference [18] "Numerical Integration" from Autar Kaw and Charlie Barker is available at http://numericalmethods.eng.usf.edu however we can provide a copy in PDF Acrobat reader of this reference for those interested.

Reference [15] was online at the time we picked it up for our records.It ceased to be online but we can provide a copy in PDF Acrobat reader of this reference for those interested.

Reference [16] we only have access to the abstract.

We performed all the numerical calculus of our simulations for both Alcubierre and Natario warp drive spacetimes using Microsoft Excel[30].We can provide our Excel files to those interested and although Excel is a licensed program there exists another program that can read Excel files available in the Internet as a freeware for those that perhaps may want to examine our files:the OpenOffice[31] at http://www.openoffice.org

[26]special thanks to Maria Matreno from Residencia de Estudantes Universitas Lisboa Portugal for providing the Second Law Of Arthur C.Clarke

[27]"Ideas And Opinions" Einstein compilation, ISBN $0-517-88440-2$, on page 226."Principles of Research" ([Ideas and Opinions],pp.224-227), described as "Address delivered in celebration of Max Planck's sixtieth birthday (1918) before the Physical Society in Berlin"

[28]appears also in the Eric Baird book Relativity in Curved Spacetime ISBN $978-0-9557068-0-6$

[29]browse Google for "Warp Field Mechanics 101" and "Warp Field Mechanics 102"

[30]Copyright(R) by Microsoft Corporation

[31]Copyright(R) by Oracle Corporation

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