Black hole as a gas of strings

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ABSTRACT- We used the micro canonical ensemble in order to evaluate the entropy of a gas of strings confined by its Schwarzschild radius. The results are compared with the entropy of the gravitational field calculated by Rothman and Anninos and with the well-known Bekenstein-Hawking entropy of a black hole.

1 - INTRODUCTION

Rothman and Anninos [1] used the relation $S = k_B \ln\Omega$ as a means to evaluate the entropy S of the gravitational field, where Ω is the phase space of the field bounded by a Hamiltonian. In their work, the phase space is calculated for gravitational waves and radiation and density perturbations in expanding FLRW spacetimes, attributing entropy to a lack of knowledge in the exact field configuration.

Meanwhile we may think of a universe populated by a gas of strings, and perhaps would be interesting to evaluate the entropy of this gas in order to compare with Rothman and Anninos calculations.

2 - THE HAMILTONIAN OF THE GAS OF STRINGS AND PHASE SPACE CALCULATION

The Hamiltonian of the gas of strings can be written as

$$H = \sum_{i=1}^{N}\{c|\mathbf{p_i}| + a\,|\mathbf{r}_i|\}, \tag{1}$$

where c is the light speed in vacuum and a is the string tension and $(\mathbf{p_i}, \mathbf{r}_i)$ are labels locating the i-particle in the space-phase. The equation for the one-dimensional space-phase reads

$$(c|\mathbf{p_i}|)/E + (a\,|\mathbf{r}_i|)/E = 1. \tag{2}$$

The space of phase area (please see figure 1) is given by

$$A = (2\,E^2)/(ca). \tag{3}$$

The hypercube volume in N-dimensions reads

$$V_N = A^N = \{2/(c\,a)\}^N\; E^{2N}. \tag{4}$$

Figure 1 (below), shows a graph of the trajectory of a particle in the space of phase. Vertical axis gives the momentum coordinate p measured in terms of E ⁄c, and horizontal axis gives the position coordinate r in terms of E⁄a. We choose arbitrary values for the vertical and horizontal intercepts, namely E ⁄c = ±1, and E⁄a = ±2.

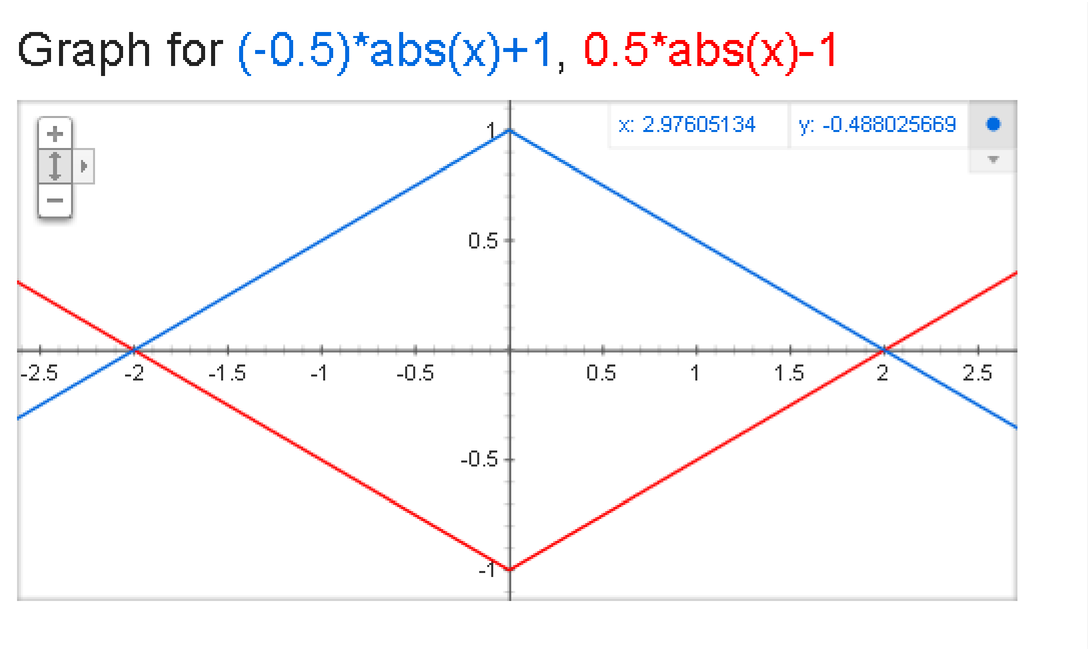

Figure 1- The 1-d space of phase

3 – BOHR-QUANTIZATION OF THE STRING HAMILTONIAN

Let us take

$$H = c\,p + a\,r, \qquad (5)$$

and the rule for Bohr-quantization

$$r\,p = n\,\hbar, \qquad n = 1,2,3,\ldots, N. \qquad (6)$$

Inserting (6) into (5) we obtain

$$H = c\,p + (na\hbar)/p. \qquad (7)$$

Next we take the minimum of (7), performing the calculation $\partial H/\partial p=0$, and find

$$p_n = [(na\hbar)/c]^{1/2}. \qquad (8)$$

Putting (8) into (6) leads to

$$E_n = H(p_n) = 2\,(a\hbar c)^{1/2}\, n^{1/2}. \qquad (9)$$

Now if we denote N the maximum quantum number of the gas of strings, we get

$$E_N = 2\,(a\hbar c)^{1/2}\, N^{1/2}, \quad \text{and} \quad (E_N)^{2N} = 4^N\,(a\hbar c)^N\, N^N. \qquad (10)$$

Inserting (10) into (4), we obtain for the volume of phase space the relation

$$V_N = (16\,\pi)^N\,\hbar^N\, N^N. \qquad (11)$$

The next step is to use the recipe to evaluate,Ω, working with the micro canonical ensemble (please see reference [2]). We have

$$\Omega = V_N/(\hbar^N N!) = \{[(16\pi)^N N^N]/N!\}. \qquad (12)$$

4 – CALCULATION OF THE ENTROPY

As a means to obtain the entropy S of the gas of strings, we use the Boltzmann recipe, namely

$$S = k_B \ln\Omega = N k_B \ln(16\,\pi\, e). \qquad (13)$$

Figure 2 shows a graph of the string potential $U(r) = a\,|r|$, and the energy constant Mc^2.

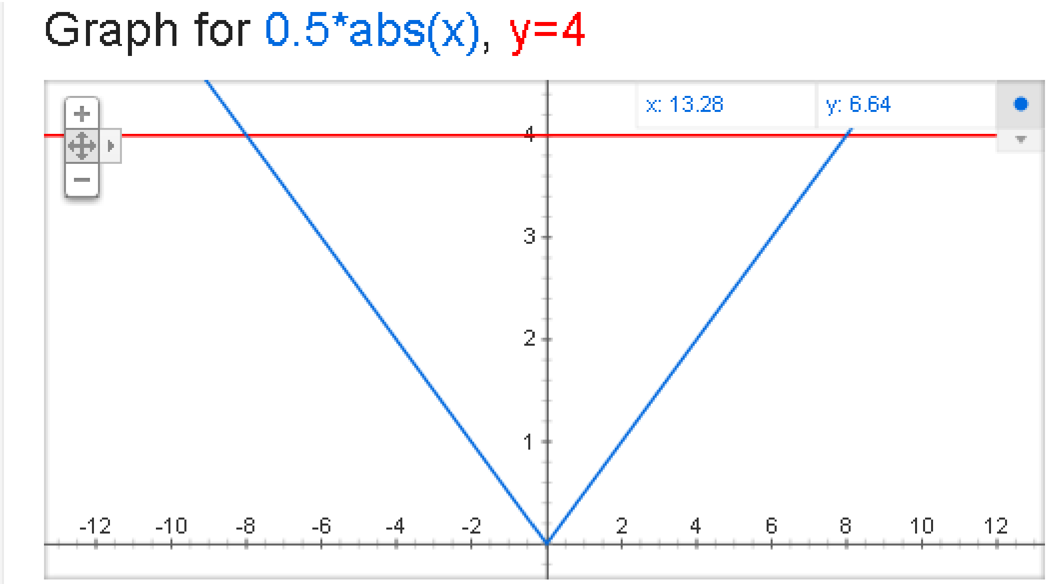

Figure 2 – The string potential $U(r) = a\,|r|$ (blue) and energy constant Mc^2 (red) are exhibited with arbitrary values of parameters, namely $a = .5$, $Mc^2 = 4$ and r_N (the classical turning point)=8.

The acknowledgment of the gas of strings as a black hole is done by making the identification the turning point r_N with the radius of Schwarzschild R_S of a black hole of mass M. Doing this, we have

$$r_N = N^{1/2}\,[(\hbar c)/a]^{1/2} = (2GM)/c^2 \equiv R_S. \qquad (14)$$

Solving (14) for N, we obtain

$$N = (4a\, G^2 M^2)/(\hbar\, c^5). \qquad (15)$$

Meanwhile the turning point condition reads

$$a\, r_n = M\, c^2, \quad \text{with} \quad r_n \equiv R_S = (2GM)/c^2. \qquad (16)$$

Relation (16) implies that

$$(2\, a\, G)/c^4 = 1. \qquad (17)$$

Inserting (17) into (15) we obtain

$$N = 2\, (M/M_{Pl})^2, \qquad (18)$$

where $M_{Pl} = (\hbar c/G)^{1/2}$ stands for the Planck's mass.

Finally by using (18) in (13) we get the entropy for this gas of strings confined in its Schwarzschild radius, namely

$$S = k_B\, [2 \ln(16\pi e)]\, (M/M_{Pl})^2. \qquad (19)$$

5- CONCLUDING REMARKS

The entropy of the confined gas of strings given by (19) must be compared with

$S_{R\text{-}A} = k_B\, [4 \ln(2\pi e)]\, (M/M_{Pl})^2$, obtained by Rothman and Anninos[1] and with

$S_{B\text{-}H} = k_B\, [8\, \pi^2]\, (M/M_{Pl})^2$, the well known result of Bekenstein-Hawking [3,4].

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