A formula that generates a type of pairs of Poulet numbers

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Abstract. Starting from the observation that the number 13^2 + 81*13 + 3*13*41 is a Poulet number (2821), and the number 41^2 + 81*41 + 3*13*41 is a Poulet number too (6601), and following my interest for the number 30, I found a formula that generates such pairs of Poulet numbers like (2821,6601).

Observation: The formula p^2 + 81*p + 3*p*q, where p is a prime of the form 30*k + 13 and q is a prime of the form 30*k + 41 (case I), or, vice versa, p is a prime of the form 30*k + 41 and q is a prime of the form 30*k + 13 (case II), generates Poulet numbers.

Examples:

: for (p,q) = (13,41), we got 2821, a Poulet number;
: for (p,q) = (41,13), we got 6601, a Poulet number;

: for (p,q) = (43,71), we got 14491, a Poulet number;
: for (p,q) = (71,43), we got 19951, a Poulet number.

Conjecture 1: There is an infinity of Poulet numbers of the form p^2 + 81*p + 3*p*q, where p is a prime of the form 30*k + 13 and q is a prime of the form 30*k + 41, where k is an integer, k >= 0.

Conjecture 2: There is an infinity of Poulet numbers of the form p^2 + 81*p + 3*p*q, where p is a prime of the form 30*k + 41 and q is a prime of the form 30*k + 13, where k is an integer, k >= 0.

Conjecture 3: If the number p^2 + 81*p + 3*p*q, where p is a prime of the form 30*k + 13 and q is a prime of the form 30*k + 41, is a Poulet number, then the number p^2 + 81*p + 3*p*q, where p is a prime of the form 30*k + 41 and q is a prime of the form 30*k + 13 is a Poulet number too (k is an integer, k >= 0).

Note: The differences between the two numbers that form such a pair might also have interesting properties; in the examples above, we have 6601 – 2821 = 3780 and 19951 – 14491 = 5460. Note that 5460 – 3780 = 1680 = 41^2 – 1.

Note: There are many Poulet numbers that can be written as p^2 + 81*p + 3*p*q, where p,q primes, but it’s not satisfied the reciprocity from the formula above.