Moutaoikil-Ripà's conjecture on Prime Numbers: $\forall\ p_0 \geq 7,\ p_0 = 2 \cdot p_1 + p_2$

Marco Ripà (July 2013)

sPIqr Society, Rome, Italy
Email: marcokrt1984@yahoo.it

Abstract

An original result about prime numbers and unproved conjectures. We show that, if the Goldbach conjecture is true, every prime number greater than 7 can be expressed as the sum of an odd prime plus twice another (different) odd prime. A computational analysis shows that the conjecture is true (at least) for primes below 7465626013.

1. Introduction

A few days ago, we discovered a quite strange, new, conjecture involving prime numbers [http://vixra.org/pdf/1307.0081v1.pdf]. It remembered us the very famous Goldbach's conjecture (by Christian Goldbach and Leonard Euler).

The statement is as follows:

Moutaoikil-Ripà's Conjecture. *For every prime number $p_0 \geq 7$, we have that $p_0 = 2 \cdot p_1 + p_2$ (where p_1 and p_2 are both primes and $p_1 \neq p_2$).*

2. An incomplete proof (assuming Goldbach's strong conjecture as true)

Let us consider a base 10 scenario and let us assume Goldbach's strong conjecture as true, we can see that we have just two cases to analyze ($p_1=2$ or $p_1>2$).

In fact, $p_0=2\cdot n+1 \rightarrow 2\cdot p_1+p_2$ is odd only if p_2 is odd (p_1 is the only prime which can be 2).

Let us assume $p_1\neq 2$, we would like to show that, assuming Goldbach's conjecture as true, the new conjecture is true as well (for any $p_0\geq 11$). It is trivial that, if $p_0=7$, there is only one possible solution (but there is one!) and it is $7:=2\cdot 2+3$.

We have the following constraints:

$$\begin{cases} p_1 \neq p_2 \\ p_1 \neq 2 \\ p_2 \neq 2 \\ p_1, p_2 \text{ are prime} \end{cases} \rightarrow \min[p_1+p_2]=3+5=8 \rightarrow \min[2\cdot n]=8 \rightarrow \min[n]=4.$$

Thus $p_0-p_1\geq 8 \rightarrow \min[p_0]$ such that $p_1=2 \rightarrow \min[p_0]=11$ (because $11=8+3$).

$p_0=2\cdot p_1+p_2 \rightarrow p_0-p_1=p_1+p_2$. But Goldbach said that $2\cdot n=p_1+p_2$, where (in our case) n is an element of $\mathbb{N}\backslash\{0,1,2,3\}$ and $p_1+p_2\geq 8$.

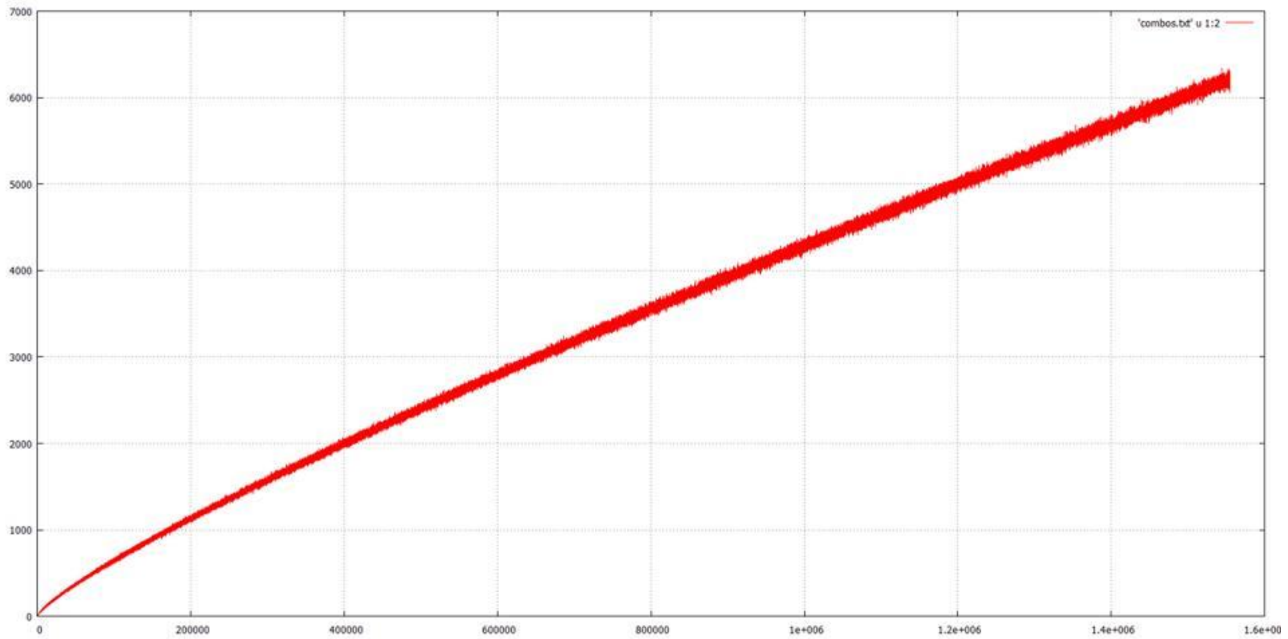
The new relation we have to "prove" is easy now: p_0-p_1 is the difference between two odd primes $\rightarrow$ it is even ($p_0-p_1\geq 8$), as we have already shown… and, on the other side of the "=", there is the sum of two distinct primes? Goldbach? Unfortunately, not. We still need to prove that, $\forall p_0>5$, there is (at least) one common element between the $\{p_1, p_2\}$ set from the new conjecture and $\{p_3, p_4\}$ from the strong Goldbach conjecture. Moreover, we need to point out that we are searching for $p_1\neq p_2$ solutions only.

With specific regard to the last point, we can see that, for every value of n we are considering ($n\geq 4$), the partition number of $2\cdot n$ is ≥ 2 (so we have at least one solution of the form $p_1\neq p_2$).

3. A computational analysis

Emanuele Dalmasso wrote a specific program to test the new conjecture for “small” values of p_0: the test has shown that the conjecture is right for any $7 \leq p_0 \leq 746562601$ ($746562601 = 2 \cdot 7 + 746562587$).

p_0 can be written in many different ways (for $11 \leq p_0$): you can see this just looking at the figure below (the number of ways such that $p_0 := 2 \cdot p_1 + p_2$ is shown on the vertical axis).



4. Conclusion

Moutaoikil-Ripà’s conjecture mainly differs from Lemoine’s one for the additional constraint $p_1 \neq p_2$. A corollary of the “proof” of the new conjecture is that, for every $p_0 \geq 11$, $p_0 = 2 \cdot p_1 + p_2$, where $p_1 > 2$ ($p_1 \neq p_2$ are both odd primes).

Lemoine’s conjecture involves any odd number above 3, while Moutaoikil-Ripà’s conjecture concerns only odd primes above 5. This is an important difference: for example, considering $n=7 \rightarrow 2 \cdot n+1=15$, we can see that the corollary stated above would be wrong. In fact, the following constraints would be taken into account: $p_1 \neq p_2$ and $p_1 \neq 2$. Therefore 15 cannot be written as $2 \cdot 2+11$, nor $2 \cdot 5+5$. Thus, there is no solution for $n=7$ and a corollary of the Moutaoikil-Ripà’s conjecture could not be satisfied.

Appendix

Basing on the Lemoine-Levy partition number development [http://arxiv.org/pdf/0901.3102v2.pdf], my second conjecture is as follows (a stronger version on the Lemoine's conjecture):

Ripà's Conjecture. *For every odd number* $2 \cdot n+1 \geq 17$ *(*$\forall\, 8 \leq n \in \mathbb{N}$*), there is (at least) a couple of odd primes,* $p_1 \neq p_2$*, such that* $2 \cdot n+1=2 \cdot p_1+p_2$*.*

A sufficient but not necessary condition to prove this conjecture is that the partition number can be proved to be (strictly) greater than 2, for every $n \geq 8$.