A Short Proof of the Laurendeau Conjecture

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Abstract

We prove the Laurendeau Conjecture.

Conjecture

Julien Laurendeau has proposed the following conjecture[1]:

Given three nonlinear nonzero functions f(x), g(x), and, h(x) with the following properties:

$2f(x) = f(x) + \big(g(x) - h(x)\big)f(x)$, (1)

$4g(x) = g(x) + \big(h(x) - f(x)\big)g(x)$, (2)

$6h(x) = h(x) + \big(f(x) - g(x)\big)h(x)$, (3)

There does not exist a solution to the following equation:

$\big(f(x) + g(x) + h(x)\big) + \big(f(x) + g(x)\big)h(x) = 2$, (4)

Proof

We begin by simplifying equations (1) to (3).

For equation (1) we subtract f(x) from each side and then divide each side by f(x), which we can do since f(x) is nonzero. These result in,

$1 = g(x) - h(x)$, (4)

And similarly,

$3 = h(x) - f(x)$, (5)

$5 = f(x) - g(x)$, (6)

Adding equations (4) and (5) gives

$4 = g(x) - f(x)$, (7)

or equivalently

$-4 = f(x) - g(x)$, (8)

However, equation (8) contradicts equation (5). Therefore there are no nonzero functions satisfying properties (1) to (3). Because there are no functions with the requisite properties there can be no solution of (4). Thus Laurendeau's Conjecture is true.

QED

[1] *Link:* http://vixra.org/pdf/1307.0159v1.pdf