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ON ANDRICA'S CONJECTURE, CRAMÉR'S CONJECTURE, GAPS BETWEEN PRIMES AND JACOBI THETA FUNCTIONS V:

(A Simple Proof for 12.10 Theorem of Ivic's Book for Difference between Consecutive Primes without Riemann's Hypothesis)

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Abstract: In this paper we developed an interesting result by constructing for differences between consecutive primes without Riemann's Hypothesis.

1. INTRODUCTION

In book *The Riemann Zeta-Function: Theory and Applications* [1, p. 321], Aleksandar Ivic posted the following theorem:

Theorem 12.10. *If the Riemann Hypothesis is true, then*

$$(1.1)\, p_{n+1} - p_n \ll p_n^{1/2} \log p_n.$$

In this paper, we prove that

$$(1.2)\, p_{n+1} - p_n \ll 0.915071 \cdot p_n^{1/2} \log p_n,$$

result is stronger than the announced by Ivic, without the Riemann Hypothesis.

Thereupon, we prove that

$$(1.3)\, p_{n+1} - p_n \ll p_n^{8371/10000},$$

which is weaker than the result of H. Iwaniec and J. Pintz (1984), [see 2]: $p_{n+1} - p_n \ll p_n^{23/42}$.

2. PRELIMINARES

The Rosser's theorem [3] states that p_n is larger than $n \log n$. This can be improved by the following pair of bounds:

$$(2.1)\, \log n + \log\log n - 1 < \frac{p_n}{n} < \log n + \log\log n,$$

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for $n \geq 6$.

3. THEOREM AND COROLLARYS

THEOREM 1.*For* $n \in \mathbb{N}_{>9}$, *then*

$$p_{n+1} - p_n \ll 0.915071 \cdot p_n^{1/2} \log p_n. \tag{3.1}$$

Proof. In previous paper [4, p.__], we encounter that

$$\sqrt{p_{n+1}} - \sqrt{p_n} < \sqrt{2}\left(\sqrt{n+1} - \sqrt{n}\right)\sqrt{n}, \tag{3.2}$$

for $n \in \mathbb{N}_{\geq 6}$. Summing $p_{n+1} - p_n$ in both members of inequality (3.2), we have

$$p_{n+1} - p_n + \sqrt{p_{n+1}} - \sqrt{p_n} < p_{n+1} - p_n + \sqrt{2}\left(\sqrt{n+1} - \sqrt{n}\right)\sqrt{n}, \tag{3.3}$$

$$\Rightarrow p_{n+1} - p_n < p_{n+1} - \sqrt{p_{n+1}} + \sqrt{p_n} - p_n + \sqrt{2}\left(\sqrt{n+1} - \sqrt{n}\right)\sqrt{n}.$$

Dividing (3.3) by $\left(p_n - \sqrt{p_n}\right)\log p_n$, we obtain

$$\begin{aligned}
&\frac{p_{n+1} - p_n}{\left(p_n - \sqrt{p_n}\right)\log p_n} \\
&< \frac{p_{n+1} - \sqrt{p_{n+1}} + \sqrt{p_n} - p_n + \sqrt{2}\left(\sqrt{n+1} - \sqrt{n}\right)\sqrt{n}}{\left(p_n - \sqrt{p_n}\right)\log p_n} \\
&= \frac{p_{n+1} - \sqrt{p_{n+1}}}{\left(p_n - \sqrt{p_n}\right)\log p_n} - \frac{p_n - \sqrt{p_n}}{\left(p_n - \sqrt{p_n}\right)\log p_n} + \frac{\sqrt{2}\left(\sqrt{n+1} - \sqrt{n}\right)\sqrt{n}}{\left(p_n - \sqrt{p_n}\right)\log p_n} \\
&= \frac{p_{n+1} - \sqrt{p_{n+1}}}{\left(p_n - \sqrt{p_n}\right)\log p_n} - \frac{1}{\log p_n} + \frac{\sqrt{2}\left(\sqrt{n+1} - \sqrt{n}\right)\sqrt{n}}{\left(p_n - \sqrt{p_n}\right)\log p_n},
\end{aligned} \tag{3.4}$$

consequently,

$$\begin{aligned}
\frac{p_{n+1} - p_n}{\sqrt{p_n}\log p_n}\left(\frac{\sqrt{p_n}}{p_n - \sqrt{p_n}}\right) &= \frac{p_{n+1} - p_n}{\left(p_n - \sqrt{p_n}\right)\log p_n} \\
&< \frac{p_{n+1} - \sqrt{p_{n+1}}}{\left(p_n - \sqrt{p_n}\right)\log p_n} - \frac{1}{\log p_n} + \frac{\sqrt{2}\left(\sqrt{n+1} - \sqrt{n}\right)\sqrt{n}}{\left(p_n - \sqrt{p_n}\right)\log p_n}.
\end{aligned} \tag{3.5}$$

On the other hand, note that $\frac{\sqrt{p_n}}{p_n - \sqrt{p_n}}$ is a decreasing function,in which the maximum occurs when $n = 1$, for $n \in \mathbb{N} - \{0\}$. In other words, $\max_{n \in \mathbb{N}_{\geq 1}} \frac{\sqrt{p_n}}{p_n - \sqrt{p_n}} = \frac{\sqrt{p_1}}{p_1 - \sqrt{p_1}} = \frac{\sqrt{2}}{2 - \sqrt{2}} = 2.41421356237\ldots$, see Table 1. But, if we consider $n \in \mathbb{N} - \{0\}$ and $n > 9$, then $\max_{n \in \mathbb{N}_{>9}} \frac{\sqrt{p_n}}{p_n - \sqrt{p_n}} = \frac{\sqrt{p_{10}}}{p_{10} - \sqrt{p_{10}}} = \frac{\sqrt{29}}{29 - \sqrt{29}} = 0.228041600254\ldots$ Therefore, for $n \in \mathbb{N} - \{0\}$ and $n > 9$, we have

$$(3.6)\ \frac{\sqrt{p_n}}{p_n-\sqrt{p_n}} \le \max_{n\in\mathbb{N}_{>9}} \frac{\sqrt{p_n}}{p_n-\sqrt{p_n}} = \frac{\sqrt{p_{10}}}{p_{10}-\sqrt{p_{10}}} = \frac{\sqrt{29}}{29-\sqrt{29}} = 0.228041600254\ldots \ll 0.23 = \frac{23}{100}.$$

Multiplying $\frac{p_{n+1}-p_n}{\sqrt{p_n}\log p_n}$ in both members of inequality (3.6), we have

$$(3.7)\ \frac{p_{n+1}-p_n}{\sqrt{p_n}\log p_n}\left(\frac{\sqrt{p_n}}{p_n-\sqrt{p_n}}\right) \le \frac{p_{n+1}-p_n}{\sqrt{p_n}\log p_n}\left(\max_{n\in\mathbb{N}_{>9}} \frac{\sqrt{p_n}}{p_n-\sqrt{p_n}}\right) = \frac{p_{n+1}-p_n}{\sqrt{p_n}\log p_n}\left(\frac{\sqrt{29}}{29-\sqrt{29}}\right) \ll \frac{23}{100}\frac{p_{n+1}-p_n}{\sqrt{p_n}\log p_n}.$$

Dividing (3.5) by (3.7), weencounter

$$(3.8)\ 1 \ll \frac{\frac{p_{n+1}-\sqrt{p_{n+1}}}{(p_n-\sqrt{p_n})\log p_n} - \frac{1}{\log p_n} + \frac{\sqrt{2}(\sqrt{n+1}-\sqrt{n})\sqrt{n}}{(p_n-\sqrt{p_n})\log p_n}}{\frac{23}{100}\frac{p_{n+1}-p_n}{\sqrt{p_n}\log p_n}} = \frac{100\sqrt{p_n}\log p_n}{23(p_{n+1}-p_n)}\left[\frac{p_{n+1}-\sqrt{p_{n+1}}}{(p_n-\sqrt{p_n})\log p_n} - \frac{1}{\log p_n} + \frac{\sqrt{2}(\sqrt{n+1}-\sqrt{n})\sqrt{n}}{(p_n-\sqrt{p_n})\log p_n}\right],$$

$$\frac{23}{100}\cdot\frac{p_{n+1}-p_n}{\sqrt{p_n}\log p_n} \ll \frac{p_{n+1}-\sqrt{p_{n+1}}}{(p_n-\sqrt{p_n})\log p_n} - \frac{1}{\log p_n} + \frac{\sqrt{2}(\sqrt{n+1}-\sqrt{n})\sqrt{n}}{(p_n-\sqrt{p_n})\log p_n}.$$

Using the Rosser's theorem in the right-hand side of inequality (3.8) for $\frac{p_{n+1}-\sqrt{p_{n+1}}}{(p_n-\sqrt{p_n})\log p_n} - \frac{1}{\log p_n}$, we find

$$(3.9)\quad \frac{23}{100}\cdot\frac{p_{n+1}-p_n}{\sqrt{p_n}\log p_n} \ll \frac{p_{n+1}-\sqrt{p_{n+1}}}{(p_n-\sqrt{p_n})\log p_n} - \frac{1}{\log p_n} + \frac{\sqrt{2}(\sqrt{n+1}-\sqrt{n})\sqrt{n}}{(p_n-\sqrt{p_n})\log p_n}$$
$$< \frac{(n+1)[\log(n+1)+\log\log(n+1)] - \sqrt{(n+1)[\log(n+1)+\log\log(n+1)]}}{\left[n(\log n+\log\log n-1) - \sqrt{n(\log n+\log\log n-1)}\right]\log[n(\log n+\log\log n-1)]} - \frac{1}{\log[n(\log n+\log\log n-1)]} + \frac{\sqrt{2}(\sqrt{n+1}-\sqrt{n})\sqrt{n}}{(p_n-\sqrt{p_n})\log p_n}.$$

We observe that $\frac{(n+1)[\log(n+1)+\log\log(n+1)]-\sqrt{(n+1)[\log(n+1)+\log\log(n+1)]}}{\left[n(\log n+\log\log n-1)-\sqrt{n(\log n+\log\log n-1)}\right]\log[n(\log n+\log\log n-1)]} - \frac{1}{\log[n(\log n+\log\log n-1)]}$ is a decreasing function, in which the maximum occurs when $n = 10$, for $n \in \mathbb{N}$ and $n > 9$; then, $\max_{n\in\mathbb{N}_{>9}}\left(\frac{(n+1)[\log(n+1)+\log\log(n+1)]-\sqrt{(n+1)[\log(n+1)+\log\log(n+1)]}}{\left[n(\log n+\log\log n-1)-\sqrt{n(\log n+\log\log n-1)}\right]\log[n(\log n+\log\log n-1)]} -\right.$

$\left.\frac{1}{\log[n(\log n+\log\log n-1)]}\right) =$
$\frac{11(\log 11+\log\log 11)-\sqrt{11(\log 11+\log\log 11)}}{\left[10(\log 10+\log\log 10-1)-\sqrt{10(\log 10+\log\log 10-1)}\right]\log[10(\log 10+\log\log 10-1)]} -$
$\frac{1}{\log[10(\log 10+\log\log 10-1)]} = 0.197000361516\ldots \ll 0.197001$, see Table 2. Thereupon, (3.9) can be written as

$$(3.10)\ \frac{23}{100}\cdot\frac{p_{n+1}-p_n}{\sqrt{p_n}\log p_n} \ll 0.197001 + \frac{\sqrt{2}(\sqrt{n+1}-\sqrt{n})\sqrt{n}}{(p_n-\sqrt{p_n})\log p_n} \Rightarrow \frac{p_{n+1}-p_n}{\sqrt{p_n}\log p_n}$$
$$\ll \frac{19.7001}{23} + \frac{100\sqrt{2}}{23}\cdot\frac{(\sqrt{n+1}-\sqrt{n})\sqrt{n}}{(p_n-\sqrt{p_n})\log p_n}$$
$$= 0.8565260869565217\ldots + \frac{100\sqrt{2}}{23}\cdot\frac{(\sqrt{n+1}-\sqrt{n})\sqrt{n}}{(p_n-\sqrt{p_n})\log p_n}$$
$$< 0.85653 + \frac{100\sqrt{2}}{23}\cdot\frac{(\sqrt{n+1}-\sqrt{n})\sqrt{n}}{(p_n-\sqrt{p_n})\log p_n}.$$

Again, using the Rosser's theorem, the inequality (2.1) in the right-hand side of (3.9),specifically in $\frac{100\sqrt{2}}{23}\cdot\frac{(\sqrt{n+1}-\sqrt{n})\sqrt{n}}{(p_n-\sqrt{p_n})\log p_n}$, we discover that

$$(3.11)\ \frac{p_{n+1}-p_n}{\sqrt{p_n}\log p_n}$$
$$\ll 0.85653 + \frac{100\sqrt{2}}{23}$$
$$\cdot\frac{(\sqrt{n+1}-\sqrt{n})\sqrt{n}}{\left(n\log n+n\log\log n-n-\sqrt{n\log n+n\log\log n-n}\right)\log(n\log n+n\log\log n-n)}.$$

On the other hand, we observe that the function $\frac{100\sqrt{2}}{23}\cdot\frac{(\sqrt{n+1}-\sqrt{n})\sqrt{n}}{\left(n\log n+n\log\log n-n-\sqrt{n\log n+n\log\log n-n}\right)\log(n\log n+n\log\log n-n)}$ is a decreasing function, in which the maximum occurs when $n = 10$, for $n \in \mathbb{N}$ and $n > 9$. In other words, $\max_{n\in\mathbb{N}_{>9}}\left(\frac{100\sqrt{2}}{23}\cdot\right.$
$\left.\frac{(\sqrt{n+1}-\sqrt{n})\sqrt{n}}{\left(n\log n+n\log\log n-n-\sqrt{n\log n+n\log\log n-n}\right)\log(n\log n+n\log\log n-n)}\right) =$
$\frac{100\sqrt{2}}{23}\cdot\frac{(\sqrt{11}-\sqrt{10})\sqrt{10}}{\left(10\log 10+10\log\log 10-10-\sqrt{10\log 10+10\log\log 10-10}\right)\log(10\log 10+10\log\log 10-10)} =$
$0.058540049371\ldots < 0.058541$ see Table 3. Thus, the inequality (3.11) can be written as

$$\frac{p_{n+1}-p_n}{\sqrt{p_n}\log p_n} \ll 0.85653 + 0.058541 = 0.915071 \Rightarrow p_{n+1}-p_n$$
$$\ll 0.915071\sqrt{p_n}\log p_n. \square$$

COROLLARY 1.*For* $n \in \mathbb{N}_{>9}$, *then*

$$p_{n+1} - p_n \ll p_n^{1/2} \log p_n. \tag{3.12}$$

Proof. We note that $0.915071 < 1$. Then, $p_{n+1} - p_n \ll 0.915071 \cdot p_n^{1/2} \log p_n < p_n^{1/2} \log p_n$; consequently, $p_{n+1} - p_n \ll p_n^{1/2} \log p_n$.□

COROLLARY 2.*For* $n \in \mathbb{N}_{>9}$, *then*

$$p_{n+1} - p_n \ll p_n^{8371/10000}. \tag{3.13}$$

Proof. We note, using the Rosser's theorem, that $0.915071 < \frac{[n(\log n + \log\log n - 1)]^{3371/10000}}{\log[n(\log n + \log\log n - 1)]} < \frac{p_n^{3371/10000}}{\log p_n}$, for $n \in \mathbb{N}$ and $n > 9$, see Table 4. Then, $p_{n+1} - p_n \ll 0.915071 \cdot p_n^{1/2} \log p_n < \frac{[n(\log n + \log\log n - 1)]^{3371/10000}}{\log[n(\log n + \log\log n - 1)]} p_n^{1/2} \log p_n < \frac{p_n^{3371/10000}}{\log p_n} p_n^{1/2} \log p_n = p_n^{1/2+3371/10000}$; consequently, $p_{n+1} - p_n \ll p_n^{8371/10000}$.□

Table 1. In this table, we have: first column: n; second column: p_n; third column: $\frac{\sqrt{p_n}}{p_n-\sqrt{p_n}}$.

1	2	2.41421356237
2	3	1.36602540378
3	5	0.809016994375
4	7	0.607625218511
5	11	0.431662479035
6	13	0.383795939622
7	17	0.320194101601
8	19	0.297716607974
9	23	0.263446887423
10	29	0.228041600254
11	31	0.218925478761
12	37	0.196743403619
13	41	0.185078105935
14	43	0.179939012483
15	47	0.170775100008
16	53	0.159232882486
17	59	0.149674926687
18	61	0.146837494598
19	67	0.139172011695
20	71	0.134659282474
21	73	0.132555607573
22	79	0.126771723299
23	83	0.123297970477
24	89	0.118567967409
25	97	0.113008935435
26	101	0.110498756211
27	103	0.109302858481
28	107	0.107019626724
29	109	0.105928763971
30	113	0.103840587613
31	127	0.0973764100761
32	131	0.095734793402
33	137	0.0934169111082
34	139	0.0926798994388
35	149	0.0892334838901
36	151	0.088588038183
37	157	0.0867305390137
38	163	0.0849823786099
39	167	0.0838725782128
40	173	0.0822845723138
41	179	0.0807813941588
42	181	0.0802979113726
43	191	0.0780014471636
44	193	0.0775648124451
45	197	0.0767125961613
46	199	0.076296646362
47	211	0.0739325668873
48	223	0.0717711014553
49	227	0.0710907928023
50	229	0.0707576576773

Table 2 In this table, we have: first column: n; second column: p_n; third column: p_{n+1}; fourth column: $\frac{p_{n+1}-\sqrt{p_{n+1}}}{(p_n-\sqrt{p_n})\log(p_n)}-\frac{1}{\log(p_n)}$; fifth column: $\frac{(n+1)[\log(n+1)+\log\log(n+1)]-\sqrt{(n+1)[\log(n+1)+\log\log(n+1)]}}{[n(\log n+\log\log n-1)-\sqrt{n(\log n+\log\log n-1)}]\log[n(\log n+\log\log n-1)]}-\frac{1}{\log[n(\log n+\log\log n-1)]}$.

1	2	3	1.68005395861	$\frac{0}{-\infty+(-i)\infty}$
2	3	5	1.07394070868	$-0.103407404743+0.7105047286i$
3	5	7	0.35750492191	$0.6817282701+7.2134385427i$
4	7	11	0.392911090466	3.36415517068
5	11	13	0.0928723458568	0.992928801663
6	13	17	0.144522096169	0.552098700798
7	17	19	0.0483569666799	0.37868977044
8	19	23	0.0826509297746	0.2883135228
9	23	29	0.0947924903736	0.233523351985
10	29	31	0.0228551690734	0.197000361516
11	31	37	0.0628049053768	0.171014028676
12	37	41	0.032959971871	0.151624011597
13	41	43	0.0143657734109	0.136622347013
14	43	47	0.0270069617393	0.12468048987
15	47	53	0.0360732725413	0.114953353485
16	53	59	0.0308446657947	0.106878839316
17	59	61	0.00894076651536	0.100069115441
18	61	67	0.0257248270611	0.0942482597957
19	67	71	0.0152011354719	0.0892147942563
20	71	73	0.00705631107655	0.0848182728157
21	73	79	0.0204516161793	0.0809441519225
22	79	83	0.0123315238729	0.0775037242845
23	83	89	0.0173854345143	0.0744272487813
24	89	97	0.0212383412249	0.0716591551294
25	97	101	0.00952863105996	0.0691546291729
26	101	103	0.00452889373479	0.066877137427
27	103	107	0.00884141277557	0.0647966033908
28	107	109	0.00421509186153	0.0628880442981
29	109	113	0.00824036236011	0.0611305384568
30	113	127	0.0276080978641	0.05950643348
31	127	131	0.00682084266855	0.0580007324349
32	131	137	0.0098495441822	0.0566006130416
33	137	139	0.00310628968586	0.0552950475109
34	139	149	0.0152669063584	0.0540744993176
35	149	151	0.00280252757998	0.052930679366
36	151	157	0.00827386007675	0.0518563484262
37	157	163	0.00788912797694	0.0508451559264
38	163	167	0.00502359257187	0.0498915075383
39	167	173	0.00731695533295	0.0489904557372
40	173	179	0.00700933419432	0.0481376088214
41	179	181	0.00224115404136	0.0473290548631
42	181	191	0.0110602376697	0.0465612978089
43	191	193	0.00207160811147	0.0458312035278
44	193	197	0.00409168891333	0.0451359540456
45	197	199	0.00199550730891	0.0444730085546
46	199	211	0.0118329819711	0.0438400700562
47	211	223	0.0110248542825	0.0432350567102
48	223	227	0.0034368739172	0.0426560771342
49	227	229	0.00168193895754	0.0421014090321
50	229	233	0.00332882229792	0.0415694806397

Table 3. In this table, we have: first column: n; second column: p_n; third column: p_{n+1}; fourth column: $\frac{100\sqrt{2}}{23}\cdot\frac{(\sqrt{n+1}-\sqrt{n})\sqrt{n}}{(p_n-\sqrt{p_n})\log p_n}$; fifth column: $\frac{100\sqrt{2}}{23}\cdot\frac{(\sqrt{n+1}-\sqrt{n})\sqrt{n}}{(n\log n+n\log\log n-n-\sqrt{n\log n+n\log\log n-n})\log(n\log n+n\log\log n-n)}$.

1	2	3	6.272587134299843	$\frac{0}{-\infty+(-i)\infty}$
2	3	5	1.9840866892171307	$0.284957455344+0.401882257901i$
3	5	7	0.6415029481034 81	28.5584210221
4	7	11	0.342624349345885 26	2.38199900062
5	11	13	0.15926803364713518	0.560183506345
6	13	17	0.12267270921353256	0.260493553292
7	17	19	0.08145670160444649	0.154796856492
8	19	23	0.06921567969328274	0.104445193557
9	23	29	0.052443335212888936	0.0761687139166
10	29	31	0.03774153236790028	0.058540049371
11	31	37	0.034436901751818134	0.0467238992906
12	37	41	0.026987421764204673	0.0383706117865
13	41	43	0.02348592801871709	0.0322193565009
14	43	47	0.022042783043103822	0.027541003079
15	47	53	0.019570039341507306	0.0238884635107
16	53	59	0.016680053359324994	0.0209743534828
17	59	61	0.014482116015245818	0.0186066327176
18	61	67	0.013870276435356464	0.0166527301043
19	67	71	0.012272487891150615	0.0150185607804
20	71	73	0.011385495549795406	0.0136357436922
21	73	79	0.01098777666277169	0.0124535383227
22	79	83	0.009923990699274668	0.0114336020945
23	83	89	0.009315802612742992	0.0105464902103
24	89	97	0.008520406322119157	0.00976926347906
25	97	101	0.007635565554398014	0.00908381898433
26	101	103	0.007255275161363983	0.00847570323531
27	103	107	0.007079118997882318	0.00793325404162
28	107	109	0.006747177453256922	0.00744697054278
29	109	113	0.006592706941280962	0.00700904427529
30	113	127	0.0063007050257034534	0.00661300565991
31	127	131	0.005440357092128137	0.00625345438267
32	131	137	0.0052341393772362445	0.00592585154612
33	137	139	0.004950004881210249	0.00562635784501
34	139	149	0.004862233095468839	0.00535170641416
35	149	151	0.004459743238734792	0.005099102064
36	151	157	0.004387230560193211	0.00486614079057
37	157	163	0.004180671911264161	0.00465074500167
38	163	167	0.0039913994543141025	0.00445111102683
39	167	173	0.003874017890551649 3	0.00426566630338
40	173	179	0.0037091856602237054	0.00409303424021
41	179	181	0.0035568799397537073	0.00393200521542
42	181	191	0.0035089891207412778	0.00378151250622
43	191	193	0.0032846763819461373	0.00364061220933
44	193	197	0.0032433113418736293	0.00350846640827
45	197	199	0.0031630094548509365	0.00338432899747
46	199	211	0.0031244090620354283	0.00326753369108
47	211	223	0.0029084075960895176	0.00315748383776
48	223	227	0.002718564860662627	0.00305364373476
49	227	229	0.00266049736742676	0.00295553119284
50	229	233	0.002632449764759305	0.00286271114864

Table 4. In this table, we have: first column: n; second column: p_n; third column: $\frac{[n(\log n+\log\log n-1)]^{3371/10000}}{\log[n(\log n+\log\log n-1)]}$; fourth column: $\frac{(p_n)^{3371/10000}}{\log p_n}$.

1	2	Indeterminate	1.82243375378
2	3	$0.320277199572 - 0.141988309367\,i$	1.31823586245
3	5	−1.51631579245	1.06892826493
4	7	1.35858857197	0.990284008887
5	11	1.04564700272	0.935886672276
6	13	0.965188468812	0.92561739844
7	17	0.934049896068	0.917286607493
8	19	0.921128220524	0.916358221138
9	23	0.916706682455	0.91776702488
10	29	0.916796176973	0.924044938646
11	31	0.919475451108	0.926700408377
12	37	0.92371653581	0.935455613605
13	41	0.928927036822	0.941624135498
14	43	0.934746387767	0.944747552697
15	47	0.940945861601	0.951013777524
16	53	0.947375692549	0.960353130585
17	59	0.953935407329	0.969518845435
18	61	0.960556360232	0.972524466632
19	67	0.967191036139	0.981376294969
20	71	0.973806277085	0.987134818096
21	73	0.98037887031	0.98997068695
22	79	0.986892604019	0.998305656515
23	83	0.99333626106	1.00372071788
24	89	0.999702226652	1.01163756407
25	97	1.00598550681	1.02182633243
26	101	1.01218302668	1.02677122375
27	103	1.01829312281	1.02920783627
28	107	1.02431517195	1.03401157009
29	109	1.03024931709	1.03637954715
30	113	1.03609626373	1.04104981543
31	127	1.04185712735	1.05674773763
32	131	1.04753331859	1.06106003078
33	137	1.05312645645	1.06739480342
34	139	1.05863830247	1.06947202789
35	149	1.06407071073	1.07961385702
36	151	1.06942558984	1.08159533292
37	157	1.0747048741	1.08745074255
38	163	1.07991050163	1.09317778661
39	167	1.08504439799	1.0969276049
40	173	1.09010846384	1.10245437188
41	179	1.09510456594	1.10786848249
42	181	1.10003453057	1.10964905412
43	191	1.10490013892	1.11837904508
44	193	1.10970312399	1.12009166299
45	197	1.11444516862	1.12348481465
46	199	1.11912790448	1.12516565649
47	211	1.12375291168	1.1350396242
48	223	1.12832171899	1.14457275303
49	227	1.13283580436	1.14767937047
50	229	1.1372965959	1.14921986787

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