An additional condition for Bell experiments for accepting local realistic theories

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We assume that one source of two uncorrelated spin-carrying particles emits them in a state, which can be described as a spin-1/2 bipartite pure uncorrelated state. We consider a Bell-Clauser-Horne-Shimony-Holt (Bell-CHSH) experiment with two-orthogonal-settings. We propose an additional condition for the state to be reproducible by the property of local realistic theories. We use the proposed measurement theory in order to construct the additional condition [K. Nagata and T. Nakamura, Int. J. Theor. Phys. **49**, 162 (2010)]. The condition is that local measurement outcome is $\pm 1/\sqrt{2}$. Otherwise, such an experiment does not allow for the existence of local realistic theories even in the situation that all Bell-CHSH inequalities hold. Also we derive new set of Bell inequalities when local measurement outcome is $\pm 1/\sqrt{2}$.

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As a famous physical theory, the quantum theory (cf. [1–5]) gives accurate and at times remarkably accurate numerical predictions. Much experimental data has fit to the quantum predictions for long time.

Fine shows [6, 7] that quantum correlation functions accept local realistic theories if and only if all Bell-Clauser-Horne-Shimony-Holt (CHSH) inequalities hold. However, is matrix theory compatible with probability theory? Matrix theory has postulates. Probability theory has postulates. We consider joint set of such postulates. Does such joint set work as new set of postulates for matrix theory and for probability theory?

Let us consider joint probability. A is an observable. B is an observable. a, b are measurement outcome in a quantum state, respectively. A and B are not commutative. Thus,

$$[A, B] \neq O. \tag{1}$$

We consider as follows: first we measure observable A and obtain the result of measurement a and next we measure observable B and obtain the result of measurement b. This joint event is different if we exchange A to B. Hence

$$P(\overbrace{A = a}^{\text{first}} \cap \overbrace{B = b}^{\text{second}}) \neq P(\overbrace{B = b}^{\text{first}} \cap \overbrace{A = a}^{\text{second}}). \tag{2}$$

On the other hand, the joint probability is depictured in terms of conditional probability:

$$\begin{aligned} &P(A = a|B = b)P(B = b) = P(A = a \cap B = b),\\ &P(B = b|A = a)P(A = a) = P(B = b \cap A = a). \end{aligned} \tag{3}$$

From postulates of probability theory, we have

$$P(A = a \cap B = b) = P(B = b \cap A = a). \tag{4}$$

We cannot assign truth value "1" for the proposition (2) and for the proposition (4), simultaneously. We are in a contradiction. It turns out that the joint set of postulates does not work as new set of postulates for matrix theory and for probability theory.

In fact, two expected values of spin-1/2 pure state (two-dimensional state) $\langle\sigma_x\rangle, \langle\sigma_y\rangle$ rule out the existence of probability space of von Neumann's projective measurement theory [8, 9]. Further, it is shown that multipartite pure uncorrelated state violates nonlocal realistic theories [10, 11].

Therefore we question if uncorrelated quantum states can generate correlations that are incompatible with any local realistic theories.

In this paper, we assume that one source of two uncorrelated spin-carrying particles emits them in a state, which can be described as a spin-1/2 bipartite pure uncorrelated state. We consider a Bell-CHSH experiment with two-orthogonal-settings. We propose an additional condition for the state to be reproducible by the property of local realistic theories. We use the proposed measurement theory [9] in order to construct the additional condition. The condition is that local measurement outcome is $\pm 1/\sqrt{2}$. Otherwise, such an experiment does not allow for the existence of local realistic theories even in the situation that all Bell-CHSH inequalities hold. Also we derive new set of Bell inequalities when local measurement outcome is $\pm 1/\sqrt{2}$.

An additional condition for the spin-1/2 bipartite pure uncorrelated state to be reproducible by the property of local realistic theories in Bell-CHSH experiments with two-orthogonal-settings is as follows:

$$(E_{LR}, E_{LR})_{\max} = (E_{QM}, E_{QM})_{\max}. \tag{5}$$

Here, the set of Bell-CHSH inequalities are

$$\begin{aligned}
-2 &\le \langle A_1B_1\rangle + \langle A_2B_2\rangle + \langle A_1B_2\rangle - \langle A_2B_1\rangle \le 2\\
-2 &\le \langle A_1B_1\rangle + \langle A_2B_2\rangle - \langle A_1B_2\rangle + \langle A_2B_1\rangle \le 2\\
-2 &\le \langle A_1B_1\rangle - \langle A_2B_2\rangle + \langle A_1B_2\rangle + \langle A_2B_1\rangle \le 2\\
-2 &\le \langle A_1B_1\rangle - \langle A_2B_2\rangle - \langle A_1B_2\rangle - \langle A_2B_1\rangle \le 2.
\end{aligned} \tag{6}$$

The above Bell inequalities hold if quantum state is a spin-1/2 bipartite pure uncorrelated state and if the measurement outcome is ± 1. According to [6, 7], we have local realistic theories for the Bell experiment in an uncorrelated state. That is, if the Bell inequalities hold then

$$\langle A_1B_1\rangle = \int d\lambda \rho(\lambda) A_1(\lambda) B_1(\lambda) \tag{7}$$

and so on. We consider two partite uncorrelated state. So the Bell inequalities hold in this case. Do we have local realistic theories for the Bell experiment? What is the relation between Fine's result and References [10, 11]?

In fact, the above situation is not satisfied when we consider the relation between (E_{LR}, E_{LR}) and (E_{QM}, E_{QM}). (E_{QM}, E_{QM}) is as follows:

$$\begin{aligned}
&(E_{QM}, E_{QM}) = \sum_{k=1}^{2}\sum_{j=1}^{2}\langle A_kB_j\rangle^2\\
&= \langle A_1B_1\rangle^2 + \langle A_2B_2\rangle^2 + \langle A_2B_1\rangle^2 + \langle A_1B_2\rangle^2\\
&= (\langle A_1\rangle^2 + \langle A_2\rangle^2)(\langle B_1\rangle^2 + \langle B_2\rangle^2) \le 1.
\end{aligned} \tag{8}$$

Hence we have the following

$$(E_{QM}, E_{QM})_{\max} = 1. \tag{9}$$

On the other hand, (E_{LR}, E_{LR}) is as follows:

$$\begin{aligned}
&(E_{LR}, E_{LR})\\
&= \sum_{k=1}^{2}\sum_{j=1}^{2}\left(\int d\lambda \rho(\lambda) A_k(\lambda) B_j(\lambda)\right)^2\\
&\le \sum_{k=1}^{2}\sum_{j=1}^{2}\\
&= 4.
\end{aligned} \tag{10}$$

The above inequality is saturated when

$$A_k(\lambda) B_j(\lambda) = 1 \tag{11}$$

for all λ. Hence we have

$$(E_{LR}, E_{LR})_{\max} = 4 \tag{12}$$

and thus

$$(E_{LR}, E_{LR})_{\max} > (E_{QM}, E_{QM})_{\max} \tag{13}$$

if the results of measurements are ± 1. Therefore we cannot consider that quantum correlation functions are reproducible by local realistic theories, that is

$$E_{QM} \neq E_{LR}. \tag{14}$$

However we have

$$(E_{LR}, E_{LR})_{\max} = (E_{QM}, E_{QM})_{\max} \tag{15}$$

if and only if the results of measurements are $\pm 1/\sqrt{2}$ [9]. Namely,

$$A_k(\lambda) = \pm 1/\sqrt{2}, B_j(\lambda) = \pm 1/\sqrt{2}. \tag{16}$$

In this case, we have the following set of Bell inequalities

$$\begin{aligned}
-1 &\le \langle A_1B_1\rangle + \langle A_2B_2\rangle + \langle A_1B_2\rangle - \langle A_2B_1\rangle \le 1\\
-1 &\le \langle A_1B_1\rangle + \langle A_2B_2\rangle - \langle A_1B_2\rangle + \langle A_2B_1\rangle \le 1\\
-1 &\le \langle A_1B_1\rangle - \langle A_2B_2\rangle + \langle A_1B_2\rangle + \langle A_2B_1\rangle \le 1\\
-1 &\le \langle A_1B_1\rangle - \langle A_2B_2\rangle - \langle A_1B_2\rangle - \langle A_2B_1\rangle \le 1.
\end{aligned} \tag{17}$$

The change of the value does not affect the quantum correlation functions E_{QM}. The above Bell inequalities hold if quantum state is a spin-1/2 bipartite pure uncorrelated state and if the measurement outcome is $\pm 1/\sqrt{2}$. Thus we have local realistic theories for the Bell experiment in the uncorrelated state in this case.

In conclusion, we have assumed that one source of two uncorrelated spin-carrying particles emits them in a state, which can be described as a spin-1/2 bipartite pure uncorrelated state. We have considered a Bell-CHSH experiment with two-orthogonal-settings. We have proposed an additional condition for the state to be reproducible by the property of local realistic theories. We have used the proposed measurement theory in order to construct the additional condition. The condition has been that local measurement outcome is $\pm 1/\sqrt{2}$. Otherwise, such an experiment have not allowed for the existence of local realistic theories even in the situation that all Bell-CHSH inequalities hold. Also we have derived new set of Bell inequalities when local measurement outcome is $\pm 1/\sqrt{2}$.

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