The cosmological constant from $AdS_5 \times S^5$

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The dark energy density of the vacuum, widely regarded as the cause of the cosmological constant, may derive from zero-point energy in the 5-sphere of AdS_5 x S^5 spacetime. The dark energy density scales inversely as the volume of the 5-sphere and has the value 1.33 x 10^{-123}, in Planck units, at the length scale of the Bohr radius, which seems to be the characteristic length scale of the vacuum. The value of dark energy density calculated in this way is consistent with the value, 1.29 ± 0.07 x 10^{-123}, calculated from the *WMAP* 9-year six-parameter ΛCDM fit. The Bohr radius, dark energy density and, therefore, cosmological constant derive from the geometry of spacetime and can be written in elegant form in Planck units.

1 Introduction

In 1998, the universe was found to have a small positive cosmological constant, Λ [1, 2]. In other words, the expansion of the universe is accelerating, suggesting the presence of vacuum energy (dark energy). The dark energy density ρ_Λ contributes around 70% of the critical density, $\rho_c = 3{H_0}^2/8\pi$ in Planck units, for an expanding flat universe; H_0 is the Hubble constant. Theoretical considerations suggest that the value of vacuum energy density should be ~1 in Planck units (c = $\hbar$ = G = 1) yet the value suggested by measurements of the cosmological constant is ~10^{-123}. Conjecturing that dark energy is zero-point energy in the 5-sphere of the string theory background $AdS_5 \times S^5$ at the length scale characteristic of the vacuum, the radius of the S^5 necessary to reproduce the measured value of dark energy density has been calculated and found to be precisely equal to the Bohr radius. Since the Bohr radius has already been found to derive from the Planck length and the geometry of spacetime [3], the dark energy density, and, consequently, the cosmological constant may be written in the same terms.

2 Background

Particle masses have been shown to occupy mass levels within three geometric sequences that descend from the Planck Mass [4, 5, 6]. Sequences 1, 2 and 3 are of common ratio 1/π, 2/π, and 1/e, respectively. The mass levels have been related to 3-brane locations in extra dimensions by way of exponential factors of distance from the Planck brane [5, 6], as in the Randall and Sundrum RS1 model [7]. We have suggested that Sequences 1 and 2 derive from the geometry of an $S^1/Z_2 \times$

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$S^1/(Z_2 \times Z_2')$ orbifold with Planck-scale compactification radii [5], while Sequence 3 may derive from the geometry of a hypersphere of Planck radius [6]; the spacetime is $AdS_6 \times S^4$.

Recently, a set of equations was discovered, in which the scales of atomic and particle physics are related to Planck scale through multiplication by powers of the quantity $(\pi/2)^{25}$ and powers of α (the fine structure constant) [3]. Those scales include the Bohr radius, the electron mass, the pion charge radius, the up and top quark masses, and the MSSM GUT scale, as shown in Table 1.

Table 1: Scales of atomic and particle physics in relation to Planck scale [3]

Scale	Relationship with Planck scale	Value from Planck relationship
Bohr radius	$a_0 = \left(\frac{\pi}{2}\right)^{125} l_P$	0.529×10^{-10} m
electron mass	$m_e = \propto^{-1} \left(\frac{\pi}{2}\right)^{-125} m_P$	0.511 MeV
pion charge radius	$r_\pi = \left(\frac{\pi}{2}\right)^{100} l_P$	0.661×10^{-15} m
up quark mass	$m_{up} = \propto \left(\frac{\pi}{2}\right)^{-100} m_P$	2.18 MeV
top quark mass	$m_{top} = \propto \left(\frac{\pi}{2}\right)^{-75} m_P$	174 GeV
MSSM GUT scale	$E_{GUT} = \propto^{-1} \left(\frac{\pi}{2}\right)^{-25} E_P$	2.09×10^{16} GeV

3 The geometry

The spacetime $AdS_5 \times S^5$ is the background for this analysis: there is a duality between Type IIA string theory on $AdS_6 \times S^4$ and Type IIB on $AdS_5 \times S^5$. We live on the vacuum brane, on a boundary of the AdS_5 spacetime, in an extra dimension that winds around an $S^1/(Z_2 \times Z_2')$ orbifold that has Planck-scale compactification radii. Massive particles live on other boundaries in the extra dimension. The characteristic length scale on a boundary is enhanced from Planck scale by an exponential factor of distance from the Planck brane in the extra dimension. The characteristic length scale on the S^5 (its radius) is equally scaled up from Planck scale. The AdS/CFT correspondence [8] has multiple manifestations, on multiple boundaries within $AdS_5 \times S^5$. The cosmological constant is assumed to derive from zero-point energy in the S^5 at the length scale of the vacuum.

4 The dark energy density

At Planck scale, the zero point energy $E_{0,P}$ is equal to ½$\hbar\omega_P$. In Planck units, as used throughout the analysis,

$$E_{0,P} = \frac{1}{2} \tag{1}$$

The vacuum energy density ρ_P at Planck scale is given by

$$\rho_P = \frac{1}{2} \tag{2}$$

The vacuum energy density scales inversely as the volume of the 5-sphere and its value ρ_B at the length scale of the Bohr radius a_0 is given by

$$\rho_B = \frac{1}{2} {a_0}^{-5} \tag{3}$$

which has the value 1.33 x 10^{-123} in Planck units. This value is consistent with the value 1.29 ± 0.07 x 10^{-123} calculated from the *WMAP* 9-year six-parameter ΛCDM fit [9]. It seems that the Bohr radius is the characteristic length scale of the vacuum. We now see that the dark energy density ρ_Λ is given by

$$\rho_\Lambda = \frac{1}{2} {a_0}^{-5} \tag{4}$$

and since the Bohr radius is given by

$$a_0 = \left(\frac{\pi}{2}\right)^{125} \tag{5}$$

it follows from (4) and (5) that

$$\rho_\Lambda = \frac{1}{2}\left(\frac{\pi}{2}\right)^{-625} \tag{6}$$

5 The cosmological constant

Since the dark energy density and the cosmological constant are related, in Planck units, by

$$\rho_\Lambda = \frac{\Lambda}{8\pi} \tag{7}$$

it follows from (4) and (7) that

$$\Lambda = 4\pi {a_0}^{-5} \tag{8}$$

and from (4) and (8) that

$$\Lambda = 4\pi\left(\frac{\pi}{2}\right)^{-625} \tag{9}$$

6 Conclusions

- Dark energy is zero-point energy in the 5-sphere of $AdS_5 \times S^5$.
- The dark energy density in the vacuum is 1.33 x 10^{-123}, in Planck units.
- The Bohr radius is the characteristic length scale of the vacuum.
- The cosmological constant is caused by dark energy.

7 References

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