A Crazy It From A Misleading Bit: How A Zero-Referenced Fundamental Theorem of Calculus Loses Information And May Be Misleading Mathematical Physics

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(Dated: June 28, 2013)

I. INTRODUCTION

Imagine for a moment an endless diamond, completely solid and pristine. No flaws or gaps in the structure. Suppose at some point in the density of the material something strange occurs in that a deformation appears out of nowhere which splits into two waves. Should these two waves interact again with each other, the deformation disappears. However should they separate enough then one of the waves, which we shall call the baryon wave, is stable by itself. This wave has the strange property that it is a traveling decrease in density of the diamond. There is no"other" material, only the decrease in something we shall have to think of as the vacuum (energy) density. The wave apparently has the ability to pass through or combine into structures with other baryon waves but has no ability to disappear back into a pristine diamond if there is no second deformation wave present, which we do not cover in this essay. Suppose that a certain combination of these first waves were to become sentient. Would they be able to detect that they are a moving wave or would their perceptions lead them to a misunderstanding of how these waves effect the very substance that they are traveling within and so not realize there exists another class of solutions for tensors (scalars, vectors and so on)? Is there a way to determine that the actual density is a fairly good model for their universe? If so, what mathematics would be required in order to describe it much better than other models which the sentient waves have based on their physical perceptions? It is due to this question that we present our proposed answer of a modification of calculus. In order to fully describe the baryon wave, the dimensions they create with their presence, the limited radius distortions they cause and even the substance itself we must re-evaluate our understanding of calculus in order to model them as the derivatives of finite Action area integrals. We propose that in order to understand how the universe stores information, we must have a foundational basis for these areas, and in order to understand how it processes information we must ensure that we have within the literature all classes of its derivatives (directional derivatives, divergence, etc.). We do not go into details in this essay, but our proposed future path is to accomplish this via a modification of Gunnar Nordström's gravitational theory, an early competing model to General Relativity worked on by Nordström, Einstein and Fokker (see [1] for a recent review). This model was discarded by Einstein and others since it did not predict gravitational lensing, a problem which our modification would seem to have the possibility of remedying (see final assertions). Therefore in this essay we introduce our different view point of calculus, named "Area" Calculus in order to distinguish the concept from the mainstream variety which we will refer to as "Single Function" Calculus.

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II. AREA CALCULUS

We will first give a simple graphical understanding of the meaning, followed by a more formulaic explanation using the same techniques as Riemann sums. We avoid a formal drawn out proof in order to keep the essay open to a wider audience but still provide the ability to intuitively understand and reproduce a proof that Single Function Calculus is a simplification of Area Calculus.

In order to do this, we need to re-examine the concept of differentiation and integration from their most basic proof in graphical form. We start with a regular rectangle where the top and the bottom are line segments such that *each* is a function of x denoted as y_1^* and y_2^* (Fig. 1).

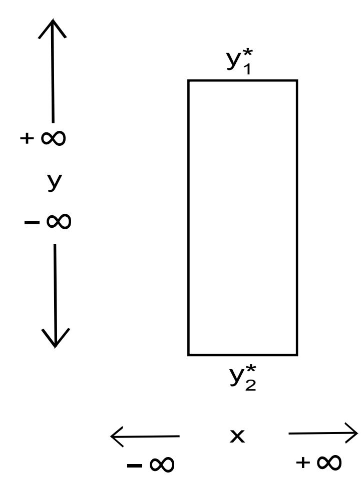

FIG. 1. Lines segments as two functions

We do not yet place these onto a coordinate plot as a solid line (Fig. 2) on the x axis can lead to an incorrect proof. We then incorporate multiple rectangles (A,B,C,D) (Fig. 3), of equal length ($\Delta x = x_2 - x_1$) and height ($\Delta y = y_1^* - y_2^*$) with an area of $((y_1^* - y_2^*) * (x_2 - x_1))$. Each horizontal line segment has a particular $|y_1^*|$,$|y_2^*|$ of $-\infty > y^* < \infty$ as measured from an unknown zero point.

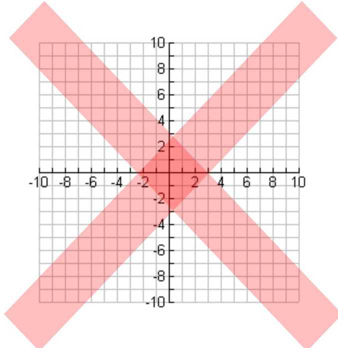

FIG. 2. Misleading solid axes

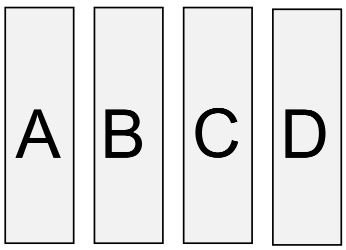

FIG. 3. Rectangles

Note that if we continue adding on rectangles with the same width and height (E,F,G,H) (Fig. 4), that

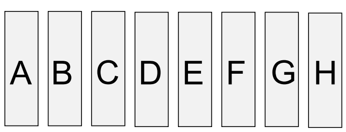

FIG. 4. Adding area via rectangles

the rate of the addition of area is constant (**rate of change of addition of area=area next-area previous**). Should we instead add on blocks of decreasing area (E',F',G',H')(Fig. 5), then the rate of the addition of area is decreasing.

From Fig. 6, we can see that a reflection of the rectangles about the lower line segment does not change the rate of the addition of area. It is still either constant, decreasing or similarly increasing (Figs. 7, 8) should we add on blocks of greater area (E",F",G",H").

We assume in this new interpretation of calculus that *area* is axiomatic such that the x and y line segments are simply the boundaries used to denote it. We contend it is extremely misleading to only incorporate these line segments (and their instantaneous rates of change) into a proof without also taking into consideration the area they bound. We designate the top line in all plots, even after reflection, as a variation of y_1 and the bottom line as a variation of y_2.

Let us now use the standard proof technique of taking the number of individual rectangles to infinity by decreasing the width of each rectangle. It is important to

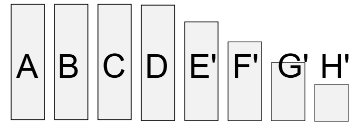

FIG. 5. Addition of area decreasing

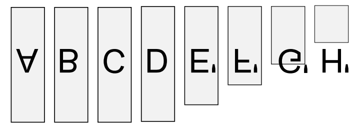

FIG. 6. Reflected but area still decreasing

always maintain in mind that whereas the width of Δx is decreasing, the distance between the line segments of Δy^* *is not* as line segments y_1^* and y_2^* are *two* separate functions of x. We must, however, denote the change in each individual line segment function as Δy_1^* and Δy_2^* as we take into account the decrease in Δx to zero (Fig. 9).

Thus we obtain an instantaneous rate of change of the addition of area (or the instantaneous change in area of rectangles of zero width) as $\frac{dy_1 - dy_2}{dx} \equiv \lim_{\Delta x \to 0} \frac{\Delta y_1^* - \Delta y_2^*}{\Delta x} = \lim_{h \to 0} \frac{(f_1(x+h) - f_1(x)) - (f_2(x+h) - f_2(x))}{(x+h) - x}$. To reiterate, although $x_2 - x_1 = \Delta x \to dx$ we must understand that $y_1^* - y_2^* = \Delta y_1^* - \Delta y_2^* \to dy_1 - dy_2$, ***not just* the single function form** dy! This would appear to be an extremely fundamental mistake.

Let us put some numerical values in to provide a better understanding. Let $y_1 = 10 - \frac{1}{x}$ and let $y_2 = 8$. The Area derivative is $\frac{dy_1 - dy_2}{dx} = \frac{d(10 - \frac{1}{x} - 8)}{dx} = \frac{1}{x^2}$. If we should reflect the area about y_2 we now have $y_1 = 8$ and $y_2 = 6 + \frac{1}{x}$. The Area derivative is unchanged at $\frac{dy_1 - dy_2}{dx} = \frac{d(8 - (6 + \frac{1}{x}))}{dx} = \frac{1}{x^2}$ despite incorporating a function with a slope of opposite sign. This is a proof that derivation happens with respect to the secondary constant function (since differentiation of $+\infty$ or $-\infty$ doesn't seem reasonable), not whether the points that make up a line are changing positively or negatively (Figs. 10 and 11). For Area figures here on out, it can be understood that a reflection of the area does not change the directional derivative.

III. ANTI-DIFFERENTIATION

Let us now take the derivative form of the previous section and use a similar technique used for Riemann Sums. Assuming that y_2 or y_1 is a constant function (we do not go into it now, but if both are constants then this would seem to be a truer definition of an Einstein manifold with $R = nk = n(y_1 - y_2)$) and the area has been reflected, then $\frac{dy_1 - dy_2}{dx} = \frac{dy_1}{dx} = \frac{-dy_2}{dx}$. Graphically, these both reproduce the same function relative to the y=0 on the x

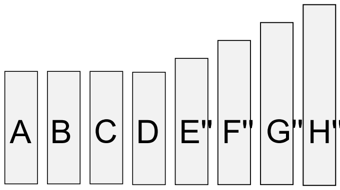

FIG. 7. Area increasing

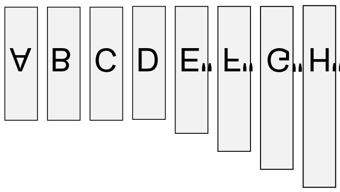

FIG. 8. Reflected but area still increasing

axis ($\frac{1}{x^2}$ in our example). During integration (mind you, we are only talking about regular Euclidean geometry here), there is no difference between the answer numerically given through standard Single Function techniques, however Area Calculus views the "area under the curve" of y_1 as instead the "area between the functions" of y_1 and y_2, providing a major proof difference during the process of anti-differentiation. From Figure 12 the x axis must be a separate function $y = 0$ which has an indefinite integral form of $\int 0dx = k$, where k is an arbitrary constant.

Thus $\int d(y_1 - y_2)dx = \int (d(y_1)) - 0)dx = (\int dy_1 dx) - (\int 0dx) = (y_1 + c) - (k)$. Noting that we can swap functions since $dy_1 = -dy_2$, this demonstrates those dual functions within Area Calculus that are viable solutions, including $((10 - \frac{1}{x}) - 8)$, $(8 - (6 + \frac{1}{x}))$ and $((0 - \frac{1}{x}) - 0)$.

IV. POISSON EQUATION

In Figure 13 we have the graphical basis of the Poisson equation where the first derivative of the field potential is force (per unit test mass) and the second derivative is the definition of energy density.

Attempts at incorporating the Cosmological Constant from the linearized field equation of $1 - 2\Phi$ into Newton's Law of Gravity using the Poisson equation leads to the equation ([2], Pg. 186) (using a positive Φ)

$$\vec{g} = -\nabla\Phi = -\frac{GM}{r^2}\hat{\vec{r}} + \frac{\Lambda c^2 r}{3}\hat{\vec{r}}.$$

There is no known way to reconcile a theoretical relationship between the observed magnitudes of M and Λ in this equation ("probably the worst theoretical prediction in the history of physics!"). However, comparing the Poisson potential plot against the requirements of Area Calculus in Figure 13 we see hidden assumptions that y_1 has been ignored, $y2 = 0 - \frac{\beta}{x}$ and that the area has no finite boundaries (not quantized) since $|(y_2)| \to 0$ as $x \to \infty$

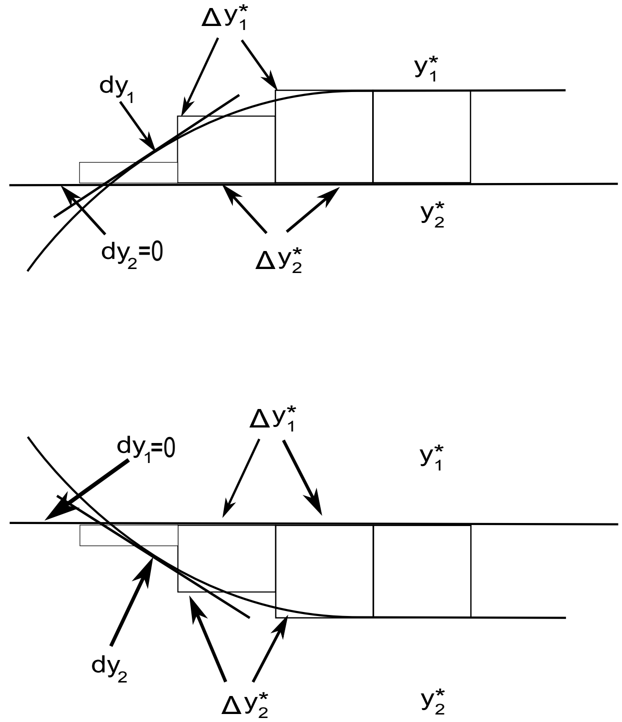

FIG. 9. Reflected line segments into continuous functions

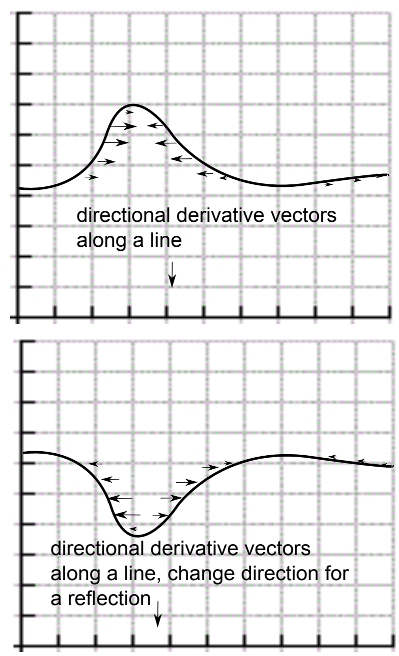

FIG. 10. Reflected line segments into continuous functions

and $|(y_2)| \to -\infty$ as $x \to 0$. We are not aware that either of these boundary conditions match any known empirical evidence, how quantized energy levels can effect out to spatial infinity nor what justification is used that all changes must happen from "zero".

Comparing the Area Calculus version of the Poisson

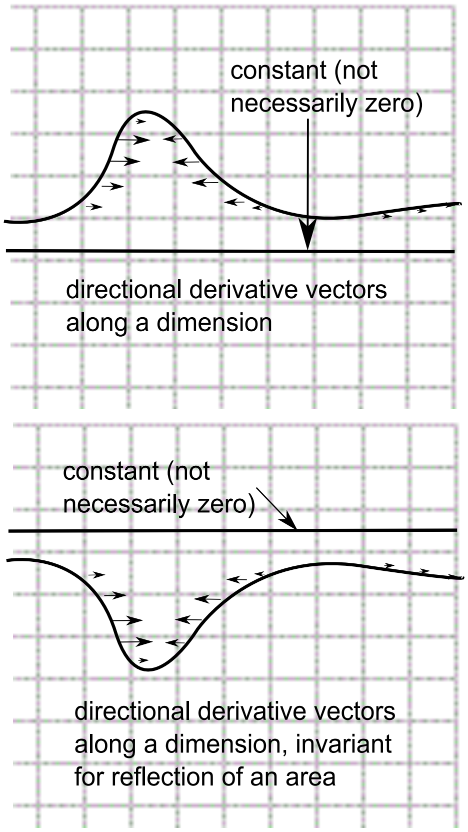

FIG. 11. Reflected line segments into continuous functions

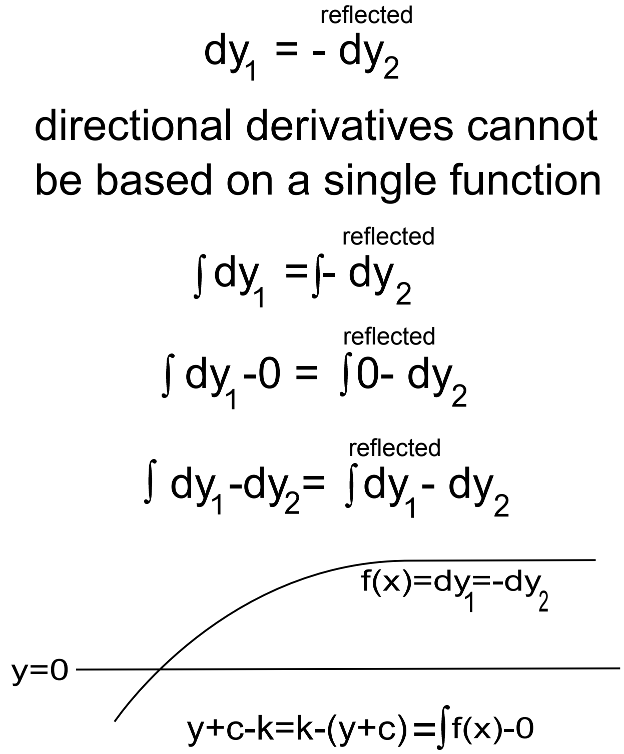

FIG. 12. Anti-differentiation of x axis

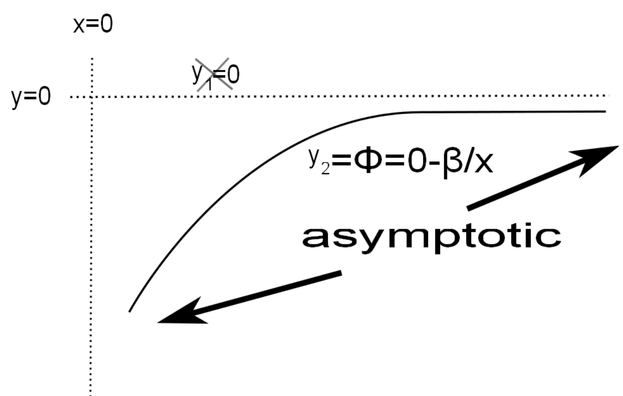

FIG. 13. Poisson field potential

equation against the linearized gravity portion demonstrates some short comings to incorporating the Cosmological Constant into GR (Fig. 14). From the most basic of linear solutions, it makes no sense where to place a multiple of the metric into a plot that already contains the metric and its perturbations. Where does Λg_{00} go in relation to the $g_{00} = 1$ line and how can it possibly be permissible for them both to occupy the same plot?

V. ACCELERATING EXPANSION

In Figures 15 and 16, we have a rough analogy of two regions when the majority of gravitational effects would appear attractive from y_2, and in Figure 17 when the majority of effects would appear repulsive from y_1. As a graphical analogy, we also offer Fig. 18. In Figure 19 we show how this corresponds to a NASA illustration of a late onset of accelerating expansion.

VI. ASSERTIONS

From the preceding sections we assert some observations for using calculus to model physical theory:

- All ranks of tensors require two functions to fully describe them, including scalars $(y_1 - y_2)$.
- Differential operators must act on two functions, not one.
- There are no single value scalar fields, only a field where one of the functions has been defined as $= 0$.
- Attraction or repulsion cannot be determined solely via the direction of a vector relative to its source.
- Single Function Calculus is a special class of calculus where one of the functions has been defined as having a magnitude of zero.

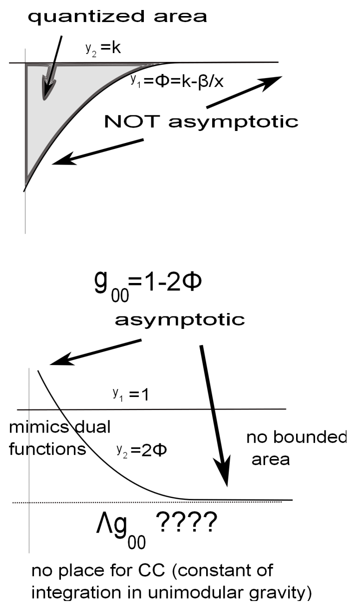

FIG. 14. Area Calculus version of Poisson field potential versus Linearized GR

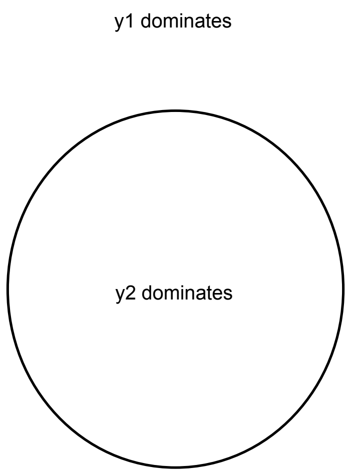

FIG. 15. Regions around matter and energy

- Differential topology, which considers only "curvature" of single lines, is fundamentally flawed as tangents are a direct consequence of a change in *area* requiring *two* lines (thus being incapable of describing any type of the energy density of a gravitational field).

- Linearized gravity utilizes two components ($1-2\Phi$, 1 and 2Φ), which appears to mimic the two functions of Area Calculus at small radii.

- Placing the Cosmological Constant into the symbolic form of Einstein's field equation is done only to conform with physical observations and is known to ruin the foundation of it being a "geometric" theory ([3] p. 410).

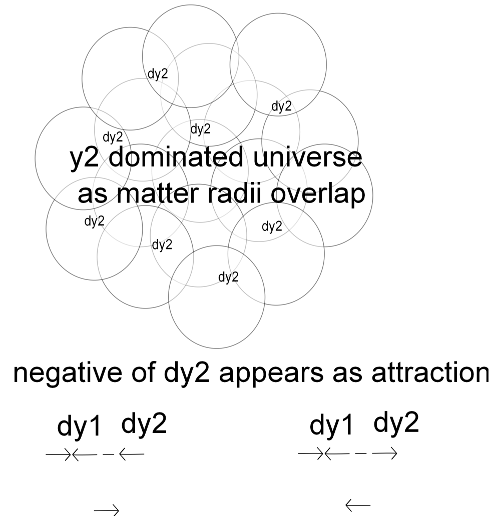

FIG. 16. Majority of gravitational effects appear attractive

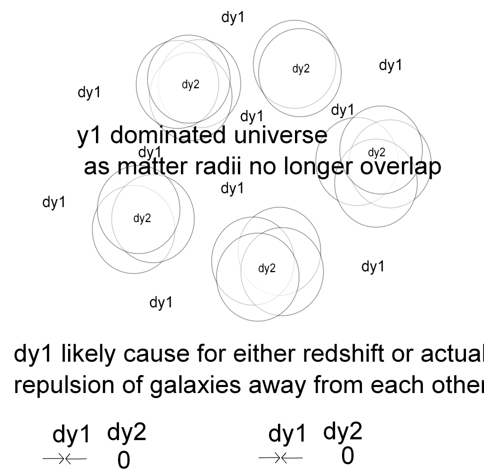

FIG. 17. Majority of gravitational effects appear repulsive

- The gravitational potential Φ within $g_{00} = 1 - 2\Phi$ and Λg_{00} must have the same units. The first derivative of Φ is the arbiter of force ($\nabla\Phi$) and the second of normal energy density ($\nabla^2\Phi$). If Λg_{00} is the energy density of the vacuum, it is reasonable to assume a model can be formed where a change in vacuum energy density is responsible for force ($\approx \nabla\Delta\Lambda$) and regular energy density ($\approx \nabla^2\Delta\Lambda$).

- The main motivation in the discarding of Gunnar Nordström's gravitational theory was that there did not seem to be a way for it to predict gravitational lensing ([1]), but a simple approximation shows how Area Calculus would predict at least gravitational redshift: $|2\Phi| \approx \frac{|k-y_2|}{k} = \Phi_{VED}$ then $1+z = \sqrt{\frac{1-2\Phi^{receiver}}{1-2\Phi^{source}}} \approx \sqrt{\frac{k-(k-y_2^{receiver})}{k-(k-y_2^{source})}} = \sqrt{\frac{1-\Phi_{VED}^{receiver}}{1-\Phi_{VED}^{source}}}$.

- We assume the area of an integral can be directly

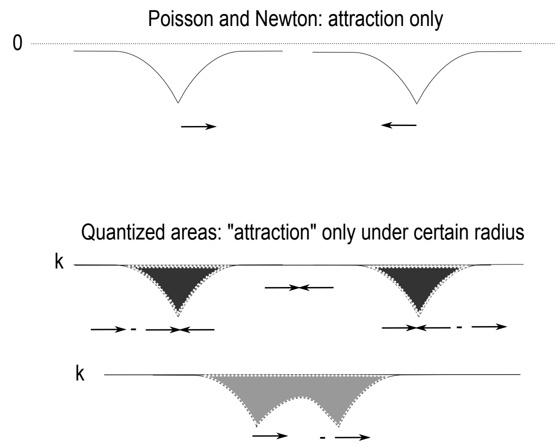

FIG. 18. Graphical analogy of difference between "attraction" and "reduced repulsion"

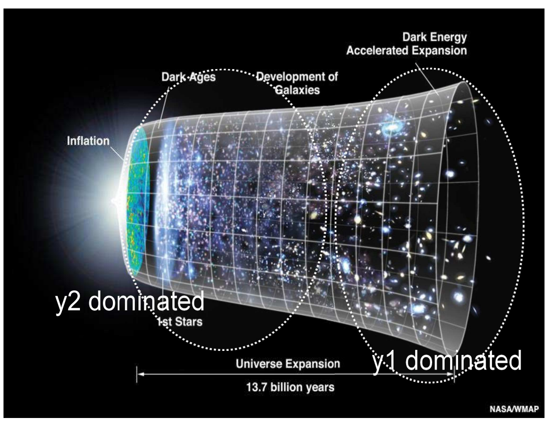

FIG. 19. Double functions corresponding to late onset of accelerating expansion, Base image: NASA / WMAP Science Team

correlated to the action of a system (Action Principle) as the foundation that calculus can fully describe physical theory, and that the universe stores information (matter and dimensions) as integrals (zero area integral=zero matter and dimensions), we experience the processing of this information as derivatives (forces, energy).

- We measure energy density from the perspective of that very same type of energy density. It would be logical that an area would have difficulty perceiving anything but another area (or four-volumes as we increase the dimensions).

Note that attractive fields in Area Calculus, from a vector viewpoint, are more accurately described as a reduced repulsion. In essence, the late appearance of an accelerating expansion would be due to the inherent quantization of energy levels, which are quantized reductions in vacuum energy density. Using GR symbology, the Poisson equation would be $\nabla^2(\Delta\Lambda_{VacuumEnergyDensity}) = \nabla^2((\Lambda-(\Lambda-\Phi_{VED})$. Within a certain radius, this would simplify to the normal Poisson equation $\nabla^2\Phi$, but outside this radius, especially if Λ is a function of the age of the Universe, then the large scale effects of the Vacuum Energy Density would dominate.

VII. CONCLUSION

So what kind of theory is this? It would appear to still be a metric field theory, although we have quite a ways to go before stating that definitively. Perhaps a more important first question to be asked is whether or not it is reasonable to re-examine how we mathematically model physical laws. If you haven't been following cosmology within the past fifteen years, then you might not be aware that our current models can only account for about 4% of our own kind of energy density in the universe ([4]) and we have no idea of what the other 96 % actually is. We conclude that *everything* is up for review including our most basic assumptions. Without a mathematical language that includes the information of area, we may not be able to describe our Universe.

[1] J. D. Norton, *Theories of Gravitation in the Twilight of Classical Physics. Part I.:EINSTEIN, NORDSTRÖM AND THE EARLY DEMISE OF SCALAR, LORENTZ COVARIANT THEORIES OF GRAVITATION*, Vol. 3 (Kluwer Academic Publishers, Netherlands, 2005) `http://www.pitt.edu/~jdnorton/papers/Nordstroem.pdf`.

[2] M. P. Hobson, G. Efstathiou, and A. N. Lasenby, *General relativity: an introduction for physicists* (Cambridge University Press, Cambridge, UK ; New York, 2006).

[3] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W.H. Freeman, San Francisco, 1973).

[4] NASA, "Wilkinson microwave anisotropy probe," (2011), accessed on 02/28/2012, last updated 11/21/2011 [online] `http://lambda.gsfc.nasa.gov/product/map/dr4/pub_papers/sevenyear/basic_results/wmap_7yr_basic_results_reprint.pdf`.