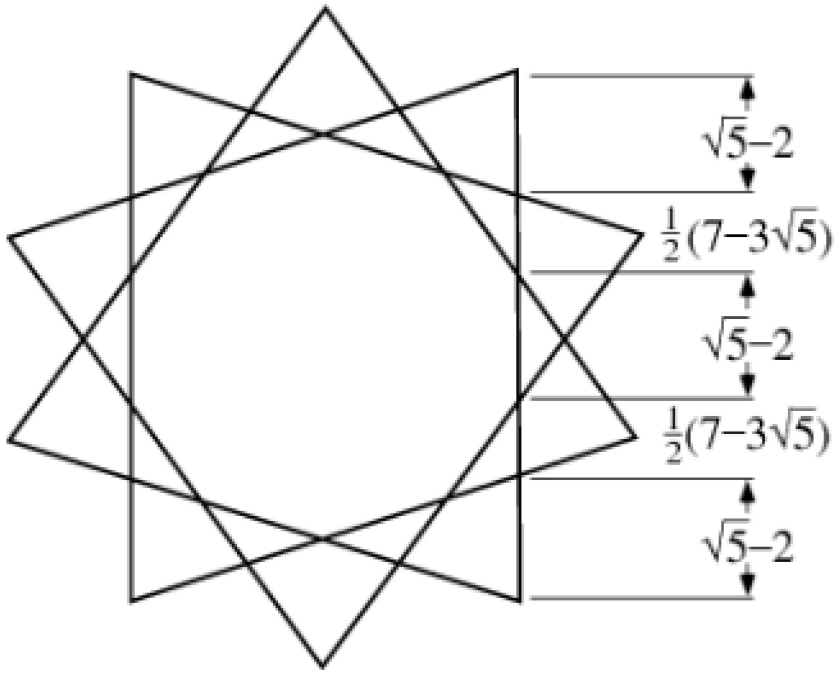
The Aikaantha State of "Black Hole" Matter and Base 60



By John Frederick Sweeney

Abstract

Space, the fundamental reality, can only exist in one state of Aikantha, the coherent state of three - dimensional, cubic resonances. The Universe remains in the Aikaantha state, fluctuating between the two values of the e logarithm and π /10. The latter value implies a circle with many trigonometric equations related to the Square Root of Five, or the Golden Section. In addition, this forms the basis for Base 60 Math in the Qi Men Dun Jia Model, and thus the icosahedron with its 60 stellated permutations. The value of π /10 thus limits the bounds of visible matter, and gives rise to Pisano Periodicity of the icosahedron. At the same time, the author dispels the current "comic book" notions of black holes: the Aikantha state is the state of black holes, yet misunderstood by modern science.

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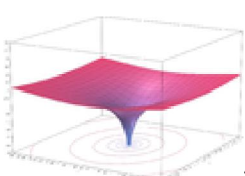
Introduction

We all have an image of what a black hole is, thanks to TV documentaries and the boyish, feverish imaginations of today's scientists. Much as the Europeans of 1400 imagined a flat earth, rimmed by dragons and sea monsters, today's dominant conception of an empty abyss that sucks in everything that gets too near indicates a complete failure to comprehend the true nature of "black holes."

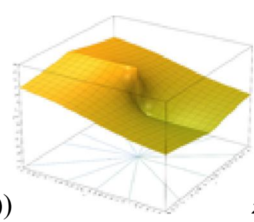
In Vedic Physics, the black hole state is referred to as the Aikaantha, or the substratum of space, which has specific qualities, including that of π /10. The Aikaantha state is maintained by two values: e and π /10 as a ratio.

Parts of this paper are edited and redacted elements from a book about Vedic Physics, in order to make the concepts more readable and easier to understand and comprehend.

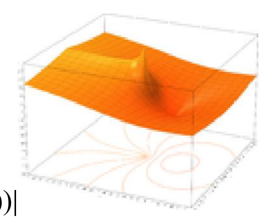
- Plots of the natural logarithm function on the complex plane (principal branch)



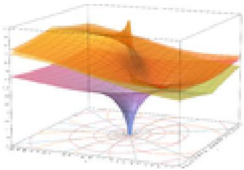
z = Re(ln(x+iy))



z = |Im(ln(x+iy))|



z = |ln(x+iy)|


Superposition of the previous 3 graphs

Three States of Matter

1	Thaama	Black Hole	compressive, dense and inelastic
2	Raja	8 x 8	resonant, shuttling and bonding and expansive
3	Sathwa	9 x 9	radiant and elastic, interactive states

According to a mathematical and scientific de-coding of Vedic literature, there exist three states of matter in the universe.

We shall discuss the Black Hole state of matter in the present paper.

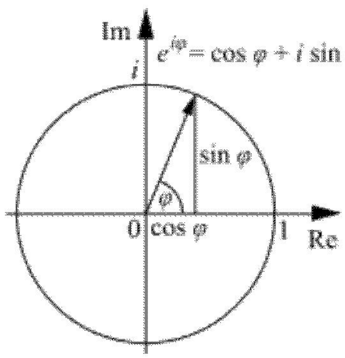
The Raja Guna equals the 8 x 8 state, which is resonant, shuttling and bonding and expansive; and is related to the I Ching, the 64 Yogini of Tantric fame, along with the Lie Algebra E8.

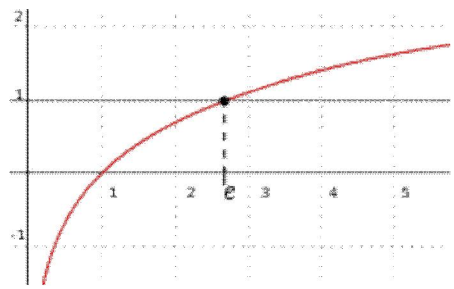
The Sathwa state corresponds to the 9 x 9 world of the Vedic Square and Vastu Shastra, Vedic Feng Shui, as well as the Tai Xuan Jing, the Dao De Jing and the 81 chapters of the Yellow Emperor's Internal Canon.

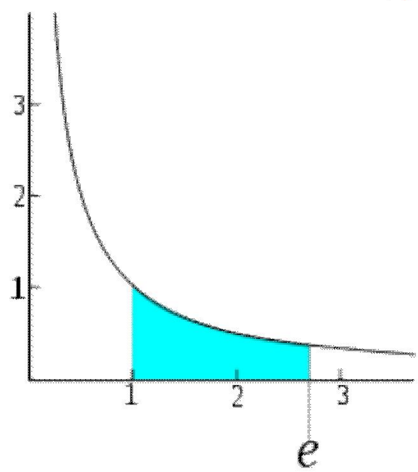
The three forms of interactive stress could only exist if the substratum has the following four qualities; namely synchronised , perpetual, dynamic in an unmanifest state of existence.

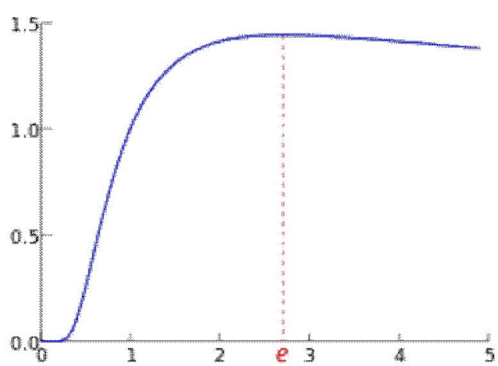
The definition for Aikaantha: is a synchronised, coherent, singular state where all components act simultaneously as a single entity. All interactions remain together in the same relative relationship of a frozen form, or move relative to each other in a cyclic period of movement, and yet remain a singular entity. Aikaantha has two mathematically limiting values, depending on its state of internal relative movement.

The two parameters e and π /10, which maintain the Aikaantha state, act with endless dynamism. The ultimate value of e is a constant, reachable only at an infinite rate of interaction. Similarly, π /10 is a transcendental number that tends to reach a constant limit at an infinite rate of interaction. Then, it ensures the ‘circularity’ of the cyclic time variation, thus ensuring a centering action. At its higher orders of interaction, π /10 remains centred and seems ‘static’.









The global maximum of $\sqrt[x]{x}$ occurs at $x = e$.

The area between the x-axis and the graph $y = 1/x$, between $x = 1$ and $x = e$ is 1.

The Natural Logarithm e

Assuming there are n components then the maximum number of possible interactive states must be N / 1. Comparing the relationship with smallest possible value of an isolated component of one unit, the number of possible interactive states become N-1 and the incremental ratio of a change, simultaneously or instantly, becomes Function 2 = (1+(1/(N-1)) N-1

(as simultaneous interactions are logarithmic): As N approaches Infinity, Function 2 equals the base of the natural logarithm e in modern terms. If all possible interactions are carried out n-1 times Simultaneously, then it will approach the value of e.Here the logarithmic sum of the incremental value and its ratio reach an asymptotic or limiting value of a transcendental number.

$$e=\left[\left(\frac{N}{N-1}\right)^{N-1}=\left(1+\frac{1}{N-1}\right)^{N-1}=2.7182818285\right]$$

the larger the number of interactive components or larger the relative volume acting as a single unit, it will always tend to equal 2.718 or e at the maximum rate of simultaneous or 'within a cycle' or instant period of interactive changes or counts. Any count of an interaction can be obtained only after the completion of the cycle and therefore the unit count per unit cycle is a relative instant.

Wikipedia entry on e

The number ***e*** is an important mathematical constant, approximately equal to 2.71828, that is the base of the natural logarithm.[1] It is the limit of $(1 + 1/n)^n$ as n approaches infinity, an expression that arises in the study of compound interest, and can also be calculated as the sum of the infinite series[2]

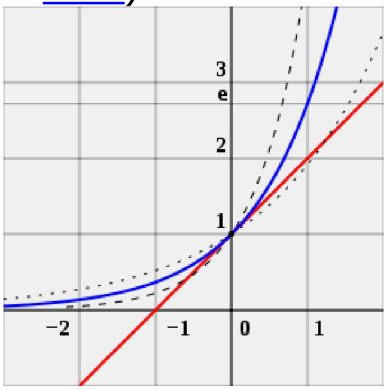
$$e = 1 + \frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \cdots$$

The constant can be defined in many ways; for example, *e* is the unique real number such that the value of the derivative (slope of the tangent line) of the function $f(x) = e^x$ at the point $x = 0$ is equal to 1.[3] The function e^x so defined is called the exponential function, and its inverse is the natural logarithm, or logarithm to base *e*. The natural logarithm of a positive number *k* can also be defined directly as the area under the curve $y = 1/x$ between $x = 1$ and $x = k$, in which case, *e* is the number whose natural logarithm is 1. There are also more alternative characterizations.

Sometimes called **Euler's number** after the Swiss mathematician Leonhard Euler, *e* is not to be confused with γ—the Euler–Mascheroni constant, sometimes called simply *Euler's constant*. The number *e* is also known as **Napier's constant**, but Euler's choice of this symbol is said to have been retained in his honor.[4] The number *e* is of eminent importance in mathematics,[5] alongside 0, 1, π and *i*. All five of these numbers play important and recurring roles across mathematics, and are the five constants appearing in one formulation of Euler's identity. Like the constant π, *e* is irrational: it is not a ratio of integers; and it is transcendental: it is not a root of *any* non-zero polynomial with rational coefficients. The numerical value of *e* truncated to 50 decimal places is

2.71828182845904523536028747135266249775724709369995...

(sequence A001113 in OEIS).



Pi Divided by Ten

The next possible variation is a relative cyclic-period difference between the interactive components which can result in a 'rotary' or cyclic periodic movement of the interactive stress forms or oscillatory waveforms. Interactive forms, despite relative movement, must remain in the same location to fulfill the condition of Aikaantha.

An interaction with a relative cyclic period difference remains in the same stationary Aikaantha state because the sum of all interactions within the cycle only add up to $\pi/10$, provided the initial displacement is x or x/2.

Even though any interaction is always directed in the line of action, the relative direction between two axes changes, with a variation in rate of interaction between the two axes.

Regardless of the rate of interactive counts per cycle, the interactive stress forms (waveforms) follow a circular path and complete the circular cycle only after 10 sequential interactions, yet remain in the Aikaantha state.

The reason is that in any confined vast field of matter, where there is no possibility of an external intervention and far from the boundary of influence, interactive exchanges can take place only by a process of simultaneous exchange of required parameters from within.

This law of Swabhava or self-similarity leads to a unique ratio that defines the value of a cycle as 10, if the Aikaantha state is to be ensured. Axiomatically the rate of variation in cyclic time between any two identical interacting components has to be 1 to 2 (See Matter Counts on Vixra by this author for more details).

Each axis has two oscillatory states, with a periodic relationship of 1 and 2 or cyclic relationship of one and half, that would allow the nodes or turning points to maintain the same locations at the same cyclic rate of two.

Instant Karma

The Aikaantha state provides the moral aspect of the universe in the sense that all actions are recorded there, including human actions. When someone does something wrong, commits a sin or transgression, then this is recorded, to be played back at another time, or even during another lifetime. This is the origin of the doctrines of karma and reincarnation in the Hindu belief system.

In other words, there is no escape in this universe from the consequences of our actions, and we get repaid in future based upon our past actions. At the same time, our actions may ameliorate past actions, by giving to the poor, repeating mantra, performing puja or homam, or acts of sadhana. In this sense the universe is moralistic, as well as holographic and combinatorial, other qualities to be discussed in future papers.

Appendices

The appendices to this paper include references from Wikipedia and Wolfram which illustrate the implications of the natural logarithm e and Pi / 10. Trigonometry Angles show the close relationship that the square root of five has to Pi /10 as well as to the Golden Ratio, and this was discussed in a previous paper by this author on Vixra. Discussion of trigonometry underscores the point made in a previous paper that none of this ancient science lay beyond the ken of ancient and medieval Indian mathematicians.

The construction of the decagon and decagonal numbers show similar relationships, as does the article on the Decagram. Finally, the section on Petrie polygons illustrates the relationship between Pi / 10 and the dodacahedron and the icosahedron, which are two key geometric figures in the Qi Men Dun Jia Model, and which are intimately related to the Golden Ration and Plato's Solids.

Conclusion

The author suggests that matter rises along the spectrum between the natural e logarithm and Pi /10 in the substratum, or black hole aspect of the universe, which remains unseen.

When matter reaches the phase of manifestation, it does so at the Pi / 10 end of the spectrum, and thus takes on characteristics of Base 60 math, including Time, as well as the Five Elements, Yin and Yang or binary qualities, in such a way that when matter finally manifests, it does so in the form of the icosahedron, or possibly in the form of the double icosahedron.

This is so since Chinese metaphysics contains the doctrines of the 60 Jia Zi, the temporal element, and the 60 Na Yin, which adds the frequency or material element, along with Five Elements and Yin and Yang qualities, when connected to the icosahedron and the 60 stellated permutations of the same. These qualities are of extreme importance to the Qi Men Dun Jia Model, which the author has been describing in this series of papers on Vixra.

The present paper demonstrates that the formation of visible matter begins in the substratum, where it takes on circular characteristics which provide a perfect fit to Base 60 math and the related concepts of Chinese metaphysics.

Appendix I

Decagonal Number

Wikipedia describes the Decagon in this way:

A **decagonal number** is a figurate number that represents a decagon. The n-th decagonal number is given by the formula

$$D_n = 4n^2 - 3n.$$

The first few decagonal numbers are:

0, 1, 10, 27, 52, 85, 126, 175, 232, 297, 370, 451, 540, 637, 742, 855, 976, 1105, 1242, 1387, 1540, 1701, 1870, 2047, 2232, 2425, 2626, 2835, 3052, 3277, 3510, 3751, 4000, 4257, 4522, 4795, 5076, 5365, 5662, 5967, 6280, 6601, 6930, 7267, 7612, 7965, 8326 (sequence A001107 in OEIS)

The n-th decagonal number can also be calculated by adding the square of n to thrice the (n—1)-th pronic number or, to put it algebraically, as

$$D_n = n^2 + 3(n^2 - n).$$

Properties

- Decagonal numbers consistently alternate parity.

This concept begins with integers. An **even number** is an integer that is "evenly divisible" by 2, i.e., divisible by 2 without remainder; an **odd number** is an integer that is not evenly divisible by 2. (The old-fashioned term "evenly divisible" is now almost always shortened to "divisible".) A formal definition of an even number is that it is an integer of the form $n = 2k$, where k is an integer; it can then be shown that an odd number is an integer of the form $n = 2k + 1$.

Examples of even numbers are −4, 0, 8, and 1734. Examples of odd numbers are −5, 3, 9, and 73. This classification applies only to integers, i.e., non-integers like 1/2 or 4.201 are neither even nor odd.

The sets of even and odd numbers can be defined as following:

- **Even** = $\{2k; \forall k \in \mathbb{Z}\}$
- **Odd** = $\{2k + 1; \forall k \in \mathbb{Z}\}$

A number (i.e., integer) expressed in the decimal numeral system is even or odd according to whether its last digit is even or odd. That is, if the last digit is 1, 3, 5, 7, or 9, then it is odd; otherwise it is even. The same idea will work using any even base. In particular, a number expressed in the binary numeral system is odd if its last digit is 1 and even if its last digit is 0. In an odd base, the number is even according to the sum of its digits – it is even if and only if the sum of its digits is even.

Appendix II

Trigonometry Angles--Pi/10

From Wolfram

$$\cos\left(\frac{\pi}{10}\right) = \frac{1}{4}\sqrt{10+2\sqrt{5}}$$

$$\cos\left(\frac{3\pi}{10}\right) = \frac{1}{4}\sqrt{10-2\sqrt{5}}$$

$$\cot\left(\frac{\pi}{10}\right) = \sqrt{5+2\sqrt{5}}$$

$$\cot\left(\frac{3\pi}{10}\right) = \sqrt{5-2\sqrt{5}}$$

$$\csc\left(\frac{\pi}{10}\right) = 1+\sqrt{5}$$

$$\csc\left(\frac{3\pi}{10}\right) = \sqrt{5}-1$$

$$\sec\left(\frac{\pi}{10}\right) = \frac{1}{5}\sqrt{50-10\sqrt{5}}$$

$$\sec\left(\frac{3\pi}{10}\right) = \frac{1}{5}\sqrt{50+10\sqrt{5}}$$

$$\sin\left(\frac{\pi}{10}\right) = \frac{1}{4}(\sqrt{5}-1)$$

$$\sin\left(\frac{3\pi}{10}\right) = \frac{1}{4}(1+\sqrt{5})$$

$$\tan\left(\frac{\pi}{10}\right) = \frac{1}{5}\sqrt{25-10\sqrt{5}}$$

$$\tan\left(\frac{3\pi}{10}\right) = \frac{1}{5}\sqrt{25+10\sqrt{5}}.$$

To derive these formulas, use the half-angle formula

$$\sin\left(\frac{\pi}{10}\right) = \sin\left(\frac{1}{2}\cdot\frac{\pi}{5}\right)$$
$$= \sqrt{\tfrac{1}{2}\left[1-\cos\left(\tfrac{\pi}{5}\right)\right]}$$
$$= \sqrt{\tfrac{1}{2}\left[1-\tfrac{1}{4}(1+\sqrt{5})\right]}$$
$$= \tfrac{1}{4}(\sqrt{5}-1)$$

$$\cos\left(\frac{\pi}{10}\right) = \cos\left(\frac{1}{2}\cdot\frac{\pi}{5}\right)$$
$$= \sqrt{\frac{1}{2}\left[1+\cos\left(\tfrac{\pi}{5}\right)\right]}$$
$$= \sqrt{\tfrac{1}{2}\left[1+\tfrac{1}{4}(1+\sqrt{5})\right]}$$
$$= \tfrac{1}{4}\sqrt{10+2\sqrt{5}}$$

$$\tan\left(\frac{\pi}{10}\right) = \sqrt{\frac{3-\sqrt{5}}{5+\sqrt{5}}}$$
$$= \tfrac{1}{5}\sqrt{25-10\sqrt{5}}\,.$$

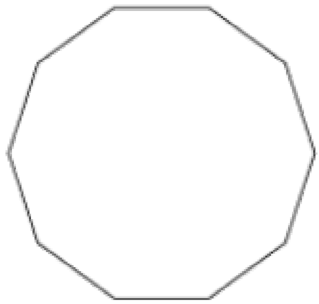
SEE ALSO:Decagon, Decagram, Trigonometry Angles, Trigonometry, Trigonometry Angles--Pi/5
CITE THIS AS:Weisstein, Eric W. "Trigonometry Angles--Pi/10." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/TrigonometryAnglesPi10.html

The side of a regular decagon inscribed in a unit circle is $\frac{-1+\sqrt{5}}{2} = \frac{1}{\phi}$, where □ is the golden ratio, $\frac{1+\sqrt{5}}{2}$.

$5 + 2\sqrt{5}$ $\tan^2(72°) = [9; [2, 8]]$	18 degrees	$\frac{\pi}{10}$

The values in the table are those angles of the form n° or aπ/b for whole numbers a n, a and b, between 0 and 90° whose sin or cosine is rational, or whose continued fraction is periodic or the square of the trig value has a periodic continued fraction.

Decagon



The constructible regular 10-sided polygon with Schläfli symbol $\{10\}$. The inradius r, circumradius R, and area can be computed directly from the formulas for a general regular polygon with side length s and $n = 10$ sides,

$$r = \tfrac{1}{2} s \cot\left(\frac{\pi}{10}\right)$$

$$= \tfrac{1}{2}\sqrt{5 + 2\sqrt{5}}\; s$$

$$R = \tfrac{1}{2} s \csc\left(\frac{\pi}{10}\right) = \tfrac{1}{2}\left(1 + \sqrt{5}\right) s$$

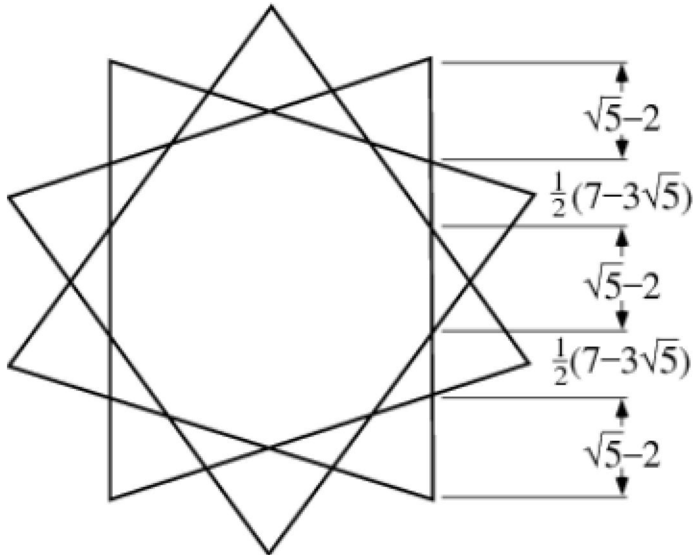
$$= \phi\, s$$

$$A = \tfrac{1}{4} n s^2 \cot\left(\frac{\pi}{10}\right)$$

$$= \tfrac{5}{2}\sqrt{5 + 2\sqrt{5}}\; s^2.$$

Here, ϕ is the golden ratio.

Appendix III Decagram

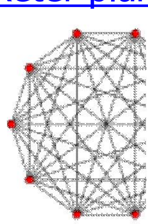
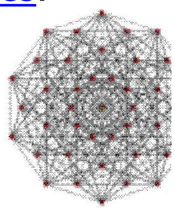
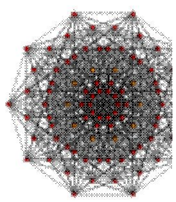
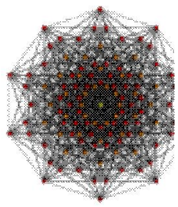
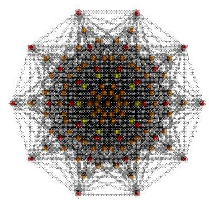


The star polygon $\{10/3\}$.
SEE ALSO:

Appendix V Petrie Polygons

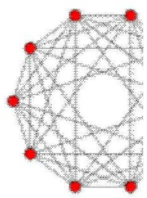
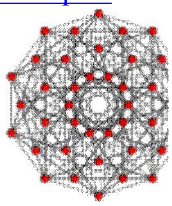
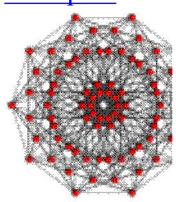
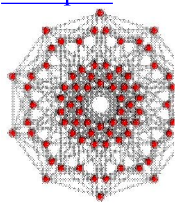
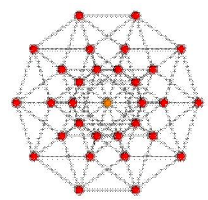
The regular decagon is the Petrie polygon for many higher dimensional polytopes, shown in these skew orthogonal projections in various Coxeter planes:

A_9

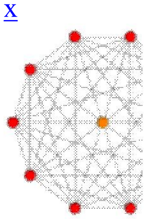
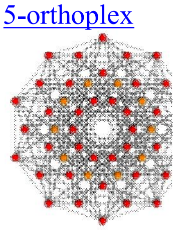
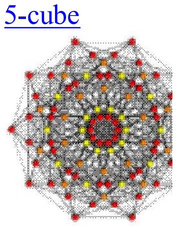
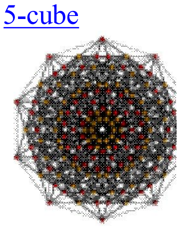
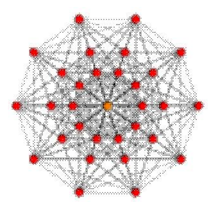
9-simplex | Rectified 9-simplex | Birectified 9-simplex | Trirectified 9-simplex | Quadrirectified 9-simplex

BC_5

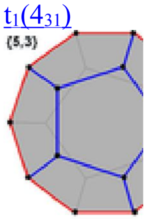
5-orthoplex | Rectified 5-orthoplex | Birectified 5-cube | Rectified 5-cube | 5-cube

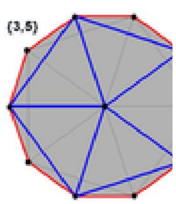
D_6

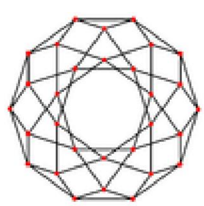






$t_1(4_{31})$ | $t_3(1_{31})$ | $t_2(1_{31})$ | $t_1(1_{31})$ | 6-demicube (1_{31})

H_3







Dodecahedron | Icosahedron

Appendix IV Golden Ratio and Pi

Phi and phi and the golden ratio values:

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Phi = Φ = 1.6180339... = (√5 + 1)/2 and phi = φ = 0.6180339... = Φ - 1 = (√5 - 1)/2cos(9°)	=	1 . 2	√	2 +	=	sin(81°)
cos(18°)	=	] . 2 =	1 . 2	√ . 2 - Φ	=	sin(72°)
cos(27°)	=	1 . 2	√	2 +	=	sin(63°)
cos(36°)	=	=	=	Φ . 2	=	sin(54°)
cos(54°)	=	] . 2 =	1 . 2	√ . 2 - φ	=	sin(36°)

cos(63°) = 1/2 √ 2 = sin(27°)

cos(72°) = = = φ/2 = sin(18°)

cos(81°) = 1/2 √ 2 = sin(9°)

This pattern uses the identities

phi = φ = √2 – Φ and Phi = Φ = √2 + φ

together with the half-angle formula for cos(A/2) (see below) starting from cos(36)=Phi/2 and cos(72)=phi/2. The pattern continues with the cosines of 4.5°, 13.5°, etc.

Wikipedia entry on natural logarithm e

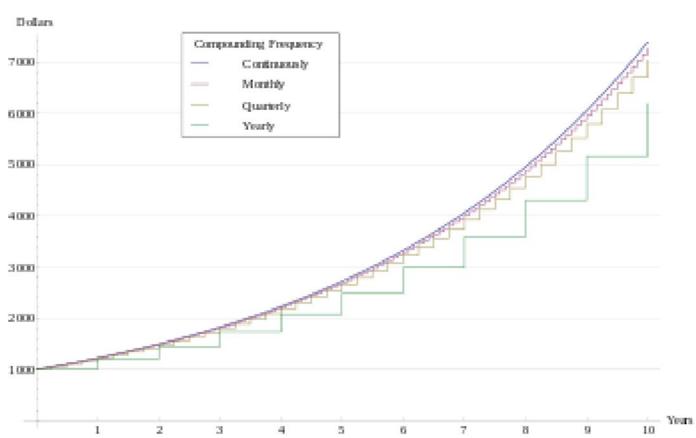
The first references to the constant were published in 1618 in the table of an appendix of a work on logarithms by John Napier.[6] However, this did not contain the constant itself, but simply a list of logarithms calculated from the constant. It is assumed that the table was written by William Oughtred. The discovery of the constant itself is credited to Jacob Bernoulli, who attempted to find the value of the following expression (which is in fact *e*):

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

The first known use of the constant, represented by the letter *b*, was in correspondence from Gottfried Leibniz to Christiaan Huygens in 1690 and 1691. Leonhard Euler introduced the letter *e* as the base for natural logarithms, writing in a letter to Christian Goldbach of 25 November 1731.[7] Euler started to use the letter *e* for the constant in 1727 or 1728, in an unpublished paper on explosive forces in cannons,[8] and the first appearance of *e* in a publication was Euler's *Mechanica* (1736). While in the subsequent years some researchers used the letter *c*, *e* was more common and eventually became the standard.

Applications

Compound interest



The effect of earning 20% annual interest on an initial $1,000 investment at various compounding frequencies

Jacob Bernoulli discovered this constant by studying a question about compound interest:[6]

> An account starts with $1.00 and pays 100 percent interest per year. If the interest is credited once, at the end of the year, the value of the account at year-end will be $2.00. What happens if the interest is computed and credited more frequently during the year?

If the interest is credited twice in the year, the interest rate for each 6 months will be 50%, so the initial \$1 is multiplied by 1.5 twice, yielding $\$1.00\times1.5^2 = \$2.25$ at the end of the year. Compounding quarterly yields $\$1.00\times1.25^4 = \$2.4414...$, and compounding monthly yields $\$1.00\times(1+1/12)^{12} = \$2.613035...$ If there are n compounding intervals, the interest for each interval will be 100%/n and the value at the end of the year will be $\$1.00\times(1 + 1/n)^n$.

Bernoulli noticed that this sequence approaches a limit (the force of interest) with larger n and, thus, smaller compounding intervals. Compounding weekly ($n = 52$) yields \$2.692597..., while compounding daily ($n = 365$) yields \$2.714567..., just two cents more. The limit as n grows large is the number that came to be known as e; with *continuous* compounding, the account value will reach \$2.7182818.... More generally, an account that starts at \$1 and offers an annual interest rate of R will, after t years, yield e^{Rt} dollars with continuous compounding. (Here R is a fraction, so for 5% interest, $R = 5/100 = 0.05$)

Bernoulli trials

The number e itself also has applications to probability theory, where it arises in a way not obviously related to exponential growth. Suppose that a gambler plays a slot machine that pays out with a probability of one in n and plays it n times. Then, for large n (such as a million) the probability that the gambler will lose every bet is (approximately) $1/e$. For $n = 20$ it is already 1/2.72.

This is an example of a Bernoulli trials process. Each time the gambler plays the slots, there is a one in one million chance of winning. Playing one million times is modelled by the binomial distribution, which is closely related to the binomial theorem. The probability of winning k times out of a million trials is;

$$\binom{10^6}{k}\left(10^{-6}\right)^k\left(1-10^{-6}\right)^{10^6-k}.$$

In particular, the probability of winning zero times ($k = 0$) is

$$\left(1-\frac{1}{10^6}\right)^{10^6}.$$

This is very close to the following limit for $1/e$:

$$\frac{1}{e}=\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n.$$

Derangements

Another application of e, also discovered in part by Jacob Bernoulli along with Pierre Raymond de Montmort is in the problem of derangements, also known as the *hat check problem*:[9] n guests are invited to a party, and at the door each guest checks his hat with the butler who then places them into n boxes, each labelled with the name of one guest. But the butler does not know the identities of the guests, and so he puts the hats into boxes selected at random. The problem of de Montmort is to find the probability that *none* of the hats gets put into the right box. The answer is:

$$p_n = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + \frac{(-1)^n}{n!} = \sum_{k=0}^{n} \frac{(-1)^k}{k!}.$$

As the number n of guests tends to infinity, p_n approaches $1/e$. Furthermore, the number of ways the hats can be placed into the boxes so that none of the hats is in the right box is $n!/e$ rounded to the nearest integer, for every positive n.[10]

Asymptotics

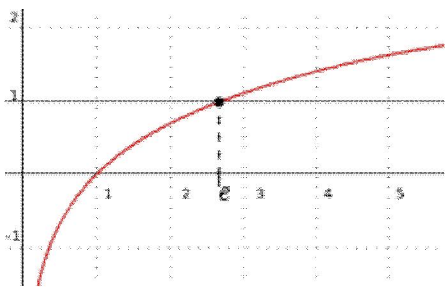
The number e occurs naturally in connection with many problems involving asymptotics. A prominent example is Stirling's formula for the asymptotics of the factorial function, in which both the numbers e and π enter:

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n.$$

A particular consequence of this is

$$e = \lim_{n\to\infty} \frac{n}{\sqrt[n]{n!}}.$$

e in calculus



The natural log at (x-axis) e, ln(e), is equal to 1

The principal motivation for introducing the number e, particularly in calculus, is to perform differential and integral calculus with exponential functions and logarithms.[11] A general exponential function $y = a^x$ has derivative given as the limit:

$$\frac{d}{dx}a^x = \lim_{h\to 0}\frac{a^{x+h}-a^x}{h} = \lim_{h\to 0}\frac{a^x a^h - a^x}{h} = a^x\left(\lim_{h\to 0}\frac{a^h-1}{h}\right).$$

The limit on the right-hand side is independent of the variable x: it depends only on the base a. When the base is e, this limit is equal to one, and so e is symbolically defined by the equation:

$$\frac{d}{dx}e^x = e^x.$$

Consequently, the exponential function with base e is particularly suited to doing calculus. Choosing e, as opposed to some other number, as the base of the exponential function makes calculations involving the derivative much simpler.

Another motivation comes from considering the base-a logarithm.[12] Considering the definition of the derivative of $\log_a x$ as the limit:

$$\frac{d}{dx}\log_a x = \lim_{h\to 0}\frac{\log_a(x+h)-\log_a(x)}{h} = \frac{1}{x}\left(\lim_{u\to 0}\frac{1}{u}\log_a(1+u)\right),$$

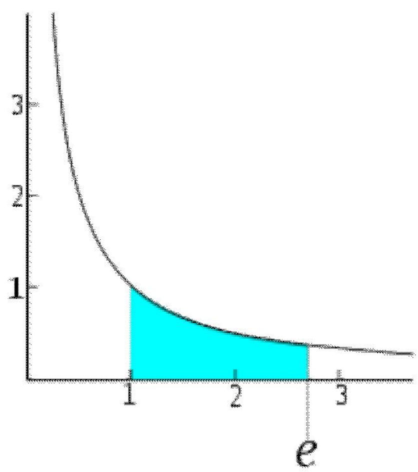
where the substitution $u = h/x$ was made in the last step. The last limit appearing in this calculation is again an undetermined limit that depends only on the base a, and if that base is e, the limit is one. So symbolically,

$$\frac{d}{dx}\log_e x = \frac{1}{x}.$$

The logarithm in this special base is called the natural logarithm and is represented as ln; it behaves well under differentiation since there is no undetermined limit to carry through the calculations.

There are thus two ways in which to select a special number $a = e$. One way is to set the derivative of the exponential function a^x to a^x, and solve for a. The other way is to set the derivative of the base a logarithm to $1/x$ and solve for a. In each case, one arrives at a convenient choice of base for doing calculus. In fact, these two solutions for a are actually *the same*, the number e.

Alternative characterizations



The area between the x-axis and the graph $y = 1/x$, between $x = 1$ and $x = e$ is 1.

See also: Representations of e

Other characterizations of *e* are also possible: one is as the limit of a sequence, another is as the sum of an infinite series, and still others rely on integral calculus. So far, the following two (equivalent) properties have been introduced:

1. The number *e* is the unique positive real number such that

$$\frac{d}{dt}e^t = e^t.$$

2. The number *e* is the unique positive real number such that

$$\frac{d}{dt}\log_e t = \frac{1}{t}.$$

The following three characterizations can be proven equivalent:

3. The number *e* is the limit

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

Similarly:

$$e = \lim_{x\to 0}(1+x)^{1/x}$$

4. The number *e* is the sum of the infinite series

$$e = \sum_{n=0}^{\infty}\frac{1}{n!} = \frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\cdots$$

where $n!$ is the factorial of n.

5. The number *e* is the unique positive real number such that

$$\int_1^e \frac{1}{t}\,dt = 1.$$

Properties

Calculus

As in the motivation, the exponential function e^x is important in part because it is the unique nontrivial function (up to multiplication by a constant) which is its own derivative

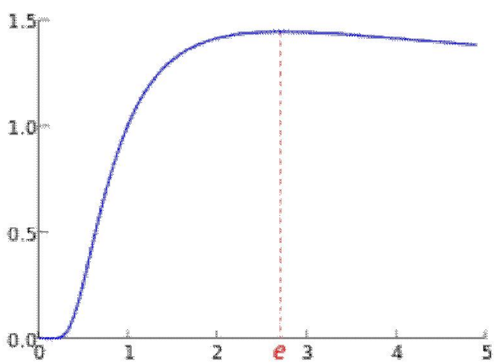
$$\frac{d}{dx}e^x = e^x$$

and therefore its own antiderivative as well:

$$e^x = \int_{-\infty}^{x} e^t\,dt$$
$$= \int_{-\infty}^{0} e^t\,dt + \int_{0}^{x} e^t\,dt$$
$$= 1 + \int_{0}^{x} e^t\,dt.$$

Exponential-like functions

See also: Steiner's problem



The global maximum of $\sqrt[x]{x}$ occurs at $x = e$.

The global maximum for the function

$$f(x) = \sqrt[x]{x}$$

occurs at $x = e$. Similarly, $x = 1/e$ is where the global minimum occurs for the function

$$f(x) = x^x$$

defined for positive x. More generally, $x = e^{-1/n}$ is where the global minimum occurs for the function

$$f(x) = x^{x^n}$$

for any n > 0. The infinite tetration

$$x^{x^{x^{\cdot^{\cdot^{\cdot}}}}} \text{ or } {}^{\infty}x$$

converges if and only if $e^{-e} \leq x \leq e^{1/e}$ (or approximately between 0.0660 and 1.4447), due to a theorem of Leonhard Euler.

Number theory

The real number *e* is irrational. Euler proved this by showing that its simple continued fraction expansion is infinite.[13] (See also Fourier's proof that *e* is irrational.)

Furthermore, by the Lindemann–Weierstrass theorem, *e* is transcendental, meaning that it is not a solution of any non-constant polynomial equation with rational coefficients. It was the first number to be proved transcendental without having been specifically constructed for this purpose (compare with Liouville number); the proof was given by Charles Hermite in 1873.

It is conjectured that *e* is normal, meaning that when *e* is expressed in any base the possible digits in that base are uniformly distributed (occur with equal probability in any sequence of given length).

Complex numbers

The exponential function e^x may be written as a Taylor series

$$e^{x} = 1 + {x \over 1!} + {x^{2} \over 2!} + {x^{3} \over 3!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Because this series keeps many important properties for e^x even when x is complex, it is commonly used to extend the definition of e^x to the complex numbers. This, with the Taylor series for sin and cos x, allows one to derive Euler's formula:

$$e^{ix} = \cos x + i\sin x,$$

which holds for all x. The special case with $x = \pi$ is Euler's identity:

$$e^{i\pi} = -1$$

from which it follows that, in the principal branch of the logarithm,

$$\log_e(-1) = i\pi.$$

Furthermore, using the laws for exponentiation,

$$(\cos x + i\sin x)^n = \left(e^{ix}\right)^n = e^{inx} = \cos (nx) + i \sin (nx),$$

which is de Moivre's formula.

The expression

$$\cos x + i \sin x$$

is sometimes referred to as cis(x).

Differential equations

The general function

$$y(x) = Ce^x$$

is the solution to the differential equation:

$$y' = y.$$

Representations

Main article: List of representations of e

The number e can be represented as a real number in a variety of ways: as an infinite series, an infinite product, a continued fraction, or a limit of a sequence. The chief among these representations, particularly in introductory calculus courses is the limit

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n,$$

given above, as well as the series

$$e = \sum_{n=0}^{\infty} \frac{1}{n!}$$

given by evaluating the above power series for e^x at $x = 1$.

Less common is the continued fraction (sequence A003417 in OEIS).

$$e = [2; 1, \mathbf{2}, 1, 1, \mathbf{4}, 1, 1, \mathbf{6}, 1, 1, ..., \mathbf{2n}, 1, 1, ...] = [1; \mathbf{0}, 1, 1, \mathbf{2}, 1, 1, \mathbf{4}, 1, 1, ..., \mathbf{2n}, 1, 1$$

[14]

which written out looks like

$$e = 2+\cfrac{1}{1+\cfrac{1}{\mathbf{2}+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\mathbf{4}+\cfrac{1}{1+\cfrac{1}{1+\ddots}}}}}}} = 1+\cfrac{1}{\mathbf{0}+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\mathbf{2}+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\mathbf{4}+\cfrac{1}{1+\cfrac{1}{1+\ddots}}}}}}}}}.$$

This continued fraction for *e* converges three times as quickly:

$$e = [1; 0.5, 12, 5, 28, 9, 44, 13, \ldots, 4(4n-1), (4n+1), \ldots],$$

which written out looks like

$$e = 1 + \cfrac{2}{1+\cfrac{1}{6+\cfrac{1}{10+\cfrac{1}{14+\cfrac{1}{18+\cfrac{1}{22+\cfrac{1}{26+\ddots}}}}}}}.$$

Many other series, sequence, continued fraction, and infinite product representations of *e* have been developed.

Stochastic representations

In addition to exact analytical expressions for representation of *e*, there are stochastic techniques for estimating *e*. One such approach begins with an infinite sequence of independent random variables X_1, X_2..., drawn from the uniform distribution on [0, 1]. Let V be the least number n such that the sum of the first n samples exceeds 1:

$$V = \min\{n \mid X_1 + X_2 + \cdots + X_n > 1\}.$$

Then the expected value of V is *e*: $\mathrm{E}(V) = e$.[15][16]

Known digits

The number of known digits of *e* has increased dramatically during the last decades. This is due both to the increased performance of computers and to algorithmic improvements.[17][18]

Number of known decimal digits of *e*Date	Decimal digits	Computation performed by
1748	23	Leonhard Euler[19]
1853	137	William Shanks
1871	205	William Shanks
1884	346	J. Marcus Boorman
1949	2,010	John von Neumann(on the ENIAC)
1961	100,265	Daniel Shanksand John Wrench[20]
1978	116,000	Stephen Gary Wozniak(on the Apple II[21])
1994 April 1	1,000,000	Robert Nemiroff & Jerry Bonnell [22]
1999 November 21	1,250,000,000	Xavier Gourdon [23]
2000 July 16	3,221,225,472	Colin Martin & Xavier Gourdon [24]
2003 September 18	50,100,000,000	Shigeru Kondo & Xavier Gourdon [25]
2007 April 27	100,000,000,000	Shigeru Kondo & Steve Pagliarulo [26]
2009 May 6	200,000,000,000	Rajesh Bohara & Steve Pagliarulo [26]
2010 July 5	1,000,000,000,000	Shigeru Kondo & Alexander J. Yee [27]

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'Other people, he said, see things and say why? But I dream things that never were and I say, why not?"

Robert F. Kennedy, after George Bernard Shaw