Fitting the Neutron and Proton Masses within Experimental Errors Exclusively to the Up and Down Quark Masses and Charges, the Fermi vev, and the CKM Quark Mixing Matrix

Jay R. Yablon
jyablon@nycap.rr.com
July 8, 2013

We report a method for expressing the neutron and proton masses within experimental errors, exclusively as a function of the up and down current quark masses and charges, the Fermi vev, and the CKM quark mixing matrix. In the process, we develop a mass and mixing matrix which may possibly be helpful for characterizing other baryon masses and better pinpointing higher-generational quark masses.

PACS: 14.20.Dh; 14.65.-q; 12.15.Ff; 14.20.-c

1. Introduction

It has been known for decades that the proton and neutron masses, $M_P = 938.272046$ MeV and $M_N = 939.565379$ MeV respectively, exceed the electron mass $m_e = 0.510998928$ MeV by a factor of just under 1840 to 1 (mass data is from [1]). Yet to date, there is no good explanation for this ratio.

Following the author's deduction in [2] of an expression for the neutron minus proton mass difference which was reported in [3] strictly as a data-fitting relationship without theoretical assertions, the possibility of explaining the neutron and proton masses themselves appeared to be a realistic possibility. Specifically, given the postulated-exact relationship

$$M_N - M_P = 0.001388449188 \text{ u} \equiv m_u - \left(3m_d + 2\sqrt{m_\mu m_d} - 3m_u\right)/\left(2\pi\right)^{\frac{3}{2}} \tag{1.1}$$

reported in (22) of [3] for a *difference* between these two masses, one needs "only" find the *sum* $M_N + M_P$ of these masses in order to then be able to deduce each of M_N and M_P separately, via a simple algebraic solution of two independent simultaneous equations for two unknowns.

This problem of finding M_N and M_P in this manner was solved by the author in [4], but the solution was based on extensive theoretical development in [5] followed by [4]. In the spirit of only reporting objective numeric relationships among phenomenological masses and energies while foregoing any theoretical assertions, the author in this letter reports his findings for the neutron and proton masses themselves as simply, directly and cleanly as possible, independently from the author's own underlying theory. As in [3], this is intended to leave latitude for others to independently form modified or alternate conceptions of the underlying physics. This letter goes beyond simply expanding with other examples upon the ideas reported in [3]. Rather, in relation to [3], this letter reports *qualitatively new results and relationships* involving empirical mass and energy and quark mixing data long-known but never before interconnected.

2. The Clue

The author concludes in [3] that Koide [6], [7] matrices of the form

$$K_{AB} \equiv \begin{pmatrix} \sqrt{m_1} & 0 & 0 \\ 0 & \sqrt{m_2} & 0 \\ 0 & 0 & \sqrt{m_3} \end{pmatrix} \tag{2.1}$$

appear to correctly capture some underlying reality as to a substantial variety of mass / energy relationships, based on its facility for characterizing both the charged lepton masses and the ^{2}H, ^{3}H, ^{3}He and ^{4}He binding data. Related to the usefulness of (2.1), is also the usefulness of taking square roots of masses / energies, as well as of forming constructs such as $\sqrt{m_u m_d}$ which appeared throughout [3], where m_u and m_d are the up and down current quark masses.

In fact, we can immediately apply these conclusions to arrive at a "ballpark" explanation for the proton and neutron masses. Using the conversion factor $1\text{ u} = 931.494061(21)\text{ MeV}$ we first convert the quark masses deduced in (23) and (24) of [3] to

$$m_u = 0.002387339327\text{ u} = 2.22379240\text{ MeV}, \tag{2.2}$$

$$m_d = 0.005267312526\text{ u} = 4.90647034\text{ MeV}. \tag{2.3}$$

Recall, these were deduced by *simultaneously* solving (1.1) together with (9) of [3], namely:

$$m_e = 0.510998928\text{ MeV}=0.000548579909\text{ u} \equiv 3\left(m_d - m_u\right)/\left(2\pi\right)^{1.5}, \tag{2.4}$$

using the empirical m_e and $M_N - M_P$. Next, we note that the Fermi vev energy v_F, defined from Fermi coupling constant G_F according to $\sqrt{2}G_F {v_F}^2 / c^4 \equiv \hbar c$, is given by (data from [1]):

$$v_F = 246219.651\text{ MeV}. \tag{2.5}$$

Finally, we use the above-noted $\sqrt{m_u m_d}$ energy together with the usefulness of taking square roots of masses and energies, to construct and evaluate:

$$\sqrt{v_F \cdot \sqrt{m_u m_d}} = \sqrt[4]{{v_F}^2 m_u m_d} = 901.835259\text{ MeV}. \tag{2.6}$$

This objective data relationship is symmetric under $u \leftrightarrow d$ interchange, as we surmise $M_N + M_P$ might be, and it differs from the observed masses, $M_P = 938.272046\text{ MeV}$ and $M_N = 939.565379\text{ MeV}$ by only about 4%. So we take this as a "clue" based on mass / energy data, that the proton and neutron masses are determined mainly by a product of $\sqrt{v_F}$ with

$\sqrt{\sqrt{m_u m_d}} = m_u^{.25} m_d^{.25}$, in the spirit of (2.1). In particular, we observe that the proton and neutron masses seem on a ratio basis to "straddle" halfway between the (much lower) quark masses and the (roughly equally higher) Fermi vev, and so appear to be determined by a hybrid combination of the Fermi vev and the up and down quark masses. We now report exactly how.

3. Fitting the Neutron plus Proton Mass sum to 6 Parts in 10,000

The Fermi vev v_F, which from (2.6) appears to play a dominant role in establishing the proton and neutron masses, is also the energy at which the electroweak interaction undergoes spontaneous symmetry breaking. Following symmetry breaking, the resulting electromagnetic interaction charge generator Q is has the value $Q_u = 2/3$ for the up quark, and $Q_d = -1/3$ for the down quark. So we start by forming "vacuum energy numbers" $v_q = Qv_F$ for each of the up and down quarks *q*, i.e., $v_u = (2/3)v_F$ for the up quark and $v_d = (-1/3)v_F$ for the down quark. We then again exercise the usefulness of taking square roots of masses and energies by forming "vacuum-enhanced masses" for each of the up and down quark (not to be confused with "constituent" quark masses), defined according to:

$$M_u \equiv \sqrt{\tfrac{2}{3} v_F m_u} = 604.175135 \text{ MeV}, \tag{3.1}$$

$$iM_d \equiv \sqrt{-\tfrac{1}{3} v_F m_d} = i \cdot 634.578446 \text{ MeV}. \tag{3.2}$$

From these we find that the square root construct:

$$\sqrt{M_u M_d} = \sqrt[4]{\tfrac{2}{9} v_F{}^2 m_u m_d} = 619.190212 \text{ MeV}. \tag{3.3}$$

Finally, we define "vacuum-enhanced Koide matrices" K for both the proton and the neutron by using (2.1) with the assignments $m_1 = iM_d; m_2 = m_3 = M_u$ for the proton (*P*) and $m_1 = M_u; m_2 = m_3 = iM_d$ for the neutron (*N*). That is, we now define:

$$\mathrm{K}_{P\,AB} \equiv \begin{pmatrix} i^{.5}\sqrt{M_d} & 0 & 0 \\ 0 & \sqrt{M_u} & 0 \\ 0 & 0 & \sqrt{M_u} \end{pmatrix}; \quad \mathrm{K}_{N\,AB} \equiv \begin{pmatrix} \sqrt{M_u} & 0 & 0 \\ 0 & i^{.5}\sqrt{M_d} & 0 \\ 0 & 0 & i^{.5}\sqrt{M_d} \end{pmatrix}, \tag{3.4}$$

while at the same time, we recall the Koide matrices defined in (3) of [3] using the current quark masses (2.2), (2.3), namely:

$$K_{P\,AB} \equiv \begin{pmatrix} \sqrt{m_d} & 0 & 0 \\ 0 & \sqrt{m_u} & 0 \\ 0 & 0 & \sqrt{m_u} \end{pmatrix}; \quad K_{N\,AB} \equiv \begin{pmatrix} \sqrt{m_u} & 0 & 0 \\ 0 & \sqrt{m_d} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix}. \tag{3.5}$$

Keeping in mind that our goal is to find $M_N + M_P$, let us take the inner product $\mathrm{K}_{PAB}\mathrm{K}_{NBC}$ using (3.4), and then form its trace $\mathrm{Tr}\left(\mathrm{K}_N \cdot \mathrm{K}_P\right) = \mathrm{K}_{PAB}\mathrm{K}_{NBA}$. This is manifestly symmetric under $N \leftrightarrow P$ interchange, as is $M_N + M_P$. To this, using (3.5), let us add the term $\mathrm{Tr}\left(K_P{}^2\right) + \mathrm{Tr}\left(K_N{}^2\right) = K_{PAB}K_{PBA} + K_{NAB}K_{NBA} = 3\left(m_u + m_d\right)$, which is simply the total of the current quark masses contained within $M_N + M_P$. The result, which is invariant under both $N \leftrightarrow P$ and $u \leftrightarrow d$ interchange, is:

$$\mathrm{Tr}\left(\mathrm{K}_N \cdot \mathrm{K}_P\right) + \mathrm{Tr}\left(K_P{}^2\right) + \mathrm{Tr}\left(K_N{}^2\right) = 3\left(i^{.5}\sqrt{M_u M_d} + m_u + m_d\right) = 3\left(i^{.5}\sqrt[4]{\tfrac{2}{9}v_F{}^2 m_u m_d} + m_u + m_d\right). \quad (3.6)$$

The negatively (-) signed charge of the down quark has, upon taking the fourth root, turned into $i^{.5} = \left(1+i\right)/\sqrt{2}$. If we use a phase $\delta = \pi/4$, we may instead write this fourth root of this negatively-signed charge as $i^{.5} = e^{i\delta}; \delta = \pi/4$. We use this to rewrite (3.6) as:

$$\mathrm{Tr}\left(\mathrm{K}_N \cdot \mathrm{K}_P\right) + \mathrm{Tr}\left(K_P{}^2\right) + \mathrm{Tr}\left(K_N{}^2\right) = 3\left(e^{i\delta}\sqrt{M_u M_d} + m_u + m_d\right) = 3\left(e^{i\delta}\sqrt[4]{\tfrac{2}{9}v_F{}^2 m_u m_d} + m_u + m_d\right). \quad (3.7)$$

Now we take the liberty to vary this phase. If we set $\delta = 0$, which amounts to ignoring the $i^{.5}$ in (3.6), or alternatively, to only considering the magnitudes but *not* the signs of the up and down quark charges Q, and if we then evaluate using v_F, m_u, m_d from (2.5), (2.2), (2.3) respectively and compare to the actual $M_N + M_P$, we find that for $\delta = 0$:

$$\begin{array}{lr} \mathrm{Tr}\left(\mathrm{K}_N \cdot \mathrm{K}_P\right) + \mathrm{Tr}\left(K_P{}^2\right) + \mathrm{Tr}\left(K_N{}^2\right) = 3\left(\sqrt[4]{\tfrac{2}{9}v_F{}^2 m_u m_d} + m_u + m_d\right) = & \mathbf{1878.96142\,MeV} \\ M_N + M_P = & \mathbf{1877.83743\,MeV}\ . \\ \text{Difference:} & \mathbf{1.12400\,MeV} \end{array} \quad (3.8)$$

This differs from the empirical $M_N + M_P$ by a mere 0.0599%. We note that embedded within, is the "clue" $\sqrt[4]{v_F{}^2 m_u m_d}$ of (2.6) multiplied by the coefficient $3\sqrt[4]{2/9} = 3\sqrt[4]{-Q_u Q_d}$, to which is added the sum $3\left(m_u + m_d\right)$ of current quark masses. If we take the predicted energy in the top line of (3.8) and divide by 2, we obtain:

$$3\left(\sqrt[4]{\tfrac{2}{9}v_F{}^2 m_u m_d} + m_u + m_d\right)/2 = 939.48071\ \mathrm{MeV}, \quad (3.9)$$

which actually "threads the needle" *between* the observed values for $M_P = 938.272046$ MeV and $M_N = 939.565379\ \mathrm{MeV}$. Given the closeness of (3.8), (3.9) to the observed proton and neutron masses and the fact that (3.8) is symmetric under both $P \leftrightarrow N$ and $u \leftrightarrow d$ interchange, we now regard (3.8) as a meaningful expression for $M_N + M_P$ to about to about 6 parts in 10,000. Thus, we now set:

$$\mathrm{Tr}\left(\mathrm{K}_N \cdot \mathrm{K}_P\right) + \mathrm{Tr}\left(K_P{}^2\right) + \mathrm{Tr}\left(K_N{}^2\right) = 3\left(\sqrt[4]{\tfrac{2}{9} v_F{}^2 m_u m_d} + m_u + m_d\right) \cong M_N + M_P \,, \tag{3.10}$$

which, again, is accurate to 6 parts in 10^4. But in arriving at (3.8) to (3.10), we have neglected the phase by setting $\delta = 0$. We now need to gain a better understanding of this phase, and in the process, see if we can close the remaining 0.06% gap to arrive at an *exact* expression for $M_N + M_P$, and therefore, for M_N and M_P separately.

4. Exact Expressions for the Proton and Neutron Masses

Working from $3\left(e^{i\delta}\sqrt{M_u M_d} + m_u + m_d\right)$ in (3.7), let us form yet another Koide matrix (2.1), this time, setting $m_1 = 3\sqrt{M_u M_d}$, $m_2 = 3m_u$ and $m_3 = 3m_d$. We then write (3.7) in 3x3 matrix form, with the phase factor separated into its own matrix, as:

$$\begin{aligned}&\mathrm{Tr}\left(\mathrm{K}_N \cdot \mathrm{K}_P\right) + \mathrm{Tr}\left(K_P{}^2\right) + \mathrm{Tr}\left(K_N{}^2\right) = 3\left(e^{i\delta}\sqrt{M_u M_d} + m_u + m_d\right)\\ &3\mathrm{Tr}\begin{pmatrix} \sqrt[4]{M_u M_d} & 0 & 0 \\ 0 & \sqrt{m_u} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix}\begin{pmatrix} e^{i\delta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \sqrt[4]{M_u M_d} & 0 & 0 \\ 0 & \sqrt{m_u} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix} \cong M_N + M_P\end{aligned} \,. \tag{4.1}$$

The middle matrix with the complex phase factor $e^{i\delta}$ seems pregnant. Specifically, we know that the unitary matrices U which are used to mix the quark and lepton generations, namely:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{pmatrix}\begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix} = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & -c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & -c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{pmatrix} \tag{4.2}$$

in the original CKM representation, also contain a middle matrix with a phase just like the middle matrix in (4.1). But these also contain $\sin\theta_1$ and $\cos\theta_1$ representing a real (Cabibbo) mixing factor. Let us therefore *hypothesize* that the middle matrix in (4.1) has a form analogous to the middle matrix in (4.2), but is merely rotated to an angle of $\theta_1 = 0$ so that this angle has thus far been hidden from view. Thus, let us introduce an analogous angle θ_1 into the middle matrix in (4.1), and allow this angle the liberty of varying just as we earlier allowed the phase δ to vary. We make no *a priori* suppositions as to the relationship, if any, between this new θ_1 and the analogous θ_1 used in CKM quark mixing. We leave it to the objective empirical data to inform us about this question. Consequently, we now rewrite (4.1) as:

$$\begin{aligned}
&\mathrm{Tr}\left(\mathrm{K}_N \cdot \mathrm{K}_P\right)+\mathrm{Tr}\left({K_P}^2\right)+\mathrm{Tr}\left({K_N}^2\right) \\
&=3\mathrm{Tr}\begin{pmatrix} \sqrt[4]{M_u M_d} & 0 & 0 \\ 0 & \sqrt{m_u} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix}\begin{pmatrix} e^{i\delta} & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{pmatrix}\begin{pmatrix} \sqrt[4]{M_u M_d} & 0 & 0 \\ 0 & \sqrt{m_u} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix} . \\
&=3\mathrm{Tr}\begin{pmatrix} \sqrt{M_u M_d}\exp(i\delta) & 0 & 0 \\ 0 & m_u\cos\theta_1 & \sqrt{m_u m_d}\sin\theta_1 \\ 0 & -\sqrt{m_u m_d}\sin\theta_1 & m_d\cos\theta_1 \end{pmatrix} \\
&=3\left(\sqrt{M_u M_d}\exp(i\delta)+(m_u+m_d)\cos\theta_1\right) \equiv M_N + M_P
\end{aligned} \tag{4.3}$$

In the very final line of (4.3), we have made one other very noteworthy change from (3.10) and (4.1), which contained "$\cong M_N + M_P$" at the very end to represent the 0.06% approximation found in (3.8). In (4.3), *in very important contrast*, we have now ended with the expression $\equiv M_N + M_P$. That is, following the introduction of this new angle θ_1, we shall now *define* both the phase δ and this new angle θ_1 *such that* the expression $3\left(\sqrt{M_u M_d}\exp(i\delta)+(m_u+m_d)\cos\theta_1\right)$ is *exactly* equal to the neutron plus proton mass sum. That is, we shall *define* δ and θ_1 via

$$M_N + M_P \equiv 3\left(\sqrt{M_u M_d}\exp(i\delta)+(m_u+m_d)\cos\theta_1\right), \tag{4.4}$$

exactly, by using the *empirical* values of M_N and M_P. Then, explaining the *exact* magnitudes of the neutron and proton masses will boil down to explaining the deduced values of δ and θ_1.

To find δ and θ_1, we first solve the simultaneous equations (1.1) for $M_N - M_P$ and (4.4) for $M_N + M_P$, to arrive for the first time at separate masses for the neutron and proton, namely:

$$M_N = \tfrac{1}{2}\left(3\left(\sqrt{M_u M_d}\exp(i\delta)+\cos\theta_1(m_u+m_d)\right)+m_u-\left(3m_d+2\sqrt{m_\mu m_d}-3m_u\right)/(2\pi)^{\frac{3}{2}}\right), \tag{4.5}$$

$$M_P = \tfrac{1}{2}\left(3\left(\sqrt{M_u M_d}\exp(i\delta)+\cos\theta_1(m_u+m_d)\right)-m_u+\left(3m_d+2\sqrt{m_\mu m_d}-3m_u\right)/(2\pi)^{\frac{3}{2}}\right). \tag{4.6}$$

The detailed calculation to deduce δ and θ_1 from (4.5) and (4.6) is shown in (6.23) to (6.30) of [4] and so will not be repeated here. But as a result of this calculation, using $M_P = 938.272046 \text{ MeV}$ and $M_N = 939.565379 \text{ MeV}$ together with (2.2), (2.3) and (3.3) we *deduce* that:

$\delta = 0$, <u>exactly</u>, and (4.7)

$$\cos\theta_1 = 0.947454124 . \tag{4.8}$$

The deduction that $\delta = 0$ removes the complex phase from (4.4) through (4.6), and is both validated and explained by empirical data which shows that the mass of the antiproton is equal to that of the proton, and that the mass of the antineutron is equal to that of the neutron, see, e.g., [8], [9]. The deduction that $\cos\theta_1 = 0.947454124$ now presents a new, empirical, "nucleon fitting angle" which, if it can be explained on some known, independent basis, would then provide a complete fitting of the proton and neutron masses to other known parameters, specifically, the up and down quark masses *and charges*, the Fermi vev, and to the extent that it can also be understood independently, the nucleon fitting angle $\cos\theta_1$.

5. Connection between the Nucleon Fitting Angle and the CKM Quark Mixing Angles

The angle θ_1 which we now seek to understand first appeared in the middle matrix in (4.2) for CKM generational mixing. So, our first avenue of inquiry should be to explore whether this angle is related in some manner to the angles used for the CKM mixing of quarks. Toward this end, we first transcribe the *empirical* values of this matrix from (11.27) of PDG's [10] as:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ -0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ -0.00867^{+0.00029}_{-0.00031} & -0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix} . \tag{5.1}$$

Because (11.27) of PDG's [10] contains *magnitudes*, but the actual mixing matrix is formed from three matrices in which $-\sin\theta$ is always a lower-left matrix entry, see the matrix product in (4.2), we have restored the negative sign in front of the three lower-left entries in (5.1) above.

Now, none of the entries in (5.1) compares directly to $\cos\theta_1 = 0.947454124$ in (4.8). But, rather than examine individual entries, we instead use mid-range entries in (5.1) to ascertain the "upper-left-to-lower-right" portion of the determinant $|V|$, which we designate as $|V|_+$ and refer to as the "major determinant." We find, from *empirical data*, and comparing $\cos\theta_1$, that:

$$\begin{aligned} &|V|_+ = V_{ud}V_{cs}V_{tb} + V_{us}V_{cb}V_{td} + V_{ub}V_{cd}V_{ts} = \mathbf{0.947535} \\ &\cos\theta_1 = \qquad\qquad\qquad\qquad\qquad\qquad\quad \mathbf{0.947454} \; . \\ &\text{Difference:} \qquad\qquad\qquad\qquad\qquad\qquad \mathbf{0.000081} \end{aligned} \tag{5.2}$$

This is a difference of a mere 8.1 parts per 100,000. The fact that the nucleon fitting angle $\cos\theta_1 = 0.947454124$ derived in (4.8) from the empirical neutron and proton masses is so close to $|V|_+$ derived from quark generation mixing, certainly warrants attention if one is objectively comparing and characterizing data. This is especially so because $|V|_+$, which is formed from all nine mixing entries in (5.1), is an *invariant* of V.

So the next question is whether these results (5.2) are truly the same within experimental errors. Happily, it turns out that they are! Specifically, we find that $\left|V\right|_+ = 0.947192 = \cos\theta_1 - 0.000262$ if we use the lower bounds of all the experimental error ranges in (5.1), and $\left|V\right|_+ = 0.947854 = \cos\theta_1 + 0.000400$ if we use upper bounds. *So this is within experimental errors.* Using $\cos\theta_1 = 0.947454$ as the baseline against which to compare $\left|V\right|_+$, we express this result as:

$$\left|V\right|_+ = \cos\theta_{1\,-0.000262}^{\;+0.000400} = 0.947454^{+0.000400}_{-0.000262}\,. \tag{5.3}$$

Because of this concurrence within experimental errors, we now establish:

$$\left|V\right|_+ \equiv \cos\theta_1 \tag{5.4}$$

as a meaningful relationship, then use (5.4) together with (4.7) to rewrite (4.4) through (4.6) as:

$$M_N + M_P \equiv 3\left(\sqrt{M_u M_d} + \left|V\right|_+ \left(m_u + m_d\right)\right), \tag{5.5}$$

$$M_N = \tfrac{1}{2}\left(3\left(\sqrt{M_u M_d} + \left|V\right|_+ \left(m_u + m_d\right)\right) + m_u - \left(3m_d + 2\sqrt{m_\mu m_d} - 3m_u\right) / \left(2\pi\right)^{\frac{3}{2}}\right), \tag{5.6}$$

$$M_P = \tfrac{1}{2}\left(3\left(\sqrt{M_u M_d} + \left|V\right|_+ \left(m_u + m_d\right)\right) - m_u + \left(3m_d + 2\sqrt{m_\mu m_d} - 3m_u\right) / \left(2\pi\right)^{\frac{3}{2}}\right). \tag{5.7}$$

The nucleon fitting angle $\cos\theta_1 = 0.947454124$ is known to *at least* three digits ($\sim 10^3$) greater accuracy than the mid-range $\left|V\right|_+ = 0.947535$ due to the former being derived from the proton and neutron masses $M_P = 938.272046$ MeV and $M_N = 939.565379$ MeV which are known to nine digits of accuracy in MeV and the quark masses (2.2), (2.3) which also become known to nine digits of accuracy in MeV because they are based on simultaneously solving (1.1) and (2.4). Thus, we may now use the far more accurate $\cos\theta_1 = 0.947454124$ to set $\left|V\right|_+ = 0.947454124$. This now becomes another empirical data point – derived ultimately from the proton, neutron and electron masses – which can then be used to fine-tune the CKM matrix entries in (5.1).

With this, we have now reached our goal of fitting the proton and neutron masses to other known parameters, and have found that these other known parameters are the up and down quark masses and charges, the Fermi vev, *and an invariant* $\left|V\right|_+$ *of the CKM quark mixing matrix*.

6. A New "Toy" for Seeking to Understand the Baryon Mass Spectrum

As a final exercise, keeping in mind that θ_1 is but one of three angles in CKM mixing, analogously to (4.3), let us form two more matrices for the second and third quark generations with *c, s, t, b* quarks, and use two more mixing matrices with angles θ_2, θ_3, as follows:

$$\begin{aligned}&3\begin{pmatrix}\sqrt{m_s} & 0 & 0\\ 0 & \sqrt[4]{M_c M_s} & 0\\ 0 & 0 & \sqrt{m_c}\end{pmatrix}\begin{pmatrix}\cos\theta_2 & 0 & \sin\theta_2\\ 0 & 1 & 0\\ -\sin\theta_2 & 0 & \cos\theta_2\end{pmatrix}\begin{pmatrix}\sqrt{m_s} & 0 & 0\\ 0 & \sqrt[4]{M_c M_s} & 0\\ 0 & 0 & \sqrt{m_c}\end{pmatrix}\\ &=3\begin{pmatrix}m_s\cos\theta_2 & 0 & \sqrt{m_c m_s}\sin\theta_2\\ 0 & \sqrt{M_c M_s} & 0\\ -\sqrt{m_c m_s}\sin\theta_2 & 0 & m_c\cos\theta_2\end{pmatrix}\end{aligned}. \tag{6.1}$$

$$\begin{aligned}&3\begin{pmatrix}\sqrt{m_t} & 0 & 0\\ 0 & \sqrt{m_b} & 0\\ 0 & 0 & \sqrt[4]{M_t M_b}\end{pmatrix}\begin{pmatrix}\cos\theta_3 & \sin\theta_3 & 0\\ -\sin\theta_3 & \cos\theta_3 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\sqrt{m_t} & 0 & 0\\ 0 & \sqrt{m_b} & 0\\ 0 & 0 & \sqrt[4]{M_t M_b}\end{pmatrix}\\ &=3\begin{pmatrix}m_t\cos\theta_3 & \sqrt{m_t m_b}\sin\theta_3 & 0\\ -\sqrt{m_t m_b}\sin\theta_3 & m_b\cos\theta_3 & 0\\ 0 & 0 & \sqrt{M_t M_b}\end{pmatrix}\end{aligned}. \tag{6.2}$$

In the foregoing, analogously to (3.1) to (3.3), we have also defined the vacuum enhanced:

$$M_c \equiv \sqrt{\tfrac{2}{3} v m_c} = 14,467\,MeV\,, \tag{6.3}$$

$$M_s \equiv \sqrt{\tfrac{1}{3} v m_s} = 2792\,MeV\,, \tag{6.4}$$

$$M_t \equiv \sqrt{\tfrac{2}{3} v m_t} = 168,758\,MeV\,, \tag{6.5}$$

$$M_b \equiv \sqrt{\tfrac{1}{3} v m_b} = 18,522\,MeV\,, \tag{6.6}$$

$$\sqrt{M_c M_s} = 6356\,MeV\,, \tag{6.7}$$

$$\sqrt{M_t M_b} = 55,908\,MeV\,. \tag{6.8}$$

These values are calculated from the PDG data [11] rounded to the nearest MeV, recognizing substantial experimental uncertainties. Also, in (6.1), (6.2), we have "cycled" the "large" square root terms involving the vacuum enhanced masses from the upper-left position in (4.3) to the lower-right position in (6.2), and have cycled the mixing angles in step with this. This is but one of several "representations" that one might choose to form.

Now, let us place the matrices (6.1), (4.3) (sans trace) and (6.2) next to one another from left to right and then multiply them to arrive at a mass and mixing matrix Θ defined as such:

$$(6.1)\cdot(4.3)\cdot(6.2)=$$

$$\frac{\Theta}{27}\equiv\begin{pmatrix} \begin{matrix} m_s m_t \sqrt{M_u M_d}\,\mathrm{c}_2\,\mathrm{c}_3\,e^{i\delta} \\ +\sqrt{m_u m_d}\sqrt{m_c m_s}\sqrt{m_t m_b}\,\mathrm{s}_1\,\mathrm{s}_2\,\mathrm{s}_3 \end{matrix} & \begin{matrix} m_s \sqrt{m_t m_b}\sqrt{M_u M_d}\,\mathrm{c}_2\,\mathrm{s}_3\,e^{i\delta} \\ -m_b\sqrt{m_u m_d}\sqrt{m_c m_s}\,\mathrm{s}_1\,\mathrm{s}_2\,\mathrm{c}_3 \end{matrix} & m_d\sqrt{m_c m_s}\sqrt{M_t M_b}\,\mathrm{c}_1\,\mathrm{s}_2 \\ -m_u\sqrt{m_t m_b}\sqrt{M_c M_s}\,\mathrm{c}_1\,\mathrm{s}_3 & m_u m_b\sqrt{M_c M_s}\,\mathrm{c}_1\,\mathrm{c}_3 & \sqrt{m_u m_d}\sqrt{M_c M_s}\sqrt{M_t M_b}\,\mathrm{s}_1 \\ \begin{matrix} -m_t\sqrt{m_c m_s}\sqrt{M_u M_d}\,\mathrm{s}_2\,\mathrm{c}_3\,e^{i\delta} \\ +m_c\sqrt{m_u m_d}\sqrt{m_t m_b}\,\mathrm{s}_1\,\mathrm{c}_2\,\mathrm{s}_3 \end{matrix} & \begin{matrix} -\sqrt{m_c m_s}\sqrt{m_t m_b}\sqrt{M_u M_d}\,\mathrm{s}_2\,\mathrm{s}_3\,e^{i\delta} \\ -m_c m_b\sqrt{m_u m_d}\,\mathrm{s}_1\,\mathrm{c}_2\,\mathrm{c}_3 \end{matrix} & m_d m_c\sqrt{M_t M_b}\,\mathrm{c}_1\,\mathrm{c}_2 \end{pmatrix}. \quad (6.9)$$

Then, let us consider the specialization where we set $\theta_2=\theta_3=0$ and $m_c=m_s=m_t=m_b=1$ and $\sqrt{M_c M_s}=\sqrt{M_t M_b}=1$. In this specialization, now taking the trace, (6.9) reduces to:

$$\begin{aligned}\mathrm{Tr}\left(\frac{\Theta}{9}\right)&=3\mathrm{Tr}\begin{pmatrix} \sqrt{M_u M_d}\exp(i\delta) & 0 & \\ 0 & m_u\cos\theta_1 & \sqrt{m_u m_d}\sin\theta_1 \\ 0 & -\sqrt{m_u m_d}\sin\theta_1 & m_d\cos\theta_1 \end{pmatrix}. \\ &=3\left(\sqrt{M_u M_d}\exp(i\delta)+(m_u+m_d)\cos\theta_1\right)=M_N+M_P\end{aligned} \quad (6.10)$$

This is synonymous with (4.3) which is simply the definition of the sum of the neutron plus proton masses which was later consummated in (5.5) by the connection to CKM mixing. Of course, one can readily see that (6.9) was constructed so as to include M_N+M_P, by design. But this is a potentially useful design.

Specifically, given that Θ in (6.9) contains all six of the quark masses and charges, the Fermi vev, three angles, and one phase, and given that in the specialization (6.10) Θ yields the mass sum M_N+M_P, it is clear that (6.9) contains within, information pertinent to the proton and neutron masses. But the proton and neutron are simply the *duu* and *udd* baryons of spin ½. Because their mass sum sits within (6.9), the question is raised whether (6.9) might be employed in other manipulations as a vehicle to characterize additional baryon masses or sums thereof. We leave this as an open question, and provide (6.9) simply as a new “toy” which individuals attempting to explain the baryon mass spectrum may wish to employ to assist their efforts.

7. Conclusion

Similarly to what was done in [3], we have simply shown how the proton and neutron masses may be fitted to the up and down quark masses and charges, the Fermi vev, and in a surprise which might not be expected *a priori*, the CKM quark mixing matrix, while forgoing any discussion of the underlying theory developed by the author successively in [12], [2], [5] and [4]. As such, while reporting data that objectively fits with empirical observation, we leave room for others in the nuclear and particle physics communities to evaluate these results based on the data alone, and perhaps develop modified or alternative theories as to the physics which might be underlying these clearly accurate relationships involving known empirical masses and energies and quark mixing matrices.

Additionally, the Θ matrix (6.9) may afford an opportunity to fit additional baryon masses beyond those of the proton and neutron together with the higher-generation quark masses. If the results in [3] are any indication, it is likely that the observed higher-generation baryon masses will be useful to better pinpoint the higher-generation quark masses, rather than vice versa, because these baryon masses are providing precise "signals" about the quark masses they confine in the same manner that the proton and neutron masses and binding energies are sending "signals" about the up and down quark masses confined within the proton and neutron.

References

[1] http://physics.nist.gov/cuu/Constants/index.html

[2] J. Yablon, *Predicting the Binding Energies of the 1s Nuclides with High Precision, Based on Baryons which Are Yang-Mills Magnetic Monopoles*, Journal of Modern Physics, Vol. 4 No. 4A, 2013, pp. 70-93. doi: 10.4236/jmp.2013.44A010. Link: http://www.scirp.org/journal/PaperInformation.aspx?PaperID=30817

[3] J. Yablon, *Fitting the ^2H, ^3H, ^3He, ^4He Binding Energies and the Neutron minus Proton Mass Difference to Parts-Per-Million based Exclusively on the Up and Down Quark Masses* (2013) (Preprint: http://vixra.org/abs/1306.0203)

[4] J. Yablon, *Predicting the Neutron and Proton Masses Based on Baryons which Are Yang-Mills Magnetic Monopoles and Koide Mass Triplets*, Journal of Modern Physics, Vol. 4 No. 4A, 2013, pp. 127-150. doi: 10.4236/jmp.2013.44A013. Link: http://www.scirp.org/journal/PaperInformation.aspx?PaperID=30830

[5] J. Yablon, *Grand Unified SU(8) Gauge Theory Based on Baryons which Are Yang-Mills Magnetic Monopoles*, Journal of Modern Physics, Vol. 4 No. 4A, 2013, pp. 94-120. doi: 10.4236/jmp.2013.44A011. Link: http://www.scirp.org/journal/PaperInformation.aspx?PaperID=30822

[6] Y. Koide, *A Fermion-Boson Composite Model of quarks and leptons*, Phys. Lett. B 120 (1983) 161.

[7] Y. Koide, *New view of quark and lepton mass hierarchy*, Phys. Rev. D 28 (1983) 252.

[8] http://cerncourier.com/cws/article/cern/29651

[9] Cresti, M., Pasquali, G., Peruzzo, L., Pinori, C., Sartori, G., *Measurement of the antineutron mass*, Phys. Lett. B, Volume 177, Issue 2, 11 September 1986, Pages 206–210

[10] http://pdg.lbl.gov/2012/reviews/rpp2012-rev-ckm-matrix.pdf

[11] http://pdg.lb.gov/2012/tables/rpp2012-sum-quarks.pdf

[12] J. Yablon, *Why Baryons Are Yang-Mills Magnetic Monopoles*, Hadronic Journal, Volume 35, Number 4, 401-468 (2012) Link: http://www.hadronicpress.com/issues/HJ/VOL35/HJ-35-4.pdf