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Quantum Mechanics

With Relativity

A Different Approach

Mourici Shachter (Shechter)

ISRAEL, HOLON

mourici@walla.co.il

mourici@gmail.com

Preface

The usual way to solve Quantum Mechanics with Relativity is using Dirac Equation. The solution of this Equation is always mathematically complicate and the physical behavior of the system is barely understood. And the Klien and Gordon Equation is "unphysical"

In this short paper I will show another procedure to solve Quantum Mechanics with Relativity. This procedure is very simple and can explain what really happens and why in the investigated system

Introduction

Quantum Mechanics, in general, is always a combination of the energy and momentum conservation in Newtonian Mechanics and De Borglie hypothesis.

From Newtonian Mechanics the total energy is the sum of kinetic and potential energy.

1] $$E = \frac{P^2}{2m} + E_p$$

2] $$\lambda = \frac{h}{P}$$ De Borglie wavelength

Where

$P = mV$ Is the linear momentum of the electron, m is the electron mass and

$\frac{P^2}{2m}$ Is the electron kinetic energy.

And $\lambda = \frac{h}{P}$ Is De Borglie wavelength.

Schrödinger in 1926 combined combined Newtonian Mechanics and De Borglie hypothesis into a linear differential equation. But is equation does not take into account relativistic effects

Klein and Gordon (1926) tried to include Einstein's Relativity in the quantum theory and developed a new equation starting from the relativistic energy momentum relation

3] $$E^2 = P^2c^2 + E_0^2$$

In this equation $P = mV$ is the linear momentum of the electron c is the speed of light and E_0 is the electron rest energy

4] $$E_0 = m_0c^2$$

And m_0 is the electron rest mass.

Klein-Gordon Equation produced solutions that seem to be “unphysical"

Dirac equation relies on Eq-3, but Dirac expanded this equation as follow

5]

$$E^2 = P^2c^2 + E_0^2 = P_x{}^2c^2 + P_y{}^2c^2 + P_z{}^2c^2 + \left(m_0c^2\right)^2 = \left(\alpha_1 P_x c + \alpha_2 P_y c + \alpha_3 P_z c + \beta m_0 c^2\right)^2$$

And α_i and β are square complex matrices. Dirac Schrödinger and Klein and Gordon Equations can of course be found in Wikipedia

I try to introduce an entirely new approach to Relativistic Quantum Mechanics that fortunately is much simpler and the physical phenomena are easily understood…

mourici@gmail.com mourici@walla.co.il

מוריצי שכטר ✡ישראל✡, חולון קרן היסוד 4 0545480550

A New Approach to Solve Relativistic Quantum Mechanics

In this short paper I will introduce another technique to solve the energy momentum equation

1] $$E^2 = \left(E_k + E_0\right)^2 = P^2c^2 + E_0^2$$

In the last equation E_k is the kinetic energy of the electron, but be careful because

2] $$E_k \neq \frac{mV^2}{2}$$

Dirac and Klein and Gordon tried to find the wave equation of E .

I am going to find the wave equation of the kinetic energy E_k , not the wave equation of E.

From Eq-1

3] $$E_k^2 + 2E_0E_k = P^2c^2$$

Since $E_0 = m_0c^2$ is constant, I can go back to Eq-1 and compute E and its wave equations.

Now let assume that the wave equation of the kinetic energy E_k is a plan wave

4] $$\Psi_k = \Psi_k\left(r,t\right) = \Psi_{k0}e^{i(kr-\omega t)} = \Psi_{k0}e^{\frac{i}{\hbar}(Pr-E_kt)}$$

In this equation I used conventional definitions like

5]

$$\lambda = \frac{h}{P} \qquad \hbar = \frac{h}{2\pi} \qquad k = \frac{2\pi}{\lambda} = \frac{P}{\hbar} \qquad \omega = 2\pi\nu = 2\pi\frac{h\nu}{h} = \frac{E_k}{\hbar}$$

Using Eq-4, The differential equation is derived as follow

6] $$\frac{\partial}{\partial t}\Psi_k = -\frac{i}{\hbar}E_k\Psi_0e^{\frac{i}{\hbar}(Pr-Et)} = -\frac{i}{\hbar}E_k\Psi_k$$

7] $$\frac{\partial}{\partial r}\Psi_k = \frac{i}{\hbar}P\Psi_{k0}e^{\frac{i}{\hbar}(Pr-Et)} = \frac{i}{\hbar}P\Psi_k$$

The relevant operators found from Eq-6 and Eq-7, are

mourici@gmail.com mourici@walla.co.il
מוריקי שכטר ✡ישראל✡, חולון קרן היסוד 4 0545480550

8] $$E_k = i\hbar\frac{\partial}{\partial t}$$

And

9] $$P_x = -i\hbar\frac{\partial}{\partial x} \qquad P_y = -i\hbar\frac{\partial}{\partial y} \qquad P_z = -i\hbar\frac{\partial}{\partial z}$$

Deriving the Differential Equation

Substituting Eq-8 and Eq-9 in Eq-3 and after few manipulations we get the wave equation

10] $$\nabla^2\Psi_k = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Psi_k - i\frac{1}{c^2}\frac{2E_0}{\hbar}\frac{\partial}{\partial t}\Psi_k$$

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The wave Equation is similar to the following Maxwell Equation

11] $$\nabla^2\varphi_k = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi_k - \frac{\rho}{\varepsilon}$$

I am not going into the consequences of this similarity in this short paper. But I found a lot of information from this similarity.

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The Solution of the Wave Equation

The wave Equation

1] $$\nabla^2\Psi_k = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Psi_k - i\frac{1}{c^2}\frac{2E_0}{\hbar}\frac{\partial}{\partial t}\Psi_k$$

Is solved using the well known technique of variable separation.

Assume

2] $$\Psi_k(r,t) = R_k(r)\Theta_k(t)$$
$$R_k = R_k(r) \qquad \Theta_k = \Theta_k(t)$$

Combining Eq-1 and Eq-2 we find that

3] $$\frac{\nabla^2 R_k}{R_k} = \frac{1}{\Theta_k}\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - i\frac{1}{c^2}\frac{2E_0}{\hbar}\frac{\partial}{\partial t}\right)\Theta_k = -k^2$$

And we get two separate equations

4] $$\nabla^2 R_k = -k^2 R_k$$

mourici@gmail.com mourici@walla.co.il
0545480550 4 מורילי שכטר ✡ישראל✡, חולון קרן היסוד

5] $$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Theta_k - i\frac{1}{c^2}\frac{2E_0}{\hbar}\frac{\partial}{\partial t}\Theta_k + k^2\Theta_k = 0$$

Eq-4 can be written in Cartesian, polar, spherical coordinates. Or even any curved coordinates. The solution of this equation supplies the **quantum numbers**

Eq-5 depends on the quantum numbers that the solution of Eq-4 supplies

To solve Eq-5 assume

6] $$\Theta_k(t) = \Theta_{k0}e^{i\omega t}$$

Substituting Eq-6 in Eq-5 and after few easy steps we get a quadratic equation

7] $$\omega^2 - \frac{2E_0}{\hbar}\omega - k^2c^2 = 0$$

The quadratic equation has two solutions

8] $$\omega = \frac{E_0}{\hbar}\left(1 \pm \sqrt{k^2\lambda_c{}^2 + 1}\right) \qquad \lambda_c = \frac{h}{m_0 c}$$

So the complete wave function, always contains two waves

9] $$\Psi_k = R_k\Theta_k = R_{k0}\Theta_{k0}e^{i\left[\frac{P}{\hbar}r - \frac{E_0}{\hbar}\left(1\pm\sqrt{k^2\lambda_c{}^2+1}\right)t\right]}$$

Conclusion

1. May be that in the spectrum of the atom, the two waves are responsible to the doublet lines.
2. The wave function has statistical and probabilistic properties like

   $P(\hat{O}) = \langle \Psi \mid \hat{O} \mid \Psi \rangle$ but is also a wave function. And describe waves with wave properties like interference, standing waves and more…

3. The wave function is composed of two waves that can produce standing waves (The consequences in detail are not discussed because any student knows wave physics)
4. A very strange results is obtained from
5. 10] $$\frac{\omega}{k} = \frac{V\sqrt{1-\beta^2}\left(1 \pm \sqrt{k^2\lambda_c{}^2 + 1}\right)}{\beta^2} \qquad \beta = \frac{V}{c}$$

mourici@gmail.com mourici@walla.co.il
מורילי שכטר ✡ישראלי✡, חולון קקן היסוד 4 0545480550

Relativistic Equation with Potential Energy $E_p(r)$

Eq-5 in the previous chapter can be written as follows

1] $$-\frac{\hbar^2}{2m_0}\nabla^2\Psi = i\hbar\frac{\partial}{\partial t}\Psi - \frac{\hbar^2}{2m_0c^2}\frac{\partial^2}{\partial t^2}\Psi$$

Let add to both side a potential $E_p(r)$

(or only to the left side)

2] $$-\frac{\hbar^2}{2m_0}\nabla^2\Psi + E_p\Psi = i\hbar\frac{\partial}{\partial t}\Psi - \frac{\hbar^2}{2m_0c^2}\frac{\partial^2}{\partial t^2}\Psi + E_p\Psi$$

The left side of EQ-2 is the Hamiltonian from Schrödinger Equation computed using rest mass

3] $$\hat{H}_0\Psi = \left[-\frac{\hbar^2}{2m_0}\nabla^2 + E_p\right]\Psi$$

From this Hamiltonian we can find the Energy Eigenvalues.

4] $$\hat{H}\Psi_{0n} = E_{0n}\Psi_{0n}$$

And substituting Eq-3 and Eq-4 in Eq-2

5] $$i\hbar\frac{\partial}{\partial t}\Psi - \frac{\hbar^2}{2m_0c^2}\frac{\partial^2}{\partial t^2}\Psi + E_p\Psi = E_{0n}\Psi$$

The last equation can be easily modified to the next form

6] $$\frac{\partial^2}{\partial t^2}\Psi - i\frac{2m_0c^2}{\hbar}\frac{\partial}{\partial t}\Psi + \frac{2m_0c^2}{\hbar^2}\left(E_{n0} - E_p\right)\Psi = 0$$

Now I use again variable separation to solve Eq-6 in the time domain

Assume the time dependent function is as in the previous chapter

7] $$\Theta(t) = \Theta_0 e^{i\omega t}$$

And making the same procedure as in the previous chapter

8] $$\left[-\omega^2 + \frac{2E_0}{\hbar}\omega + \frac{2E_0}{\hbar^2}\left(E_{n0} - E_p\right)\right]\Theta_0 = 0$$

This Eq can be rewritten as follows

9] $$\omega^2 - \frac{2E_0}{\hbar}\omega - k_v{}^2c^2 = 0 \qquad k_v{}^2c^2 = \frac{2E_0}{\hbar^2}\left(E_{n0} - E_p\right)$$

mourici@gmail.com mourici@walla.co.il
מורילי שכטר ✡ישראלי✡, חולון קקן היסוד 4 0545480550

This Equation is identical to Eq-7 in the previous chapter and has the same form of solution

10] $$\omega = \frac{E_0}{\hbar}\left(1 \pm \sqrt{k_v^{\,2}\lambda_c^{\,2}+1}\right) \qquad k_v \neq k \qquad \lambda_c = \frac{h}{m_0 c}$$

Again the wave function contain two waves

Conclusion

The suggested procedure is based on Schrödinger Equation. The easiest way to understand what happens in the investigated system is, to find the wave function and the energy states of the kinetic energy

1. The external potential bends space. The form of the result is as in free space but appear to us as viewed in a bend mirror
2. There are always two waves so , there is also standing waves and interference and other familiar wave phenomena
3. The two waves can explain "Spin" and "Entanglement" because they exist at the same time and have the same direction the two waves explains also the doublets in the spectrum.
4. May be that Relativity is a wave theory and duality does not exist at all.
5. There is a similarity between Maxwell equation and Relativistic Quantum Mechanics for this reason statistic and waves relates to each other.

The process introduced in this paper is very simple and I hope that student will adopt it with computer aid to solve complicate quantum problems and still understand and system behavior.

Please send comment to the mail mourici@walla.co.il mourici@gmail.com

sof

mourici@gmail.com mourici@walla.co.il
מורילי שכטר ✡ישראל✡, חולון קקן היסוד 4 0545480550