A Modification of Cosmological Friedman-Robertson-Walk Metric

ChiYi Chen[a*]
[a] *chenchiyi@hznu.edu.cn*
Hangzhou Normal University, Hangzhou 310036, China

In this paper we firstly present an explicit dynamical equation which satisfies the general principle of relativity under the framework of classical mechanics. In light of this fact, the necessity of Einstein's equivalence principle for the gravity being geometrized should be reexamined. Especially, Einstein's (strong) equivalence principle claims that the inertial force is equivalent to the gravitational force on physical effect. But in fact the new dynamical equation proves that the essence of the inertial force is the real force exerted on the reference object, which can actually be all kinds of forces such as the gravitational force, electromagnetic force and so on. Therefore, in this context we only retain the numerical equality between the inertial mass and gravitational mass and abandon Einstein's (strong) equivalence principle. Consequently, the candidate for the standard clock should be corrected into the mathematical clock which duplicates the real clock equipped by the observer himself. Then an adjusted physical picture for how to convert the gravitational force into a geometric description on space-time is presented. On the other hand, we point out that all cosmological observations are made by the observer at the present time on the earth, instead of any other observers including the comoving observers in the earlier unverse. On this basis, we introduce an extra factor $b(t)$ in FRW cosmological metric to depict the gravitational time dilation effect since the local proper clock may run in a faster and faster rate with the expanding of the universe. In this way, we may obtain a positive value of $\rho + 3p$ and avoid the introduction of dark energy in the current universe.
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1 INTRODUCTION

As an alternative theory for Newton's gravity law, Einstein's geometric theory of gravity was used in studying the universe shortly after the theory was established in 1915, thus initiated the beginning of modern cosmology. In Einstein's geometric theory of gravity, the curvature of the space-time is determined by energy and momentum distributions[1–4]. The predicted gravitational redshift effect is proved by a series of gravity tests in the solar system[5–7]. This effect has adequately illustrated that geometric description is more accurate to describe gravity than Newton's gravity law. In recent decades, cosmology has gradually developed into a science [8, 9] including observations of supernova, cosmic microwave background, etc. However, as soon as current standard cosmology was compared with cosmological observation data, it was confronted with some rigorous challenges such as dark energy[10, 11], dark matter[11]. All of these challenges have shaken the existing knowledge of physics. Especially dark energy, which may drastically change the direction of scientific research. Further considering that the dynamics of the universe can not be repeated in laboratory, therefore it is necessary for us to make repeated inspection on the foundation of the standard cosmology.

In this paper, we would like study the formulism of space-time metric for cosmological observation theory. The paper is organized as follows. In Sec.2, we prove that the general principle of relativity can actually be realized even under the framework of classical mechanics by presenting an explicit form of new dynamical particle equation. For the physics indicated by this new equation, Einstein's (strong) equivalence principle is given up. Besides, we also point out a question exists in the cosmology: who is the real observer for cosmological observation. This is not a trivial question since the observer at the present time on the earth is not equivalent to the comoving observer after Einstein's (strong) equivalence principle is abandoned. In Sec.3, we propose a modification on the cosmological Friedman-Robertson-Walk metric based on the consideration of gravitational time dilation effect. New dynamical equation for cosmology and some possible new changes are also presented under this modified cosmological metric. In Sec.4, we give a short summary of physical logic as a conclusion.

2 MOTIVATIONS AND REASONS

2.1 an explicit realization for general principle of relativity

General principle of relativity can actually be realized in Classical Mechanics. In the classical framework, a particle dynamical equation which satisfies the general principle of relativity can be constructed as[12],

$$m_p \mathbf{a}|_{p-O} = \mathbf{F}|_p - \frac{m_p}{m_O}\mathbf{F}|_O \equiv \mathbf{f}|_{p-O}. \tag{1}$$

Here $\mathbf{F}|_p$ denotes the total forces from the whole universe exerted on the particle p, and $\mathbf{F}|_O$ denotes the total forces from the whole universe exerted on the reference object O. Both of them can be calculated respectively according to existing knowledge of all types of interactions. $\mathbf{a}|_{p-O}$ denotes the relative acceleration between particle p and reference object O. m_p and m_O respectively denote the mass of the particle p and the reference object O. In principle, any real object in the universe is qualified to be the reference object of a reference frame O in above equation. In the mean time, any physical reference frame must be fixed by at least one real object. Therefore, the equation (1) can be directly applied in any irrotational reference frames, and naturally interprets the nature of the inertial force. According to the equation (1), the inertial force is essentially the real force exerted on the reference object that must be deducted in the relative counting of forces.

The general principle of relativity is essentially a practical requirement. On one hand, we are never able to know about the actual state of motion of our terrestrial reference frame where our observers exist. On the other hand, we are always able to determine the rotation of any reference frame relative to the spatial background of the whole universe by resorting to the galaxies far enough away. Therefore, what the practical observation really requires is that dynamical laws must keep form invariant to any arbitrary reference object. Furthermore, in particle dynamics, the rotation phenomena can always be attributed to the relative motion between different particles. But for single particle, there is no concept of rotation. Just as the problem of variable mass system under the framework of Newtonian mechanics, there the variable mass phenomena should be attributed to the relative motion between different particles in the system of particles. Thus the fundamental law for particle dynamics is still $\mathbf{F}|_p = m_p \mathbf{a}|_{p-O}$, while $\mathbf{F}|_p = \frac{d(\mathbf{p}|_{p-O})}{dt}$ can be generalized from the former equation. Therefore, the problem of reference frames' rotation is actually a mathematical problem. Ultimately, the problem of the rotation of reference frames can be separated from the problem of dynamical relativity. What general principle of relativity can be naturally realized is that dynamical laws keep form invariant in all irrotational reference frames.

2.2 Einstein's (strong) equivalence principle is given up

A nontrivial significance may be illustrated by the equation (1) is that the general principle of relativity for particle dynamics can be realized in a very simple and natural approach, which is obviously different with Einstein's view[13]. Because the concept of inertial force still exists in Einstein's special theory of relativity, and even in his general principle of relativity Einstein's (strong) equivalence principle still claims that the inertial force is physically equivalent to the gravitational force. However, it has been indicated by the new dynamical equation (1) that, the nature of the inertial force is the real force exerted on the reference object, which can actually be all kinds of interactions such as the gravitational interaction, electromagnetic interaction and so on. Therefore, the necessity of Einstein's equivalence principle for the physical picture of gravity being geometrized should be scrutinized. The discussion in this paper is just made on the basis of giving up Einstein's (strong) equivalence principle and only accepting the equality between the gravitational mass and inertial mass. Consequently, the running rate for a clock free falling under gravity should no longer be assumed to be a constant. In principle, the gravitational field is responsible to change the rate of local clocks since gravitational time dilation effect has been verified at least in solar gravity tests[5–7]. In light of this new physics, here the standard clock is no longer the usual free falling clocks in Einstein's gravity theory. What is the qualified candidate for the standard clock which is defined to make the comparison on the running rate of local clocks? Our new approach to realize the general principle of relativity has demonstrated an one to one correspondence between the counting of forces and the selection of reference frame. Once the counting of forces exerted on the particle under study being given, we have the only one option for the reference frame. In other words, it is not able for us to make a further transformation on observers. Besides, according to the gravitational time dilation effect, the running rate of local clocks depends on the strength of gravitational field at their positions. There are two reasons. On the one hand, the realization of general principle of relativity has no longer need of Einstein's (strong)equivalence principle, so theoretically it is no longer necessary to assume that the running rate of all free falling clocks under changing gravity are the same constant. On the other hand, the clock which is relatively rest in the gravitational field and the clock which is free falling under changing gravity, they differ only in a non-gravitational force and the resulting acceleration. If there is really no gravitational time dilation effect which exists for the free falling clock under changing gravity, it must imply that a non-gravitational force and the resulting acceleration are also able to bring a time dilation effect for clocks. However, by now there is no such a sign which has been observed and verified in all past experiments. Therefore, in this paper we suppose that any clock in the gravitational field, regardless of its state whether it is free falling or not, its running rate will change with the strength of gravitational fields. For above reasons, we also propose that the clock equipped by the observer himself is the only qualified candidate for the standard clock which is used as the reference to make the comparison on the running rate of local clocks.

For the advantage in understanding, it is necessary to establish a rigid and homogeneous coordinate system for the reference frame on the spatial region under study, the space-time scale inside it is determined by observer's own clock and ruler. However the gravitational effect on the space-time geometry has been verified in many gravity tests, hence such an observer's coordinate system for the reference frame can only be mathematical. Therefore, the space-time interval measured by this set of coordinate system is mathematically equivalent to be measured by the observer's own clock and ruler.

2.3 the existence of an absolute cosmic spatial background

The new approach to realize the general principle of relativity implies the existence of an absolute cosmic spatial background. Now we try to interpret the new dynamical equation (1) at the level of natural philosophy. Firstly, we suppose that the particle dynamics is certainly to be a theory with causal principle. Under the framework of classical mechanics, we regard forces as the cause, and regard accelerations as the effect(result). Then in the process of derivation, the only one most fundamental principle which can be based on is the causal consistency. To the total force exerted on the particle p from the whole universe, how should we express the corresponding effect? The total force exerted on a single particle should be objective, namely it will not change with the replacement of observers. Therefore, the corresponding effect should also be objective, and not relevant with any reference frame. In this way, a completely objective acceleration can only be expressed as the acceleration in the absolute background of the whole universe (called cosmic spatial background),

$$\mathbf{F}|_p = m_p \frac{d^2}{dt^2}(X,Y,Z)|_p. \tag{2}$$

Here the objective position of the particle p in the cosmic spatial background is particularly denoted by $(X,Y,Z)|_p$. In fact, the cosmic spatial background can be intuitively understood. As the name implies, the spatial background is just what still exist in a space region after all objects inside it were moved away. And the cosmic spatial background is just what still exist in the whole universe after all concrete objects in the universe were moved away. If we believe that there must exist an objective motion state for any object in the universe behind innumerous relatively measured motion states, we may naturally admit the existence of an absolute basis for the motion of every object. Therefore, we postulate that there exists an absolute and eternally unchangeable spatial background for the whole universe. Why the background of the universe should be absolute and eternally unchangeable? The reason is that the causal consistency requirement will not be reached between the total force exerted on the particle and the acceleration of the particle "relative" to the background, if the background is not absolute or some kinematical evolution still exists in itself.

The motion of all objects in the universe must be included into this common absolute three-dimensional space background, because only on this basis then the existence of an objective dynamical law is possible. Strictly speaking, the existence of an absolute comic spatial background is an assumption at the natural philosophy level. The key point which must be distinguished here is that the existence is proposed for absolute background, instead of absolute space-time. Here *absolute* means that the cosmic spatial background has no dynamical evolution and is irrelative to the existence and motion of any object. The reason is that both Newton's view of absolute space-time and Einstein's space-time theory of relativity are mainly concerned with the change rules of space-time scales. But the space-time scales are radically defined by the intervals of proper events occurred on objects, and so belong to the property of concrete objects. However, the cosmic spatial background itself do not contain any concrete object. Therefore, the existence of absolute cosmic spatial background will not be conflicted with the core ideas of modern physics, and we must distinguish the spatial background of the universe from the spatial intervals in Einstein's theory of relativity. Definitely, an absolute cosmic spatial background can be included into Newton's view of absolute space-time. But in contrast, the former do not require that the space-time intervals are absolute, which is the key point only for the latter.

Logically, the absolute cosmic spatial background should exist for our universe although it is not concrete like general objects. For example, we have known that the dimensions of cosmic spatial background should be of three. But there is still a problem that the objective position in cosmic spatial background can not be directly measured. What we can really measure is the difference between any two objective positions, which substantially constructs a mathematical vector,

$$\mathbf{x}|_{p-O} = (X, Y, Z)|_p - (X, Y, Z)|_O. \tag{3}$$

All objects in the universe, including objects under study (p) and reference objects (O), should be of equal status in the most fundamental law of dynamics. For example, the dynamics of any real reference object should also satisfy $\mathbf{F}|_O = m_O \frac{d^2}{dt^2}(X, Y, Z)|_O$. The introduction of reference frames is just to make relative measurements on kinematical quantities. As a causal correspondence, the forces should also be relatively counted in nature.

$$m_O \mathbf{F}|_p - m_p \mathbf{F}|_O = m_p m_O \frac{d^2}{dt^2}((X, Y, Z)|_p - (X, Y, Z)|_O) = m_p m_O \frac{d^2 \mathbf{x}|_{p-O}}{dt^2}. \tag{4}$$

Here the reference object O naturally corresponds to the origin point of a reference frame. Finally, we obtain

$$\frac{\mathbf{F}|_p}{m_p} - \frac{\mathbf{F}|_O}{m_O} = \mathbf{a}|_{p-O}. \tag{5}$$

This is the another equivalent form of the new dynamical particle equation (1). Therefore, the new dynamical equation (1) can be logically and exactly derived, only if we postulate the existence of an absolute cosmic spatial background. The key point which must be distinguished here is that the existence is proposed for absolute background, instead of absolute space-time intervals. Therefore in this sense, the expansion of the universe is not an expansion of the cosmic background.

2.4 a modified physical picture for gravity being geometrized

In light of above explicit realization of general principle of relativity, we only retain the assumption that the gravitational mass equals the inertial mass. In fact, this most weak equivalence principle is already enough to account for why the gravity can be depicted by a geometric theory. For example, we may illustrate this idea by using the trajectory of satellites around the earth. To simplify the question, we assume the trajectory is a circle. According to Newtonian mechanics, we have $\frac{GMm}{r^2} = m\frac{v^2}{r} \Rightarrow v^2 = \frac{GM}{r}$. It is obvious to see that the unique parameter for the trajectory (r) does not depend on the mass of the satellite. That is to say, the dynamical law for satellites in gravity field has no direct correlation with the magnitude of gravitational forces exerted on these satellites. Therefore, the dynamical law of satellites can be equivalently depicted by a kinematical law. This is a concrete example to account for that the physical effect of gravity is the geometry of space-time.

On the other hand, according to gravitational time dilation effect which has been verified by the experiments on the solar system, the running rate of clocks is related to the strength of local gravitational field. But on the logic, the validity of Einstein's field equation does not depend on Einstein's equivalence principle[14]. Therefore, in this paper we still adopt Einstein's gravitational field equation to be a mathematical correct equation in converting gravity into geometric description of space-time.

According to the experiments on the solar gravity, the space-time interval of local proper events occurred on some objects under gravitational field will show the difference compared with that occurred on the reference object. Therefore, a consistent approach to convert gravity into geometric description of space-time should be carried out as follows. First, a suitable reference frame such as sun-centered reference frame must be selected, which will bring both the relative measurement of distance and the relative counting of gravitational forces. Second, a rigid and homogeneous coordinate system should be established. The reference object is usually defined as the origin point of coordinate system. At the same time, the space and time scale in this coordinate system is defined by the clock and the ruler equipped by the observer himself and duplicated into the whole coordinate system. Finally, according to the comparison between the local proper clock (ruler) located at every position in gravitational field and the mathematical clock duplicated from the observer himself, the curved geometry of space-time is thereby determined meanwhile this observer-dependent coordinate system is provided as the background. Furthermore, if we mathematically retain

Einstein's field equation to be the correct equation converting gravity into geometric description of space-time, the geometry of the curved space-time can be solved quantitatively. In fact, if we recall the solar gravity test, we will find that above approach for converting gravity into geometric description of space-time can self-consistently explain the gravitational redshift effect. Besides, there is a different point which distinguish our present physics picture from that of traditional Einstein's theory of gravity. In present physical picture for gravity being geometrized, the strength of gravitational field should be counted relatively, rather than the total contribution from all the matter in the universe. More specifically, in the derivation of Schwarzschild metric, the counting of the gravitational force is actually restricted to that exerted by objects inside the solar system. Meanwhile, the reference origin is fixed at the center of the solar system. Therefore, what the solar gravity test has essentially satisfied is our present approach to convert gravity into geometric description of space-time.

2.5 who is the real observer for cosmology

Compared with the whole evolution history of our universe, the cosmological observer is the observer only at the present time on the earth, rather than the comoving observer which is evolving with the universe. The new approach to realize the general principle of relativity has shown that the counting of forces must be in an one-to-one correspondence with the selection of the reference frame. Therefore, any arbitrary choice of the reference frame would not affect the validity of dynamical equations, but every term in dynamical equations is closely related to the concrete reference object. To the geometric theory of gravity, reference object dependence is changed to be observer's space-time scales dependence. Therefore, it is very important to make clear the dynamical state of the observer himself for the sake of correct application of Einstein's field equation.

Firstly, all observations from cosmology and all determinations on the redshift values are processed by the observer at the present time, instead of a comoving observer which evolves with the expanding of the universe. Therefore, in cosmology there is one important feature should be emphasized that all observations are actually made at "the present time". Not only the observer is at the present time, But also the determination of redshift values is implemented at the present time. In other words, cosmological observer is not the observer at any other time of the universe, especially not the comoving observer on the earth.

Considering that the strength of gravitational field in the universe is changing continuously, the clock equipped by the comoving observer on the earth should run faster and faster with the expanding of the universe, compared with that of the observer at the present time on the earth. Therefore, in constructing a rigid and homogeneous reference coordinate system for cosmology, the duplicated clock (ruler) should be in strict accordance with the clock (ruler) equipped by the observer at the present time on the earth.

3 NEW COSMOLOGICAL METRIC AND DYNAMICAL EQUATIONS[15]

According to previous discussion, there are two points of physical considerations should be fully incorporated in the construction of cosmological metric. Firstly, it has been demonstrated by an explicit equation that the general principle of relativity can be realized in a very concise picture, so Einstein's (strong) equivalence principle is no longer indispensable for the geometric theory of gravity. We only retain the numerical equality between the inertial mass and gravitational mass since it has a solid foundation from experiments. But the assumption that all free falling clocks under gravity run in a uniform rate is given up. Therefore, the candidate for the standard clock which is used to compare the rate of clocks should also be changed. According to the gravitational time dilation effect and the new realization approach for the general principle of relativity, the running rate for the clock equipped by the observer himself must be set as the reference scale in cosmological metric. In mathematics, the clock equipped by the observer himself can be duplicated by imagination on to all space-time points. In theory, the geometrical effect resulted by the gravitational field is depicted by the difference between above mathematically defined standard clock and local clocks. Secondly, we know the matter density in the universe has changed a lot from the beginning of the unverse, so the strength of gravitational field has also changed appreciably. There exists an evolution of the running rate for every local clock fixed on the comoving galaxies of the universe. Therefore, relative to the long evolution history of the universe studied in cosmology, the construction of cosmological metric must exactly distinguish the present observer's clock on the earth from the comoving observer's clock on the earth.

3.1 the coordinate system of reference frame for cosmological metric

First, as for spatial coordinates of cosmological metric, there is a Hubble's principle which predicts a predominate spatial coordinate system, which can be named as spatial comoving coordinate system. In principle the observer can duplicate his ruler at every spatial point and build up a rigid and homogeneous spatial coordinator system. We give the ruler of the observer at the present time on the earth the name of "the cosmological observation ruler". The spatial interval in the comoving coordinate system is just measured by this cosmological observation ruler,

$$dl^2 = a^2(t)[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2]. \tag{6}$$

Although the comoving coordinates are selected as spatial coordinates in cosmological metric, but the cosmological observer is still fixed to be the observer at the present time on the earth. Therefore, the distance between cosmological comoving coordinate points should be measured by the present observer's ruler on the earth. In principle we can only study the local space-time property by solving cosmological dynamics equations around the comoving point where the earth located. But by resorting to the cosmological principle, the solved local space-time property can be easily extended into the global space-time property for the whole universe.

Second, as for the coordinate time, we must also define it from the point of observers' view. We all know that the study of the universe is mainly based on the observation of light signals emitted from the earlier universe. The redshift of light signals is a particularly important quantity to investigate the evolution of the universe. The redshift is determined by comparing the received light signal with the same type of light signals on the earth at the present time. Therefore, the coordinate time should be defined resorting to the clock of the observer at the present time on the earth. We take the present observer's clock rate as the standard scale, duplicate this clock rate on every time-point and build up a rigid and homogeneous coordinate time system. We give this time system the name of "the cosmological observation clock". We use t to denote the reading number of the cosmological observation clock. On the other hand, we know the matter density in the universe has changed a lot from the beginning of the unverse, so the strength of gravitational field has also changed appreciably. Hence a local (proper) clock in the earlier universe, regardless of its state whether it is free falling or not, will run in a different rate relative to the clock under null gravity. To be distinguished from the cosmological observation clock, the reading number of the local (proper) clock which is fixed on the comoving galaxy in the earlier universe is denoted by τ. The time dilation effect is expressed by

$$d\tau = b(t)dt. \tag{7}$$

It must be noticed that if we set $b(t_0) = 1$, which also means that the coordinate time t is equivalently measured by the proper clock of the present observer on the earth. In other words, the interval of coordinate time is actually a kind of proper time intervals but all of them are measured by the clock at the present time. More specifically, for the observer at the present time, dt can always be measured as his proper time interval.

According to above definition of the time and spatial coordinates, a general cosmological metric can be written as[15],

$$ds^2 = -b^2(t)dt^2 + a^2(t)[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2]. \tag{8}$$

In fact, it is also the most general form for cosmological metric under the condition of the cosmological principle. We propose the metric (8) to replace the well known Friedman-Robertson-Walk (FRW) metric in processing the observation data. The reason is what we reiterated in this paper that cosmological observations are always made by the observer at the present time on the earth, instead of any other observers including the local observer in the earlier unverse.

3.2 the dynamical equation of cosmology

According to the new cosmological metric (8), The non-zero components of Ricci tensor can be easily derived:

$$R_{00} = -3\frac{\ddot{a}}{a} + 3\frac{\dot{a}\dot{b}}{ab}, \tag{9}$$

$$R_{ij} = \frac{1}{b^2}(2\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a} - \frac{\dot{a}\dot{b}}{ab})g_{ij} + \frac{2k}{a^2}g_{ij}. \tag{10}$$

A consistent energy-momentum tensor is given by the following form due to the cosmological principle[13],

$$T_{\mu\nu} = \rho U_\mu U_\nu + p(\eta_{\mu\nu} + U_\mu U_\nu). \tag{11}$$

Substituting the equations (9-11) into Einstein's gravitational field equation, the fundamental equations of cosmology are derived out[15]:

$$\frac{\dot{a}^2}{a^2b^2} + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \tag{12}$$

$$\frac{1}{b^2}(\frac{\ddot{a}}{a} - \frac{\dot{a}\dot{b}}{ab}) = -\frac{4\pi G}{3}(\rho + 3p). \tag{13}$$

where k is the curvature of space. In essence, k is not a dynamical variable but a certain value maybe decided by the primary conditions. The cosmic energy density $\rho(t)$ and pressure $p(t)$ can be treated in principle as the functions of two geometrical variables $a(t)$ and $b(t)$. As a complementarity, we need to put forward a specific method to solve above system of equations in further research. However, since the expansion of the universe soon becomes slow enough, in a tentative investigation we might suppose $b(t) = \sqrt{1 - \frac{k'}{a(t)}}$ in analogous to the counterpart $b(r) = \sqrt{1 - \frac{2GM}{rc^2}}$ in Schwarzschild metric. Here k' is also an undetermined constant.

3.3 physical predicts under the new cosmological metric

Without explicitly solving equations (12-13), we are still able to understand the kinematic effects of the expansion (see details in [9]) from the metric (8). We have a redshift value from geometrical analysis (analogous to the analysis in [9]),

$$1 + z' = \frac{a(t_0)}{a(t_1)}. \tag{14}$$

This is just the same result in the standard cosmology. However, above redshift effect should be only attributed to the kinematical effect from the expansion which can only be attributed to the Doppler effect, rather than gravitational redshift effect. A complete observational redshift effect in cosmology should further incorporate the gravitational time dilation effect, which should be counted into when the light signals being emitted and received. Analogous to the gravitational time dilation effect in the solar gravity test, we have

$$1 + z'' = \frac{b(t_0)}{b(t_1)}. \tag{15}$$

Then we obtain the final value for redshift[15]:

$$1 + z = (1 + z')(1 + z'') = \frac{b(t_0)}{b(t_1)}\frac{a(t_0)}{a(t_1)}. \tag{16}$$

Since $b(t_0)/b(t_1)$ is greater than 1 and increase with the decrease of the time t_1 when the universe is expanding, the fitted value of apparent coordinate acceleration d^2a/dt^2 may be changed according to the current observation data[9, 10].

Furthermore, we discuss the mathematical expression for the acceleration of the universe. As we know, acceleration is not invariant under coordinate transformation, and it will change explicitly in different coordinate reference frames. Once the cosmological metric (8) is adopted, since t is the only proper time coordinate for the observer at the present time on the earth, the definition of the phenomenological acceleration should be revised to be $\frac{d^2a}{dt^2}$, instead of $\frac{d^2a}{d\tau^2}$. $\frac{d^2a}{dt^2}$ may be called as the coordinate acceleration and $\frac{d^2a}{d\tau^2}$ may be called as the proper acceleration. The relationship between these two definitions of acceleration is given by[15]

$$\frac{d^2a}{d\tau^2} = \frac{1}{b^2(t)}\frac{d^2a(t)}{dt^2} - \frac{1}{b^3(t)}\frac{da(t)}{dt}\frac{db(t)}{dt}. \tag{17}$$

In considering that all comparisons of the frequency of light signals are implemented at the present time, then all the redshift are intrinsically measured by the clock of the present observer on the earth. That is to say, all redshifts are essentially evaluated by the coordinate time t, which has been defined to run at the same rate with the clock of the present observer on the earth. Hence the value of the phenomenological acceleration of expanding, which can be resulted from current observational data, is directly related to $\frac{d^2a}{dt^2}$. It has been shown by the equation (17) that the sign of this formula may be in different with that of $\frac{d^2a}{d\tau^2}$. To illustrate this point, we may investigate the evolution property of $b(t)$ in analogy to the gravitational time dilation effect in Schwarzschild metric. There the time dilation factor $\sqrt{1-\frac{2GM}{rc^2}}$ increase with the distance r. So we expect that the factor $b(t)$ may increase with the decrease of the gravitational field strength. With the expanding of the universe, the gravitational field strength decreases, then $b(t)$ will increase with time. It indicates $\frac{db(t)}{dt}>0$. On the other hand, we have $\frac{da(t)}{dt}>0$ for an expanding universe. Hence it is possible to have a negative $\frac{d^2a}{d\tau^2}$ according to the equation (17) even $\frac{d^2a}{dt^2}>0$ holds. It further means a possibility that $\rho+3p>0$ according to the equation (13). Therefore, we must keep in mind that there are two different concepts of the acceleration when we talk about the accelerated expansion of the universe. The acceleration indicated directly from the practical observation data is $\frac{d^2a}{dt^2}$, rather than $\frac{d^2a}{d\tau^2}$.

4 CONCLUSION

In this paper we present a feasible and natural approach to realize the general principle of relativity. In light of such a realization, we propose a consistent physical picture in converting gravity into geometric description of space-time. Firstly, according to the observer, a spatial region from which gravitational forces are counted should be firstly determined. And the clock and the ruler equipped by the observer himself should be duplicated all over the selected spatial region. Thus a rigid and homogeneous coordinate system is established. Secondly, only based on the background of this mathematical coordinate system, the relatively counted gravity can be correctly calculated by Einstein's field equation. Besides, since we abandon the assumption that all free falling clocks under different gravity still run at the same rate, the standard clock is changed to be the mathematical clock which duplicates the observer's clock all over the spatial region.

Finally, we investigate the foundation of cosmology and introduce a new cosmological metric in which the coordinate time is defined referring to the proper clock equipped by the observer at present on the earth. The present observer on the earth is the only qualified reference observer to determine the redshift value of light signals that were emitted from the earlier universe. Furthermore, we know that the matter density in the universe has changed a lot from the beginning of the universe, so the strength of gravitational field has also changed appreciably. Therefore the proper clock at the present time on the earth must run at a different rate comparing with that in the earlier universe because of the existence of gravitational time dilation effect. Consequently, there are two different concepts of acceleration introduced to describe the evolution of the universe. One is coordinate acceleration d^2a/dt^2, the other is proper acceleration $d^2a/d\tau^2$. It is significant for us to distinguish these two accelerations. Because even a positive coordinate acceleration is directly evaluated from current observation data, it is still possible to have a negative proper acceleration according to gravitational time dilation effect. Then $\rho+3p$ will turn out to be a positive value according to the equation (13). In this way, the dark energy is possible to be ultimately removed from the present universe.

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* Electronic address: chenchiyi@hznu.edu.cn

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