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# Energy gap of superconductor close to $T_c$ without $CC^+$

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**Abstract** We derived the energy gap of superconductor close to $T_c$, without using the usual methods of creation-annulation operators $CC^+$.
our approximations ar in good agreement with the numerical estimates and theoretical results.

**Keywords** superconductor · energy gap · cooper pair · coherence lengths

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## 1 Introduction

the superconductors coherence length $\xi(T)$ can be treated as the size of the cooper pair[1] and $\xi(0)$ represents the smallest size of the wave packet that the superconducting charge carries can form.a fact which allows the calculation of the energy cost of breaking the cooper-pairs in radius $\xi$.
the minimum energy $E_g = 2\Delta(T)$ should be required to break a pair,creating two quasiparticle excitations.this$\Delta(T)$ was predicted to increase from zero at $T_c$ to a limiting value $E_g(0) = 2\Delta(0)$.[2]
in this work, we proposed a simple method to obtain $\Delta(T)$ close to $T_c$ as function in the interacting range, where the interaction effect is modeled by a complex term added to the hamiltonian.We also discuss the conditions which must be satisfied for our method to be applicable.

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## 2 Calculation

we will try to determine the exchange energy of particle by enclosing it in Box and letting the volume $v$ of the box shrink ($\delta v \prec 0$) or expands ($\delta v \succ 0$) under the influence of the interaction. the normalization condition $\int |\Psi|^2 dv = 1$ expresses the fact that the probability of finding the particle any where is just 1. particles are temporally trapped in the volume $v$, to create a short-lived intermediate state, if the life time of such a state is $\tau$, we can write close the particle position $x$ :

$$\Psi(x,t)\Psi(x,t)^* \sim v^{-1}. \tag{1}$$

the Hamiltonian of the system is given by sum of $H_0$ and $H_I$ where:

$$H_I\Psi \sim (\delta\varepsilon_0 + i\delta\varepsilon)\Psi \tag{2}$$

now let us acting on both sides of equation (1) by the operator $i\hbar\partial_t$, we obtain:

$$i\hbar\partial_t\Psi\Psi^* + i\hbar\partial_t\Psi^*\Psi = i\hbar\partial_t v^{-1}. \tag{3}$$

hence, as the box becomes smaller the kinetic energy and momentum will grown according to the position-momentum uncertainty relation and vice versa [3]. there for the complex variation in eigenvalue can now be written as follows:

$$\varepsilon - \varepsilon_0 = \delta\varepsilon_0 + i\delta\varepsilon \tag{4}$$

where $\delta\varepsilon_0$ its real part and $\delta\varepsilon$ its imaginary part, using equations from (2)to(4) and the time dependent Schrodinger equation we obtain:

$$\delta\varepsilon = -\frac{\hbar}{2}\partial_t logv. \tag{5}$$

particle is temporally trapped in the volume $v$ means $\hbar\overrightarrow{k}.\overrightarrow{\nabla}v = 0$, where $\hbar\overrightarrow{k}$ is the particle's momentum[4]. we can easily verify that:

$$\frac{\partial v}{\partial t} = \frac{\delta v}{\delta t}. \tag{6}$$

substituting equation (6) into equation (5) gives:

$$\delta t\delta\varepsilon = -\frac{\hbar}{2}\frac{\delta v}{v}. \tag{7}$$

from (7) we can see that, if $\delta v$ is positive then $\delta\varepsilon \prec 0$ : the system (volume) loses (emitted) energy,and vice versa if $\delta v$ is negative then $\delta\varepsilon \succ 0$ the system gain (absorbed) energy. the expectation value of $\delta\varepsilon$ in interval of time $\tau$ is defined by:

$$\triangle E = \int_0^\tau \frac{\delta t}{\tau}\delta\varepsilon = -\frac{\hbar}{2\tau}\int_{v_0}^{v}\frac{\delta v}{v}. \tag{8}$$

the energy-time relationship can then be written as:

$$\triangle E = -\frac{\hbar}{2\tau} log \frac{v}{v_0}. \tag{9}$$

let us consider the macroscopic volume $V$,which contains $N$ microscopic volume $V = Nv$ . when neglecting the interactions between $N$ volume $v$, we find that the change in internal energy of the volume $V$ is $\triangle U = N\triangle E$ . using equation (9) we find:

$$\triangle U = -N\frac{\hbar}{2\tau} log \frac{V}{V_0} \tag{10}$$

the comparison of (10) with the adiabatic change in internal energy of an ideal gas[5]gives an equation with the same chape of energy-time uncertainty relation but with thermodynamic energy $k_B T$ [6–11]:

$$\tau.k_B T = \frac{\hbar}{2} \tag{11}$$

where $T$ is the temperature and $k_B$ is the Boltzman constant.

## 3 Energy gap

the BCS and GL coherence lengths [12] are related by:

$$\xi \sim \xi_0 \left(1 - t\right)^{-\frac{1}{2}} \tag{12}$$

where $t$ being the reduced temperature $t = \frac{T}{T_c}$.the system (volume) looses energy $\Delta E = -\triangle$ to create one quasiparticle.using equation (9) and (11) with volumes $v \sim \xi^3$ and $v_0 \sim (\xi - \xi_0)^3$ (see Figure.1) we obtain:

$$\triangle \sim 3k_B T log \left(1 - \sqrt{1-t}\right)^{-1} \tag{13}$$

at the limit when $t \to 1$, $(T \sim T_c)$, the equation (26) becomes:

$$\triangle \sim 3k_B T_c \sqrt{1-t} \tag{14}$$

the energy gap is very good agreement with these other results [13] where :

$$\triangle \sim 3.06 k_B T_c \sqrt{1-t} \tag{15}$$

## 4 Discussion

Energy-Time uncertainty relation determine the conditions on $t$ under which equations (13) and (14) are true.if we consider equation (9) as energy-time uncertainty relation $-2\triangle E\tau \sim \hbar$, then we must have $v \sim ev_0$ where $e \simeq 2.718$ this means that there are a critical region, using equation (12) we obtain $t \sim 0.92$ for $v \sim \xi^3$ and $v_0 \sim (\xi - \xi_0)^3$.

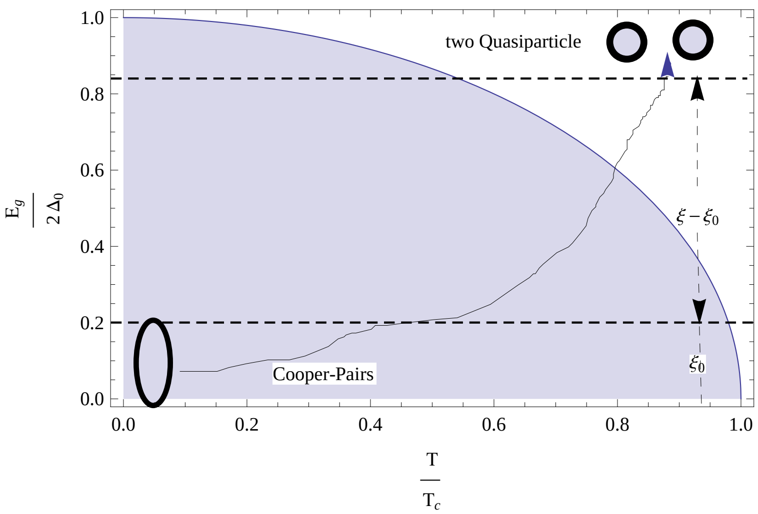


**Fig. 1** A schematic view to show the dimension order of two free volumes:$v \sim \xi^3$ where the breaking of cooper-pair in radius $\xi$ at energy $\sim 2\Delta(0)$, and $v_0 \sim (\xi - \xi_0)^3$ before the breaking.

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