A possible generic formula for Carmichael numbers

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Abstract. To find generic formulas for Carmichael numbers (beside, of course, the formula that defines them) was for long time one of my targets; I already found such a formula, based on Korselt's criterion; I possible found now another such a formula.

Conjecture:
Any Carmichael number can be written as (n^2*p^2 – q^2)/(n^2 – 1), where p and q are primes or power of primes or are equal to 1 and n is positive integer, n > 1.

The first Carmichael number, 561,
can be written as (4*p^2 – q^2)/3 for [p,q] = [29,41], [41,71], [7^2,89], [421,29^2]; it can also be written as (16*p^2 – q^2)/15 for [p,q] = [23,7], [29,71] etc.

The second Carmichael number, 1105,
can be written as (4*p^2 – q^2)/3 for [p,q] = [29,7], [31,23], [53,89], [59,103], [67,11^2], [829,1657]; it can also be written as (9*p^2 – q^2)/8 for [p,q] = [37,59], [7^2,113], [61,157] etc.

The third Carmichael number, 1729,
can be written as (4*p^2 – q^2)/3 for [p,q] = [37,17], [43,47], [67,113], [73,127], [103,193], [433,863], [1297,2593]; it can also be written as (9*p^2 – q^2)/8 for [p,q] = [43,53], [53,107], [67,163], [167,487], [1153,3457]; it can also be written as (16*p^2 – q^2)/15 for [p,q] = [41,31], [47,97], [97,353], [157,657], [173,673], [251,991]; it can also be written as (25*p^2 – q^2)/24 for [p,q] = [41,23], [61,227], [151,727], [347,1723] etc. (seems that the famous Hardy-Ramanujan number can set a record for how many ways can be written this way).

Few subsets of Carmichael numbers:
A subset of Carmichael numbers C has the following property: C = (4*p^2 – q^2)/3, where q is the smaller prime that verify the relation q > sqrt (3*C/4), and p is prime or a power of prime; few such numbers are:

1105, 1729, 6601, 41041, 75361, 340561, for corresponding [p,q] = [7,29], [17,37], [19,71], [71,179], [239,7^2], [509,11^4].

Another subset of Carmichael numbers C has the following property: C = (n^2*p^2 – 1)/(n^2 – 1), where p is the smaller prime that verify the relation p > sqrt (3*C/4); few such numbers are:
2465, 8911, 10585, 15841, 162401, for corresponding [n,p] = [2,43], [3,89], [3,97], [2,109], [2,349].

Another subset of Carmichael numbers C (but this time only related to the formula above) has the following property: C = (4*p^2 – 7153)/3, where p is prime; such numbers are: 561, 488881, for corresponding p = 47, 607 (interesting that 607 – 47 = 560 and 561 is the first Carmichael number).

Another subset of Carmichael numbers C (this time too only related to the formula above) has the following property: C = (p*q^2 – 1723^2)/(p – 1), where p and q are primes or power of primes; few such numbers are: 1105 for [p,q] = [1249,59], 1729 for [p,q] = [5^2,347], 2465 for [p,q] = [7^2,251].

Note: The formula based on Korselt’s criterion that I was talking about in Abstract is: C = p^k + n*p^2 - n*p (if C > p^k) or C = p^k - n*p^2 + n*p (if p^k > C) for any p prime divisor of C and any k natural number. See the sequence A213812 that I submitted to OEIS.