

Extended Electron in Constant Electric Field

Part 1 : The net electric force \mathbf{F}_e produced on the extended electron

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Abstract : When an extended electron is subject to an external constant electric field \mathbf{E} , the net electric force \mathbf{F}_e is produced on it . The analysis on the extended model of the electron shows that \mathbf{F}_e is the resultant of two opposite forces \mathbf{F} and \mathbf{F}' (i.e., $\mathbf{F}_e = \mathbf{F} + \mathbf{F}'$), where \mathbf{F} is the resultant of all elementary forces \mathbf{f}_e which are produced on *surface dipoles* of the electron ($\mathbf{F} = \sum \mathbf{f}_e$), and \mathbf{F}' is the electric force produced on *the core* ($-q_0$) of the electron . We will calculate \mathbf{f}_e , \mathbf{F} and \mathbf{F}' by applying boundary conditions on the surface of the extended electron . The effective electric charge Q of the electron will be deduced from the expression $\mathbf{F}_e = Q \mathbf{E}$. Part 2 discusses the radiation process of the extended electron in the electric field .

Keywords : electric dipoles , surface dipoles , the core , electric and magnetic boundary conditions , radiation by forces , radiation cone , conditions for radiation .

1. Introduction :

The readers are recommended to read the previous article ^{1(a)} : "A new extended model for the electron " to have a view on the *extended model* of the electron and the *assumptions for calculations* ; since all the calculations in this article will be based on this model and the assumptions on the electric and magnetic boundary conditions . In a nutshell , the extended model of the electron is a spherical composite structure consisting of the point-charged core ($-q_0$) which is surrounded by countless electric dipoles ($-q,+q$) as schematically shown by Fig.1 and other figures in this article .

Since this article " **Extended electron in constant electric field** " is rather long , it will be divided into two parts : Part 1 : Determination of the net electric force \mathbf{F}_e ,
Part 2 : The radiation of the extended electron in electric field .

To reduce the length of the main text , long calculations are put into the **appendices** , only the results of these calculations are shown in the main text . (Appendices A and B are placed at the end of part 2 , the readers can read them later if they want to) .

When a *point electron* of electric charge e is subject to an external electric field \mathbf{E} , the Lorentz force $\mathbf{F}_L = e \mathbf{E}$ is a single force ; i.e., it cannot be decomposed into simpler components . But for an *extended electron* , the net force \mathbf{F}_e is more complicated because it consists of countless point charges : the electric dipoles ($-q$, $+q$) and the core ($-q_0$) , and hence lots of single forces are produced on it when it is subject to the external field .

To determine the net electric force \mathbf{F}_e produced on the extended electron , we will first calculate the "elementary" force \mathbf{f}_e which is developed on a *surface dipole* , then the resultant $\mathbf{F} = \sum \mathbf{f}_e$ of all forces \mathbf{f}_e produced on all surface dipoles of the electron (section 2) . Then we will calculate the electric force \mathbf{F}' which is produced on the core ($-q_0$) of the electron (section 3) , and finally the net force $\mathbf{F}_e = \mathbf{F} + \mathbf{F}'$ (section 4) .

Section 5 discusses the variability of the relative permittivity ϵ of the extended electron .

Section 6 discusses the variability of the effective electric charge Q of the extended electron .

Section 7 discusses the equation of motion of the extended electron in the external electric field .

2. Determination of the elementary force \mathbf{f}_e produced on a surface dipole and the resultant $\mathbf{F} = \sum \mathbf{f}_e$.

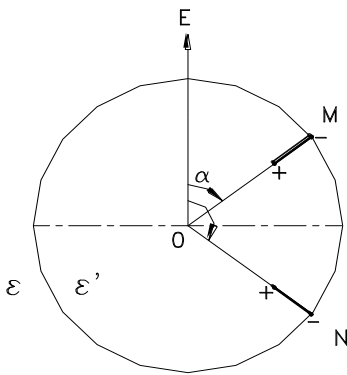
Let \mathbf{E} be the external electric field which exerts on the extended electron ; \mathbf{E} creates the electric field \mathbf{E}' inside the extended electron . Boundary conditions ^{1(a)} on the electric field allow us to determine the electric field \mathbf{E}' from the components of the external field \mathbf{E} which is supposed to be a known field . First , let us calculate two elementary electric forces \mathbf{fe} produced on *two arbitrary surface dipoles* M and N on the *upper and lower hemispheres* of the electron , respectively . (Fig.1)

For the surface dipole M on the upper hemisphere, the angle α is $0 < \alpha < \pi/2$ (Figs. 4 and 5) . The force \mathbf{fe} is the resultant of two forces \mathbf{fn} and $\mathbf{f'n}$ which are developed on two ends $-q$ and $+q$ of the dipole M , respectively . The calculations are shown in the **Appendix A** . The results are :

* when $\varepsilon < 1$ (Fig. 4) , $fe = f'n - fn = (1/\varepsilon - 1) q E \cos \alpha$, \mathbf{fe} is centrifugal , (1-a)

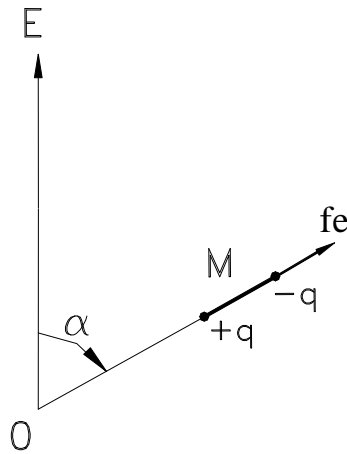
* when $\varepsilon > 1$ (Fig. 5) , $fe = fn - f'n = (1 - 1/\varepsilon) q E \cos \alpha$, \mathbf{fe} is centripetal , (1-b)

Where ε is *the relative permittivity* of the extended electron (ε') to the free space (ε_0) ; i.e., $\varepsilon = \varepsilon' / \varepsilon_0$.

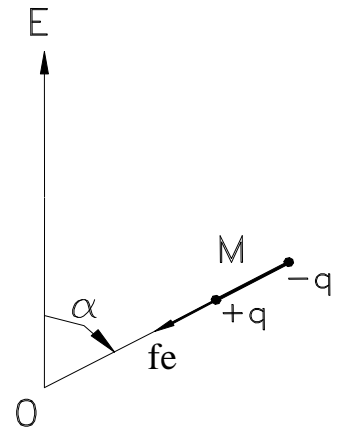


F.1 : Two surface dipoles M and N on the upper and lower hemispheres of the extended electron .

(Error : permittivity of free space outside the electron is ε_0 , not ε as shown in F.1).



F.4: When $\varepsilon < 1$: the resultant force \mathbf{fe} acting on dipole M is centrifugal .

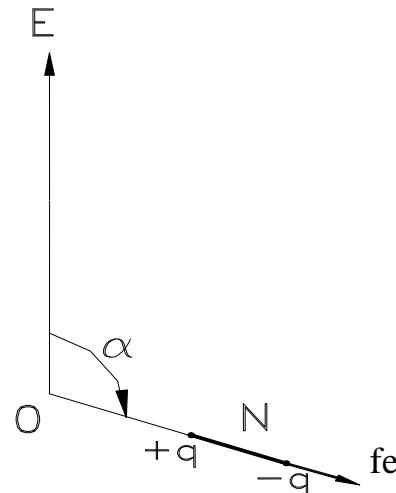
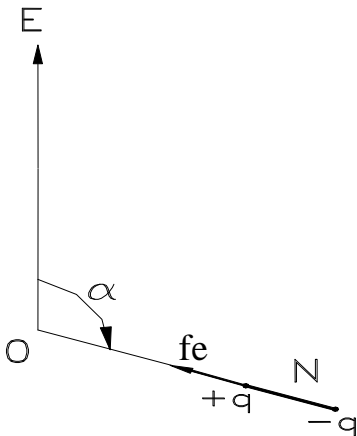


F.5 : When $\varepsilon > 1$: the resultant force \mathbf{fe} acting on dipole M is centripetal .

Now let us calculate the resultant force \mathbf{fe} acting on the surface dipole N on the *lower hemisphere* of the electron (Figs. 8 and 9) : $\pi/2 < \alpha < \pi$, (hence $\cos \alpha < 0$) (See **Appendix A**) . The results are :

* when $\varepsilon < 1$ (Fig. 8) : $fe = f'n - fn = (1/\varepsilon - 1) q E (-\cos \alpha)$, \mathbf{fe} is centripetal , (1-c)

* when $\varepsilon > 1$ (Fig. 9) : $fe = fn - f'n = (1 - 1/\varepsilon) q E (-\cos \alpha)$, \mathbf{fe} is centrifugal . (1-d)



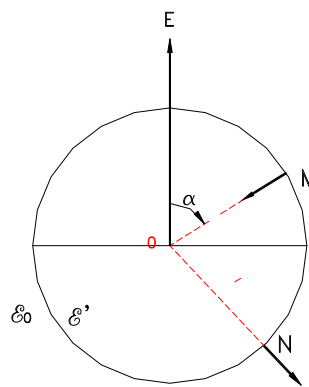
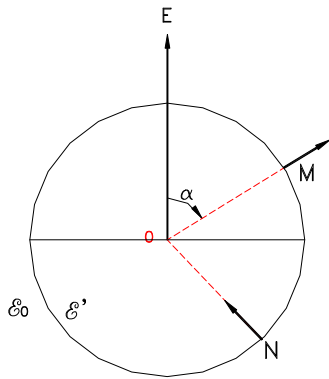
F.8 : When $\varepsilon < 1$: the resultant force \mathbf{fe} acting on dipole N is centripetal .

F.9 : When $\varepsilon > 1$: the resultant force \mathbf{fe} acting on dipole N is centrifugal .

Now let us combine the results from four figures above to show the directions of the elementary forces \mathbf{fe} produced on two surface dipoles M and N :

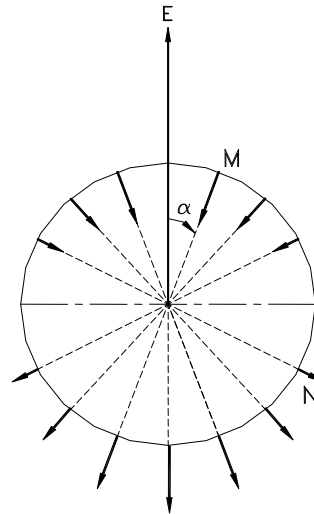
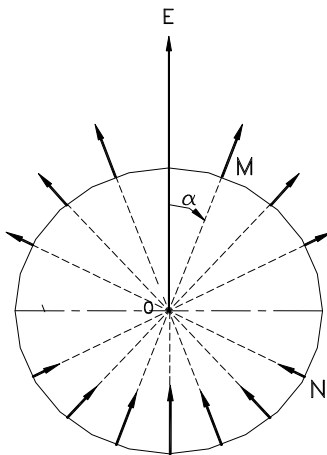
* when $\varepsilon < 1$: Fig. 10 shows two forces \mathbf{fe} at M and N ; and Fig.10-a (from Fig.10) shows all forces \mathbf{fe} on the upper and lower surface of the electron .

* when $\varepsilon > 1$: Fig. 11 shows two forces \mathbf{fe} at M and N ; and Fig.11-a (from Fig.11) showing all forces \mathbf{fe} on the upper and lower surface of the electron.



F.10 : When $\varepsilon < 1$:
 - on the upper hemisphere , \mathbf{fe} is centrifugal
 - on the lower hemisphere , \mathbf{fe} is centripetal

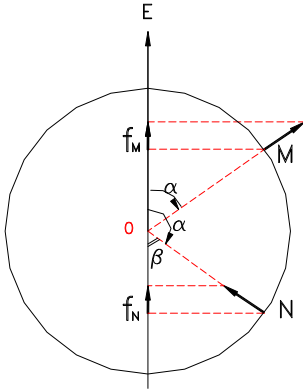
F.11 : When $\varepsilon > 1$:
 - on the upper hemisphere , \mathbf{fe} is centripetal
 - on the lower hemisphere , \mathbf{fe} is centrifugal



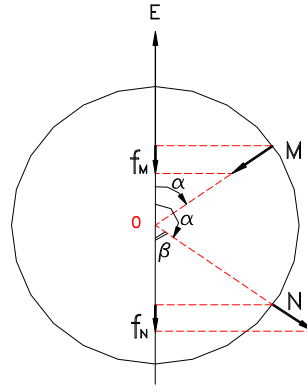
F.10-a (from F.10) When $\varepsilon < 1$:
 - on the upper hemisphere : all \mathbf{fe} are centrifugal
 - on the lower hemisphere : all \mathbf{fe} are centripetal

F.11-a (from F.11) When $\varepsilon > 1$:
 - on the upper hemisphere : all \mathbf{fe} are centripetal
 - on the lower hemisphere : all \mathbf{fe} are centrifugal

We have finished determining the magnitude and the direction of the elementary forces \mathbf{f}_e in two cases when $\varepsilon < 1$ and $\varepsilon > 1$. Now we will determine the direction and magnitude of the resultant force $\mathbf{F} = \sum \mathbf{f}_e$ which exerts on the surface dipoles of the extended electron in these two cases.



F.12: $\varepsilon < 1$:
Projections of \mathbf{f}_e at M and N onto the axis \mathbf{OE} : \mathbf{f}_M and \mathbf{f}_N point upwards .



F.13: $\varepsilon > 1$:
Projections of \mathbf{f}_e at M and N onto the axis \mathbf{OE} : \mathbf{f}_M and \mathbf{f}_N point downwards .

Because all forces \mathbf{f}_e are symmetric about the axis \mathbf{OE} (as shown in two Figs.10-a and 11-a), their resultant force $\mathbf{F} = \sum \mathbf{f}_e$ lies on \mathbf{OE} and is equal to the sum of all projections of \mathbf{f}_e onto \mathbf{OE} . So , to determine the magnitude of \mathbf{F} , we have to calculate first the projection of \mathbf{f}_e onto \mathbf{OE} .

Let \mathbf{f}_M be the projection of \mathbf{f}_e at M onto the axis \mathbf{OE} and \mathbf{f}_N , the projection of \mathbf{f}_e at N onto \mathbf{OE} .

When $\varepsilon < 1$ (Fig. 12), \mathbf{f}_M and \mathbf{f}_N point in the direction of \mathbf{E} ; their magnitudes are :

$$f_M = f_e \cos\alpha = (1/\varepsilon - 1) q E \cos\alpha \cos\alpha = (1/\varepsilon - 1) q E \cos^2\alpha \quad (1)$$

$$f_N = f_e \cos\beta = f_e (-\cos\alpha) = (1/\varepsilon - 1) q E (-\cos\alpha)(-\cos\alpha) = (1/\varepsilon - 1) q E \cos^2\alpha \quad (2)$$

The magnitudes of \mathbf{f}_M and \mathbf{f}_N in two Eqs. (1) and (2) have identical form but in Eq.(1) : $0 < \alpha < \pi/2$ and in Eq.(2) : $\pi/2 < \alpha < \pi$. So, $\mathbf{F} = \sum \mathbf{f}_e = \sum \mathbf{f}_M$ if α varies from 0 to π ; that is

$$\mathbf{F} = \sum_{i=1}^n (1/\varepsilon - 1) q E \cos^2\alpha_i = (1/\varepsilon - 1) q E \sum_{i=1}^n \cos^2\alpha_i \quad (3)$$

\mathbf{F} points in the direction of \mathbf{E} (Fig.14)

where n is number of surface dipoles on the surface of the electron ;

α_i is the angular position of a surface dipole , varying from 0 to π ;

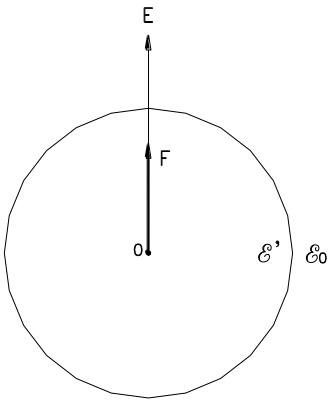
ε is the relative permittivity of the electron : $\varepsilon = \varepsilon' / \varepsilon_0$;

q is electric charge at the end of a surface dipole ;

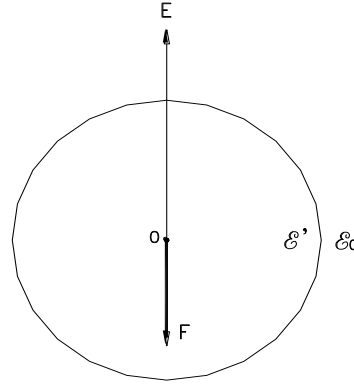
When $\varepsilon > 1$ (Fig. 13) \mathbf{f}_M and \mathbf{f}_N point in the opposite direction to \mathbf{E} , so the resultant $\mathbf{F} = \sum \mathbf{f}_e = \sum \mathbf{f}_M$ also points in the opposite direction to \mathbf{E} . The same reasoning as above gives

$$\mathbf{F} = \sum_{i=1}^n (1 - 1/\varepsilon) q E \cos^2\alpha_i = (1 - 1/\varepsilon) q E \sum_{i=1}^n \cos^2\alpha_i \quad (4)$$

\mathbf{F} points in the opposite direction to \mathbf{E} (Fig.15)

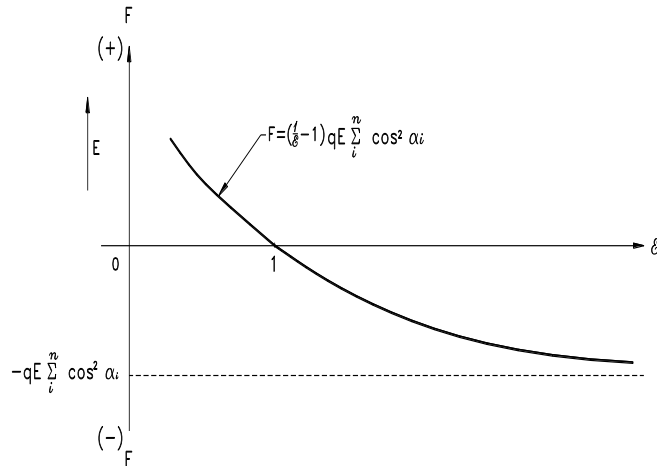


F.14 : $\varepsilon < 1$: the resultant force $\mathbf{F} (= \Sigma \mathbf{f}_e)$ points in the direction of \mathbf{E} .



F.15 : $\varepsilon > 1$: the resultant force $\mathbf{F} (= \Sigma \mathbf{f}_e)$ points in the opposite direction to \mathbf{E} .

In two equations (3) and (4) , the sum $\sum_{i=1}^n \cos^2 \alpha_i$ depends on the number n of surface dipoles and the angular position α_i of the surface dipole , they represent the *physical structure* of the electron . The magnitude and direction of \mathbf{F} thus depend on ε as shown in Eqs. (3) & (4) and two Figs.14 & 15. The magnitude and direction of \mathbf{F} (with respect to the direction of \mathbf{E}) is graphically shown in Fig.16 We have finished determining the magnitude and direction of the resultant $\mathbf{F} (= \Sigma \mathbf{f}_e)$.



F.16 : The smooth curve shows the *magnitude and direction* of \mathbf{F} versus ε : Eq. (3)

- for $\varepsilon < 1$: \mathbf{F} is positive ; i.e., $\mathbf{F} \uparrow \uparrow \mathbf{E}$ as shown by F. 14
- for $\varepsilon > 1$: \mathbf{F} is negative ; i.e., $\mathbf{F} \downarrow \uparrow \mathbf{E}$ as shown by F. 15

From two equations (3) and (4) we note that \mathbf{F} exists only if $\varepsilon \neq 1$; this means that the absolute permittivity ε' of the extended electron is different from the permittivity ε_0 of the surrounding free space ($\varepsilon' \neq \varepsilon_0$) ; that is, they are two different media . Otherwise, if $\varepsilon = 1$, then $\varepsilon' = \varepsilon_0$ and $\mathbf{F} = 0$; this physically means that the electron has no electric dipoles around its core ; i.e., only the core exists : this is a point electron . Therefore , mathematically , the point electron is a particular case of the extended electron when $\varepsilon = 1$. For the extended electron $\varepsilon < 1$ as we will see below .

3. Determination of the net electric force produced on all interior dipoles and the electric force \mathbf{F}' produced on the core ($-q_0$) of the electron

3.1 The net electric force produced on all interior dipoles is zero .

The external electric field \mathbf{E} produces the field \mathbf{E}' inside the extended electron ; all *interior dipoles* and *the core* ($-q_0$) are thus subject to the field \mathbf{E}' . Because the dipole is extremely small , two ends $-q$, $+q$ of an interior dipole are exerted upon by the same field \mathbf{E}' and hence two electric forces \mathbf{f}_e and \mathbf{f}'_e are *equal and opposite* . The dipole may be slightly re-oriented (or polarized) but the resultant force $\mathbf{f}_e + \mathbf{f}'_e$ produced on the dipole is cancelled out . This explanation applies to all interior dipoles , which are possibly polarized by the field \mathbf{E}' , but the net electric force produced on all interior dipoles is zero .

3.2 Determination of the electric force \mathbf{F}' produced on the core ($-q_0$) of the electron .

The core is a point charge ($-q_0$) which is subject to the field \mathbf{E}' . To determine the electric force \mathbf{F}' produced on the core , we have to calculate field \mathbf{E}' first . The calculation is shown in **Appendix B** . It shows that the electric field \mathbf{E}' is parallel to the external field \mathbf{E} and has magnitude equal to :

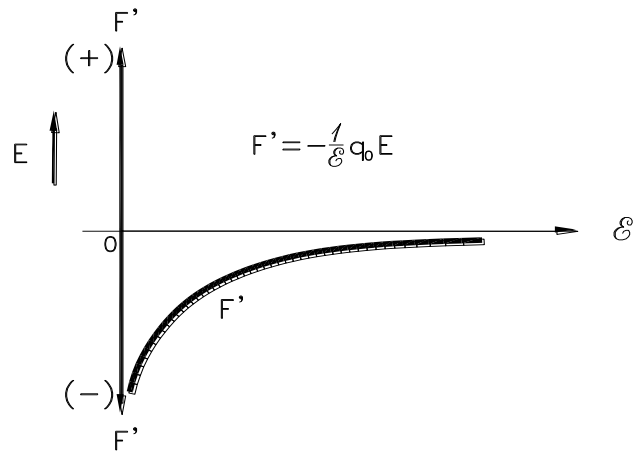
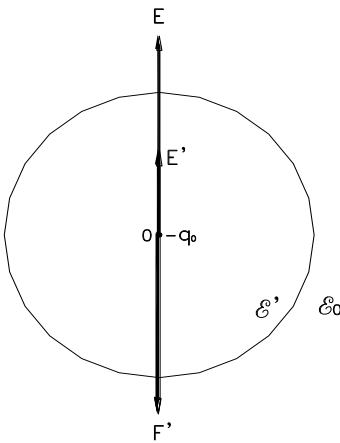
$$\mathbf{E}' = (1/\varepsilon) \mathbf{E} \quad (12)$$

So , the electric force \mathbf{F}' produced on the core ($-q_0$) is

$$\mathbf{F}' = -q_0 \mathbf{E}' = - (1/\varepsilon) q_0 \mathbf{E} \quad (13)$$

\mathbf{F}' is thus always negative ; i.e., it always points in the opposite direction to \mathbf{E} (Fig.17) .

Fig.18 shows the variation of \mathbf{F}' vs ε according to Eq.(13) .



F.17 : \mathbf{E} is the external electric field ;
 \mathbf{E}' is the electric field produced on the core ($-q_0$),
 \mathbf{F}' is the electric force produced on the core ,
it is always negative ; i.e., $\mathbf{F}' \downarrow \uparrow \mathbf{E}$.

F.18 : The curve represents the magnitude
and direction of the electric force \mathbf{F}' versus ε .
 \mathbf{F}' is always negative ; i.e., $\mathbf{F}' \downarrow \uparrow \mathbf{E}$.

4. Calculation of the net electric force \mathbf{F}_e produced on the extended electron

The net electric force \mathbf{F}_e which is developed on the extended electron when it is subject to a constant electric field \mathbf{E} is the sum $\mathbf{F}_e = \mathbf{F} + \mathbf{F}'$.

\mathbf{F} is given by Eq.(3) and \mathbf{F}' by Eq.(13). Since \mathbf{F} and \mathbf{F}' are collinear, their algebraic sum is

$$\mathbf{F}_e = \left(\frac{1}{\varepsilon} - 1 \right) q E \sum_i^n \cos^2 \alpha_i - \frac{1}{\varepsilon} q_0 E \quad (14)$$

or

$$\mathbf{F}_e = \left[\left(\frac{1}{\varepsilon} - 1 \right) (q/q_0) \sum_i^n \cos^2 \alpha_i - \frac{1}{\varepsilon} \right] q_0 E \quad (15)$$

Let's set $a \equiv (q/q_0) \sum_i^n \cos^2 \alpha_i$, (it is a factor inside the above square bracket) (16)

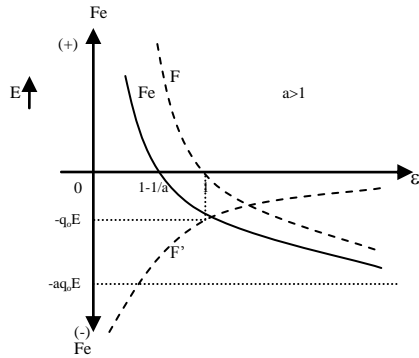
'a' is thus a dimensionless positive number since q , q_0 and $\sum_i^n \cos^2 \alpha_i$ are positive numbers;

Eq.(15) becomes

$$\mathbf{F}_e = \left(\frac{a-1}{\varepsilon} - a \right) q_0 \mathbf{E} \quad (\text{Fig. 19}) \quad (17)$$

Note : If we insert 'a' from (16) into the expression of \mathbf{F} in Eq.(3), we get a simpler expression for \mathbf{F} :

$$\mathbf{F} = \left(\frac{1}{\varepsilon} - 1 \right) a q_0 \mathbf{E}, \text{ in which the parameter 'a' absorbs the charge } q \text{ and the sum } \sum_i^n \cos^2 \alpha_i.$$



$$\mathbf{F} = \left(\frac{1}{\varepsilon} - 1 \right) a q_0 \mathbf{E} \quad , \quad (\text{F.16})$$

$$\mathbf{F}' = - (1/\varepsilon) q_0 \mathbf{E} \quad , \quad (\text{F.18})$$

$$\mathbf{F}_e = \mathbf{F} + \mathbf{F}' = \left(\frac{a-1}{\varepsilon} - a \right) q_0 \mathbf{E} \quad , \quad (\text{F.19})$$

F.19 : $\mathbf{F}_e = \mathbf{F} + \mathbf{F}'$.

For $\varepsilon > 1$: \mathbf{F} and \mathbf{F}' are both negative ; \mathbf{F}_e is always negative , i.e., \mathbf{F}_e points in opposite direction to \mathbf{E}

For $1 - 1/a < \varepsilon < 1$: \mathbf{F}_e is negative and tends to zero as $\varepsilon \rightarrow 1 - 1/a$

For $\varepsilon < 1 - 1/a$: \mathbf{F}_e becomes positive ; i.e., \mathbf{F}_e points in the direction of \mathbf{E} : the extended electron behaves like a positron .

We have thus calculated the net electric force \mathbf{F}_e that is produced on the extended electron when it is subject to the external electric field \mathbf{E} . The magnitude and direction of \mathbf{F}_e depend on two parameters ε and 'a' ; their actual (or physical) intervals of variation are discussed in the section 5 below .

We notice that for a *point electron* $\epsilon = 1$, $\mathbf{F} = 0$, $\mathbf{F}' = -q_0 \mathbf{E}$, and hence $\mathbf{F}\mathbf{e} = \mathbf{F} + \mathbf{F}' = -q_0 \mathbf{E}$. This is the familiar value of the electric force produced on the point electron: $\mathbf{F}\mathbf{e} = e \mathbf{E}$.

But for an *extended electron* the magnitude of $\mathbf{F}\mathbf{e}$ is modified by the factor $(\frac{a-1}{\epsilon} - a)$ which depends

on two parameters 'a' and ϵ . And thus the electric force $\mathbf{F}\mathbf{e}$ does not vary linearly with \mathbf{E} (as the Lorentz classical equation $\mathbf{F}\mathbf{e} = e \mathbf{E}$).

The parameter 'a', as defined by the expression (16), characterizes *the physical structure* of the electron, and depends on q , q_0 , n and α_i

- . q , the magnitude of electric charge on the end of a dipole;
- . q_0 , the magnitude of electric charge of the core;
- . n , total number of surface dipoles on the surface of the electron;
- . α_i , representing the angular distribution of the surface dipole i on the spherical surface of the electron; the angle α varies from 0 to π .

5. The actual (physical) intervals of variation of ϵ and a

Let us examine Fig.19 which shows the variation of the magnitudes of \mathbf{F} , \mathbf{F}' and their sum $\mathbf{F}\mathbf{e}$ versus ϵ . If $\epsilon = 1$, $\mathbf{F} = 0$, $\mathbf{F}' = -q_0 \mathbf{E}$, $\mathbf{F}\mathbf{e} = \mathbf{F} + \mathbf{F}' = -q_0 \mathbf{E} = \mathbf{F}'$: the net force $\mathbf{F}\mathbf{e}$ is thus reduced to the force \mathbf{F}' which is produced on the core of the electron; this means that when $\epsilon = 1$ the extended electron is mathematically equivalent to a point charge which is the core (as already noted at the end of section 2).

So, for the extended electron, $\epsilon \neq 1$; that is, either $\epsilon > 1$ or $\epsilon < 1$.

For $\epsilon > 1$, both \mathbf{F} and \mathbf{F}' are negative, and hence $\mathbf{F}\mathbf{e}$ is always negative. This means that the net electric force $\mathbf{F}\mathbf{e}$ produced on the electron is always different from zero, and as a consequence, from the Newton's second law of motion, the acceleration is always different from zero; and hence the velocity of the electron will increase to infinity with time: this is the runaway behaviour. Since the velocity of any particle must be limited to c , so we reject the interval of variation of $\epsilon > 1$

For $\epsilon < 1$, \mathbf{F} becomes positive while \mathbf{F}' is always negative; $\mathbf{F}\mathbf{e} = \mathbf{F} + \mathbf{F}'$: (Fig.19)

- * $\mathbf{F}\mathbf{e} < 0$ in the interval $(1 - 1/a) < \epsilon < 1$: $\mathbf{F}\mathbf{e}$ points in the opposite direction to \mathbf{E} ; this means that the extended electron behaves like a real electron.
- * $\mathbf{F}\mathbf{e} > 0$ for $\epsilon < (1 - 1/a)$: $\mathbf{F}\mathbf{e}$ points in the direction of \mathbf{E} ; i.e., the extended electron behaves like a positive electron or a **positron**, so we reject this interval.

Thus we accept the interval $(1 - 1/a) < \epsilon < 1$, for which $\mathbf{F}\mathbf{e}$ is negative, it accelerates the electron in the opposite direction to the external field \mathbf{E} . This interval will be written shortly as $\epsilon < 1$.

Figs. 4, 8, 10, 10-a, 12, and 14 satisfy this condition ($\epsilon < 1$). They show the directions of the elementary forces $\mathbf{f}\mathbf{e}$ and the resultant \mathbf{F} ($= \sum \mathbf{f}\mathbf{e}$): \mathbf{F} is upward (positive).

Fig. 17 shows \mathbf{F}' always downward (negative). Since \mathbf{F} and \mathbf{F}' vary with ϵ , when $\epsilon = 1 - 1/a$, $\mathbf{F} = -\mathbf{F}'$ and $\mathbf{F}\mathbf{e} = \mathbf{F} + \mathbf{F}' = 0$ (Fig.19): the electron reaches the speed limit c .

We note that the speed limit c is a *postulate of the special theory of relativity*: it has no physical interpretation. But here we can explain the physical reason why the extended electron's speed tends to c : when it is accelerated by an external electric field \mathbf{E} , the magnitudes of two opposite forces \mathbf{F} and \mathbf{F}' tend to be equal when $\epsilon = 1 - 1/a$, and the resultant force $\mathbf{F}\mathbf{e} \rightarrow 0$. From the Newton's second law of motion: if $\mathbf{F}\mathbf{e} = m \, d\mathbf{v}/dt \rightarrow 0$, then $d\mathbf{v}/dt \rightarrow 0$ and \mathbf{v} tends to a constant that must be c because the electron is being accelerated. So, the variability of two opposite forces \mathbf{F} and \mathbf{F}' (with ϵ) helps explain why the speed limit c is reached.

In short, since ϵ varies in the interval $(1 - 1/a) < \epsilon < 1$, we come to the following results:

- The correction factor $\left(\frac{a-1}{\epsilon} - a\right)$ of the electric force \mathbf{F}_e in Eq.(17) varies from -1 to 0 .
- The magnitude of the net electric force \mathbf{F}_e varies from $-q_0E$ to 0.
- The velocity of the electron varies with ϵ and tends to c as $\epsilon \rightarrow 1-1/a$.
- Since the relative permittivity ϵ is a positive number , $\epsilon \rightarrow 1-1/a$ means that $1-1/a > 0$ i.e., '**a**' is greater than unity (**a > 1**) .

Four remarks related to the values of ϵ :

1. The positron . In this article we only investigate the properties of the electron (not the positron) ; this is when ϵ varies in the interval $(1-1/a) < \epsilon < 1$, where $a > 1$. In the following , this interval is written shortly as $\epsilon < 1$. From Fig.19 we notice that when $\epsilon < 1-1/a$, the electron is accelerated in the direction of the applying field \mathbf{E} ; i.e., the electron behaves like a positron . This transition suggests that e^- and e^+ can be transformed from one to the other if its relative permittivity ϵ is being changed around the value $(1-1/a)$ where $\mathbf{F} = -\mathbf{F}'$, $\mathbf{F}_e = 0$ and its speed is about c .

[For comparison , let us recall that Dirac revealed from his relativistic equation of the electron that the electron can have two energy states : positive and negative . He wrote in his Nobel lecture ²:
"if we disturb the electron, we may cause a transition from a positive-energy state of motion to a negative-energy one ... ; they correspond to the motion of an electron with a positive charge instead of the usual negative one - what the experimenters now call a positron" . Thus , for Dirac , e^- and e^+ are not independent particles since e^- can be transformed to e^+ by a disturbance (he did not specify what kind of disturbance) ; in other words , the positron has its origin from the electron] .

The calculations of forces on the extended electron lead to Fig.19 which shows the variation of the net electric force \mathbf{F}_e with ϵ ; it is the change of ϵ about the value $(1-1/a)$ that transforms e^- to e^+ and vice versa , so the disturbance here is the abrupt change of ϵ . Moreover , the positron is not created alone, but together with the electron in the pair production $e^- - e^+$. This suggests that the positron may have the origin from the electron .

2. Why $\epsilon < 1$?

We have come to the result $\epsilon < 1$ such that the velocity of the electron is limited to c when it is accelerated in \mathbf{E} ; (otherwise , if $\epsilon \geq 1$, it will runaway ; i.e., its velocity tends to infinity) . Moreover , when $\epsilon < 1$, \mathbf{F}_e is negative , it accelerates the electron in the opposite direction to the applying field \mathbf{E} , this is a feature of the real electron .

This value of relative permittivity $\epsilon < 1$ is specific to the "material" of the extended electron because common materials such as water , glass , mica ... all have $\epsilon \geq 1$; e.g., for air $\epsilon = 1$, water (81) , glass (5-10) , mica (6) and **no material with $\epsilon < 1$** is mentioned or listed in the current textbook ³ . This may be because the extended electron is composed of *electric dipoles* meanwhile common materials are composed of *atoms or molecules* .

3. The orientation (or polarization) of electric dipoles

Now from Eq. (12): $\mathbf{E}' = (1/\epsilon) \mathbf{E}$, since $\epsilon < 1$, $\mathbf{E}' > \mathbf{E}$. This means that when an external electric field \mathbf{E} is applied on the extended electron , it creates an electric field \mathbf{E}' parallel and stronger inside the electron . The reason for this is that the applying field \mathbf{E} polarizes all electric dipoles inside the electron , giving rise to the field \mathbf{E}' greater than \mathbf{E} . This polarization causes the permittivity ϵ of the electron to vary in the interval $(1-1/a) < \epsilon < 1$. We can say that ϵ is a measure of the sensibility of the structure of the electron to the applying field \mathbf{E} .

Since electric dipoles contain both charges – and + , the polarization of the ensemble of electric dipoles in the extended electron creates an *electric dipole moment* for the extended electron ^{4,5} . A point electron with only negative charge cannot have an electric dipole moment .

4. The narrow interval of variation of the relative permittivity ϵ

The parameter ‘ a ’ is a dimensionless positive number defined by the expression (16) :

$$a \equiv (q / q_0) \sum_i^n \cos^2 \alpha_i$$

The magnitude of ‘ a ’ increases with the number ‘ n ’ of *surface dipoles* because $\sum_i^n \cos^2 \alpha_i$ is the sum of n positive terms ($\cos^2 \alpha_i$). For an extended electron , the number of surface dipoles is expected to be large ($n \gg 1$); and hence , ‘ a ’ is also expected to be a large number ($a \gg 1$). If $a \gg 1$, then $1/a$ is an infinitesimal number , and hence ($1 - 1/a$) is infinitesimally less than 1 ; therefore , the interval $(1 - 1/a) < \epsilon < 1$ becomes a very narrow interval ($1 - \epsilon < 1$). And hence , we can say that ϵ is approximately constant and equal to 1- , ($\epsilon \approx 1-$). (This approximation is needed for calculations in the **Appendix C**).

We conclude that when the extended electron is subject to the external field \mathbf{E} , its permittivity ϵ varies in the narrow interval $(1 - 1/a < \epsilon < 1)$ which causes the net electric force \mathbf{F}_e to change in the interval $(-q_0 E , 0)$ and the effective electric charge Q of the extended electron varies in the interval $(- q_0 , 0)$ as we see below .

6. The variability of the electric charge of the extended electron

From Eq.(17) which gives the net electric force \mathbf{F}_e

$$\mathbf{F}_e = \left(\frac{a-1}{\epsilon} - a \right) q_0 \mathbf{E} \quad (\text{shown in Fig.19})$$

the effective electric charge Q of the electron can be deduced as

$$Q = \left(\frac{a-1}{\epsilon} - a \right) q_0 \quad a > 1 \quad (18)$$

From Fig.19 , for the electron (\mathbf{F}_e is negative) , ϵ varies in the interval $(1 - 1/a) < \epsilon < 1$ and hence Q is negative and varies in the interval $(- q_0 , 0)$.

To find the correspondence between the velocity v with ϵ and Q , let us recall that on page 6 of the previous article entitled ^{1(b)} “ **A Foundational Problem in Physics : Mass versus Electric Charge** ” , we have obtained the general expression for the effective electric charge of the electron when it is subject to an external field :

$$Q = - (1 - v^2 / c^2)^{N/2} q_0 \quad (19)$$

where $N \geq 0$ is a real number representing the applying electric or magnetic field .

A minus sign is added on the right hand side of Eq.(19) to indicate the negative charge Q of the electron . From (18) and (19) we get

$$\frac{a-1}{\varepsilon} - a = - (1 - v^2/c^2)^{N/2} \quad (20)$$

or
$$\varepsilon = (a-1) / [a - (1 - v^2/c^2)^{N/2}] \quad (21)$$

So , when $v \ll c$, $\varepsilon \approx 1$, $Q \approx -q_0$ and $F_e \approx -q_0 E$ (equivalent to a point electron) and as $v \rightarrow c$, $\varepsilon \rightarrow 1 - 1/a$, $Q \rightarrow 0$ and $F_e \rightarrow 0$.

So , when it moves at $v \ll c$, the extended electron is mathematically equivalent to a point electron (in terms of electric charge and force , both are constant) . But as $v \rightarrow c$ it behaves differently from the point electron : its effective charge Q and the net force F_e change with velocity , i.e., with time , while it is accelerated in the electric field E .

We note that Eq.(20) provides a relationship between three parameters 'a' , ε and the velocity v of the electron . The **Appendix C** shows that this equation can suggest a way to relate the radiation and absorption of light of the extended electron to its velocity ; that is , when the electron radiates , it slows down ; and when it absorbs light , it speeds up .

7. Equation of motion of the extended electron in external electric field E

Equation of motion of the extended electron in the form of the Newton's 2nd law of motion is

$$m \mathbf{v}' = \mathbf{F}_e = \mathbf{F} + \mathbf{F}' = \left(\frac{a-1}{\varepsilon} - a \right) q_0 \mathbf{E} \quad \text{from Eq.(17)} \quad (22)$$

or
$$m \mathbf{v}' = - (1 - v^2/c^2)^{N/2} q_0 \mathbf{E} \quad \text{from Eq.(20)} \quad (23)$$

where $\mathbf{v}' (= d^2x/dt^2)$. Hence , the equation (23) is a second order differential equation which obeys the Newton's first law of motion (law of inertia) and avoids the problems of runaway and preacceleration .

We should note that the **radiation reaction force F_{rad}** (which is the recoil force produced by the radiation of an accelerating point charge) **does not exist in the radiation of the extended electron** . The force F' which is produced on the core ($-q_0$) , is not the radiation reaction force produced by the radiation of the extended electron .

Meanwhile the equation of motion of the *point electron* (Lorentz-Abraham-Dirac equation) including the radiation reaction force F_{rad} is a third order differential equation

$$m \mathbf{v}' = \mathbf{F}_{ext} + \mathbf{F}_{rad} \quad (24)$$

where $F_{rad} = 2q^2 \mathbf{v}'' / 3c^3$ (in cgs) ; $\mathbf{v}'' (= d^3x/dt^3)$ is the second time derivatives of the velocity \mathbf{v} . Eq.(24) does not obey the law of inertia and is involved in many problems including the runaway and preacceleration⁶ .

Summary and conclusion

More than 100 years ago , many physicists had proposed various extended model for the electron : Lorentz (1904) , Poincaré (1906) ... All these models contained only negative charges .

The new extended model contains both types of charges (+ and -) in its volume. **It is a version^{1(a)} of the image of the screened electron** by vacuum polarization such that calculations can be performed on it to determine its effective electric charge . We have come to the following results :

When the electron is subject to an external electric field \mathbf{E} , two opposite electric forces \mathbf{F} and \mathbf{F}' are produced on it : \mathbf{F} is the resultant of all elementary forces \mathbf{fe} , \mathbf{F}' is produced on the core ($-q_0$). The net force $\mathbf{F}_e = \mathbf{F} + \mathbf{F}'$ accelerates the electron in the opposite direction to \mathbf{E} , varies with velocity and tends to zero as $v \rightarrow c$. The effective electric charge Q changes with the velocity and tends to zero as $v \rightarrow c$.

The investigation on this extended model leads us to following ideas :

- 1/ The concept of *screened electron by vacuum polarization*^{1(a)} should be abandoned since it does not provide any form of calculation to determine the effective electric charge of the electron. The virtual pairs of (e^- , e^+) are fictitious, they must not have real effect on the electron. Whereas electric dipoles or photons are real particles (as conceived by Feynman), which constitute the real part of the electron around its central core ($-q_0$) instead of those fictitious pairs of (e^- , e^+).
- 2/ The old-fashioned *concept of mass varying with velocity*^{1(b)} should be abandoned and replaced by the *concept of varying effective charge* which is derived from the new extended model of the electron.
- 3/ The extended model of the electron can help explore more experimental properties of the electron, such as its radiation and spin in external electric and magnetic fields.

Part 2 : Radiation of the extended electron in constant electric field .

1. Introduction : radiation by forces

Radiation process is a tough topic to discuss because of the dual nature of light and the unknown structure of the electron that emits light. In this part 2, the extended model of the electron^{1(a)} will be used in the discussion of the radiation process. As for light, it is assumed that the electron emits light, and light may be **particles** (called photons), as conceived by **Feynman*** or it may be a **chunk of self-sustaining field**** which detaches from the electron and travels through space, as conceived by the classical theory of radiation.

In this article and subsequent ones, we choose to consider **light as particles** which are identified as tiny needles carrying two opposite charges $-q$ and $+q$ at their two ends; these "**electric dipoles**" form the outer part of the extended electron^{1(a)}. When the electron emits its electric dipoles into the surrounding space, we say it is radiating.

The following sections will introduce the readers to *a novel way of explaining the radiation process : this is radiation by forces*.

2. Radiation of the extended electron by electric forces

Before discussing the radiation process and defining the conditions for radiation, let us review two electric forces \mathbf{fe} and \mathbf{G} which are produced on *surface dipoles* of the extended electron when it is subject to an external constant electric field \mathbf{E} . These two forces will help explain the radiation process. First, the elementary forces \mathbf{fe} produced on all surface dipoles are shown in F.10-a when $\epsilon < 1$: \mathbf{fe} are centrifugal on the upper hemisphere and centripetal on the lower hemisphere; their magnitude is

* **Feynman** : " I want to emphasize that light comes in this form – particles . It is very important to know that light behaves like particles , especially for those of you who have gone to school , where you were probably told something about light behaving like waves . I'm telling you the way it does behave – like particles . " (Optics , E.Hecht , p.138)

** " If the point charge is subjected to a sudden acceleration caused by some external force , then pieces of the electric and magnetic fields break away from the point charge and propagate outward as a self-supporting electromagnetic wave pulse . " (Classical Electrodynamics , 1988 , H. C. Ohanian , p. 411)

$$f_e = (1/\epsilon - 1) q E \cos \alpha, \quad \epsilon < 1, \quad 0 < \alpha < \pi$$

Second are the cohesive forces \mathbf{G} which attract all surface dipoles toward the core of the electron ; i.e., they are centripetal and produced by the self-field \mathbf{E}_0 of the electron ; their magnitude is

$$G = [(1/\epsilon) - 1] q E_0, \quad \text{Eq.(16)}$$

which had been calculated in Fig.19 of the article ^{1(a)} "A New Extended Model for the Electron ".

These two forces f_e and \mathbf{G} are shown together in Fig.20 and Fig.21 below .

On the upper hemisphere, while f_e are centrifugal , the cohesive forces \mathbf{G} are centripetal . So , if the magnitude of f_e is stronger than that of \mathbf{G} ($f_e > \mathbf{G}$) , these surface dipoles can break away from the surface of the electron ; that is, the electron emits these electric dipoles upwards . This means that the electron radiates upwards , from the upper hemisphere , while the electron is moving downwards in the opposite direction to the applying field \mathbf{E} .

Meanwhile , on the lower hemisphere, since both f_e and \mathbf{G} are centripetal , these surface dipoles cannot break free from the surface of the electron ; that is , there is no radiation from the lower hemisphere

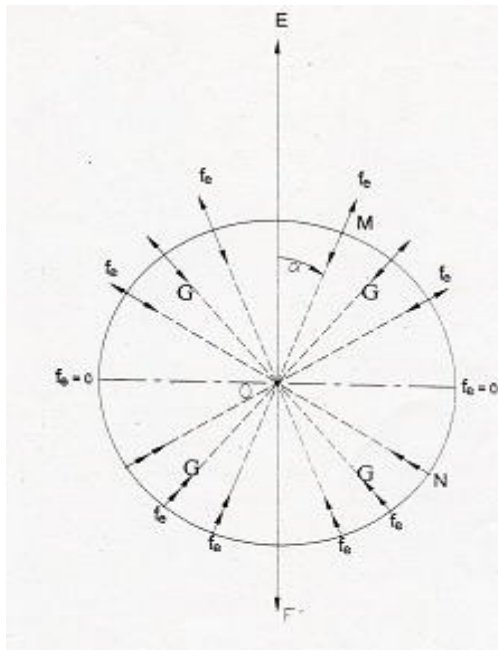
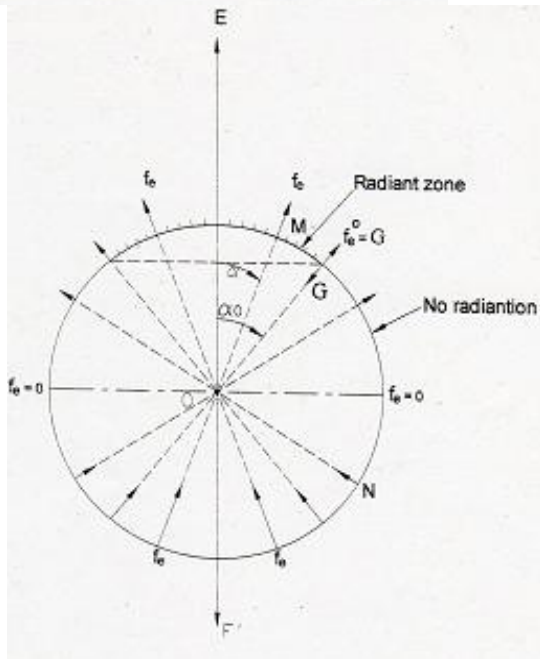


Fig. 20 shows when $\epsilon < 1$:

- electric forces f_e : centrifugal on the upper hemisphere and centripetal on the lower hemisphere ,
- cohesive forces \mathbf{G} : all are centripetal ,
- electric force \mathbf{F}' acting on the core ($-q_0$) ,
- the direction of the applying field \mathbf{E} .



F.21 shows when $\epsilon < 1$:

the radiant zone limited by the angle α_0 where $f_e^0 = \mathbf{G}$.

The cone of radiation is upward (in the direction of \mathbf{E}) while the electron moves downwards .

In short , the extended electron does not radiate from its entire surface , but only from a limited zone around the north pole , on the upper hemisphere of the electron as shown in Fig.21 .
In the following , we will determine this radiant zone around the north pole and define the conditions for radiation of the extended electron in the electric field \mathbf{E} .

3. The cone of radiation - Conditions for radiation .

Now let us determine the radiant zone on the upper hemisphere as shown in Fig.21 .

Let us recall that the magnitude of \mathbf{f}_e produced on a surface dipole M of the upper hemisphere has been calculated in part 1 (Fig.4) ; \mathbf{f}_e is centrifugal , its magnitude is

$$f_e = \left(\frac{1}{\varepsilon} - 1 \right) q E \cos \alpha \quad , \quad \text{where } \varepsilon < 1 \quad \text{and} \quad 0 < \alpha < \pi/2 \quad (25)$$

\mathbf{f}_e thus depends on the angular position α of the surface dipole .

$$\text{Let } \alpha_0 \text{ be the value of the angle } \alpha \text{ at which } \mathbf{f}_e^0 = \mathbf{G} \quad (\text{in magnitude}) \quad (26)$$

then the condition $f_e > G$ (for the radiation to occur as stated above)
is rewritten as $f_e > f_e^0$

$$\text{or} \quad \left(\frac{1}{\varepsilon} - 1 \right) q E \cos \alpha > \left(\frac{1}{\varepsilon} - 1 \right) q E \cos \alpha_0 \quad (27)$$

$$\text{or} \quad \cos \alpha > \cos \alpha_0$$

$$\text{or} \quad \alpha < \alpha_0 \quad (28)$$

Fig.21 shows *the radiant zone* on the upper hemisphere , restricted in a zone around the north pole of the electron , limited by the angle α_0 which is defined by the relation (26).

All rays radiating from the radiant zone are contained inside *a radiation cone* of half- angle α_0 at the vertex O . So , the existence of the angle α_0 means the existence of the radiation cone .

Now let us determine the condition for the existence of the angle α_0 . From (26) we have

$$\left(\frac{1}{\varepsilon} - 1 \right) q E \cos \alpha_0 = G \quad \text{where } \mathbf{E} \text{ is the applying electric field .}$$

The magnitude of \mathbf{G} has been calculated to be equal to

$$G = \left(\frac{1}{\varepsilon} - 1 \right) q E_0 \quad \text{where } \mathbf{E}_0 \text{ is the self field of the electron .}$$

$$\text{Hence } \left(\frac{1}{\varepsilon} - 1 \right) q E \cos \alpha_0 = \left(\frac{1}{\varepsilon} - 1 \right) q E_0 \quad \text{or} \quad \boxed{\cos \alpha_0 = E_0 / E < 1} \quad (29)$$

$$\text{The condition for the radiation to occur is thus} \quad \boxed{E > E_0} \quad (30)$$

Therefore ,

- i) if $E < E_0$, $\cos \alpha_0 > 1$; α_0 does not exist : no radiation emits from the electron while it is accelerated in \mathbf{E} ; this also means that in free space ($\mathbf{E} = 0$) the electron cannot radiate .
- ii) if $E = E_0$, $\cos \alpha_0 = 1$, $\alpha_0 = 0$: the radiation cone reduces to a ray (or a beam) of light emanating from the north pole of the electron .
- iii) if $E > E_0$, $\cos \alpha_0 < 1$, $0 < \alpha_0 < \pi/2$: radiation occurs around the north pole of the

electron ; a radiation cone emanates from the electron as shown in Fig.21 .

- iv) if $E \rightarrow \infty$, $\cos \alpha_0 \rightarrow 0$, $\alpha_0 \rightarrow \pi/2$: the radiant zone expands to cover the entire upper hemisphere (this is only a limit case since E can never tend to infinity).

Thus $E > E_0$ is the condition for the existence of the angle α_0 (which implies the condition for the emission of radiation). So, while it is accelerated in E , the extended electron can or cannot radiate * · ** depending on the strength of the applying field E compared to that of its self- field E_0 , **but not on its acceleration** ***.

We note that in the case i) $E < E_0$: the electron is accelerated by the applying field E but it does not radiate because $\cos \alpha_0 > 1$ (i.e., α_0 does not exist) ; therefore, the radiation of the extended electron does not depend on its acceleration .

(The **Larmor 's formula** links the power radiated ($P = 2q^2 a^2 / 3c^3$) by a radiating *point electron* to its acceleration ; this formula does not applied to the extended electron).

Following are statements of three eminent physicists about the radiation and acceleration :

- * **Pearle**⁷ : “ *A point charge must radiate if it accelerates , but the same is not true of an extended charge distribution .* ”
- ****Jackson**⁸ : “ *Radiation is emitted in ways that are obscure and not easily related to the acceleration of a charge .* ”
- *** **Feynman** : “ *We have inherited a prejudice that an accelerating charge should radiate .* ”

We also note that the **radiation reaction force** F_{rad} (which is the recoil force produced by the radiation of a radiating point electron) **does not exist in the radiation of the extended electron** .

The force F' produced on the core (- q_0) , although pointing in the opposite direction to the radiation , *is not produced* by the radiation of the extended electron .

As mentioned in section 7 of part 1 , if the radiation reaction force F_{rad} is included in the equation of motion of the electron , the equation becomes a third order differential equation ; this is Lorentz-Abraham-Dirac equation for the point electron which is involved in such problems as runaway and preacceleration .

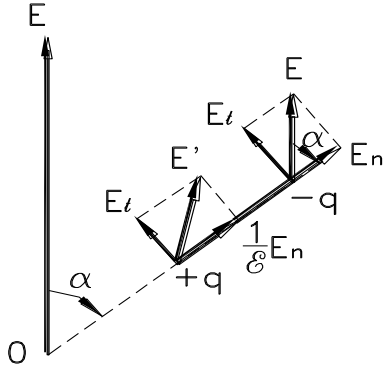
Conclusion

This article presents the theory on a new extended model of the electron ^{1(a)} , it reveals some features which are quite different from those of the point electron .

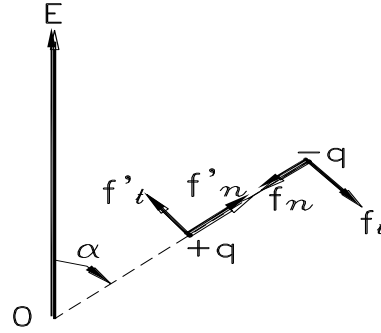
In part 1 , the net electric force F_e developed on the extended electron is not a single force , but the resultant of two opposite forces F and F' ; F_e and the effective electric charge Q change with velocity of the electron and tend to zero as $v \rightarrow c$.

Part 2 introduces a novel way of explaining the radiation process of the extended electron : radiation by electric forces . When the extended electron is subject to an external field E which is stronger than the self-field E_0 of the electron , the electron radiates in the direction of E by the elementary forces f_e that emanate from the upper hemisphere of the extended electron (Fig. 21) ; the radiation does not depend on the acceleration of the electron and produces no radiation reaction force .

Appendix A : Calculation of the elementary force f_e



F.2 : Normal and tangential components of \mathbf{E} and \mathbf{E}' on two ends of surface dipole M



F.3 : Normal and tangential components of the electric forces on two ends of surface dipole M

First , we use boundary conditions to determine the electric field \mathbf{E}' inside the electron , then we calculate the electric force f_e .

Fig. 1 shows two arbitrary surface dipoles M and N : M on the upper hemisphere, N on the lower hemisphere .

Boundary conditions applied on dipole M give (Fig. 2)

$\mathbf{E} = \mathbf{E}_n + \mathbf{E}_t$ acting on the negative charge $-q$ of the surface dipole M ,

$\mathbf{E}' = \mathbf{E}'_n + \mathbf{E}'_t = (1/\epsilon) \mathbf{E}_n + \mathbf{E}_t$ acting on the positive charge $+q$,

where $E_n = E \cos \alpha$, $0 < \alpha < \pi/2$ and $E_t = E \sin \alpha$.

Since $\mathbf{E}_t = \mathbf{E}'_t$, two tangent forces f_t and f'_t produced on two ends $-q$ and $+q$ of dipole M are equal and opposite (Fig. 3) : $f_t = f'_t = q E_t = q E \sin \alpha$.

Two normal forces f_n and f'_n have magnitudes

$f_n = q E_n = q E \cos \alpha$, f_n is centripetal ,

$f'_n = q E'_n = (1/\epsilon) q E_n = (1/\epsilon) q E \cos \alpha$, f'_n is centrifugal .

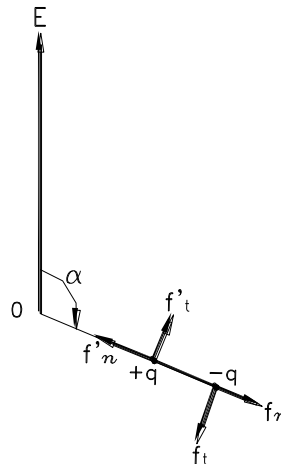
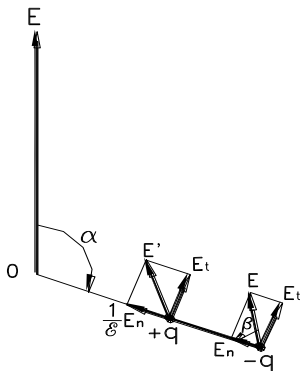
Since the dipole is extremely small , these forces can be considered as if they apply on the same point on the dipole M . Their resultant force f_e acting on the dipole M is thus

$f_e = f_n + f'_n + f_t + f'_t = f_n + f'_n$ since f_t and f'_t are equal and opposite .

Since f_n and f'_n are in opposite directions, the magnitude and direction of f_e depend on ϵ :

* when $\epsilon < 1$ (Fig. 4) , $f'_n > f_n$, hence : $f_e = f'_n - f_n = (1/\epsilon - 1) q E \cos \alpha$, f_e is centrifugal , (1-a)

* when $\epsilon > 1$ (Fig. 5) , $f_n > f'_n$, hence : $f_e = f_n - f'_n = (1 - 1/\epsilon) q E \cos \alpha$, f_e is centripetal , (1-b)



F.6 : Normal and tangential components of \mathbf{E} and \mathbf{E}' on two ends of surface dipole N

F. 7 : Normal and tangential components of the electric forces on two ends of surface dipole N

Now we calculate the resultant force \mathbf{fe} acting on the dipole N , on the lower hemisphere of the electron : $\pi/2 < \alpha < \pi$, (hence $\cos \alpha < 0$), < Fig. 6 & Fig.7 .

Since $\alpha + \beta = \pi$, $E_n = E \cos \beta = -E \cos \alpha$ and $E_t = E \sin \beta = E \sin \alpha$.

Boundary conditions give $\mathbf{Et} = \mathbf{E}'t$; so, two tangent forces \mathbf{ft} and $\mathbf{f}'t$ produced on two ends $-q$ and $+q$ of the dipole N are equal and opposite : $f_t = f'_t = q E \sin \alpha$.

Two normal forces \mathbf{fn} and $\mathbf{f}'n$ have magnitudes

$f_n = q E_n = q E (-\cos \alpha)$, \mathbf{fn} is centrifugal,

$f'_n = q E'_n = (1/\epsilon) q E (-\cos \alpha)$, $\mathbf{f}'n$ is centripetal.

Since $\cos \alpha < 0$, $(-\cos \alpha)$ is a positive number; the resultant force \mathbf{fe} acting on the dipole N is

* when $\epsilon < 1$ (Fig. 8): $f_e = f'_n - f_n = (1/\epsilon - 1) q E (-\cos \alpha)$, \mathbf{fe} is centripetal, (1-c)

* when $\epsilon > 1$ (Fig. 9): $f_e = f_n - f'_n = (1 - 1/\epsilon) q E (-\cos \alpha)$, \mathbf{fe} is centrifugal. (1-d)

We have used four expressions (1-a), (1-b), (1-c) and (1-d) in the determination of \mathbf{fe} in the main text .

Appendix B : Determination of the force \mathbf{F}' produced on the core of the electron

The core is a point charge ($-q_0$) subject to the field \mathbf{E}' . Because of the spherical symmetry of the structure of the of the electron, \mathbf{E}' at the core must be parallel to the external field \mathbf{E} . And hence we can write

$$\mathbf{E}' = k \mathbf{E} \quad (5)$$

where k is a positive number different from 1 : $0 < k \neq 1$.

k is positive because \mathbf{E}' is parallel to \mathbf{E} ; $k \neq 1$ because if $k = 1$, Eq.(5) becomes $\mathbf{E}' = \mathbf{E}$; and this means that the electric field inside the electron is independent of the material of the electron. But since the extended electron has a permittivity (ϵ') different from that of the surrounding free space (ϵ_0), it is reasonable to think that \mathbf{E}' depends on the medium of the electron; i.e., \mathbf{E}' differs from \mathbf{E} in magnitude or $k \neq 1$.

Now let's apply boundary conditions to two points on both sides of the interface : M on the surface of the electron and the center O

$$\text{At } M : \mathbf{E} = \mathbf{E}_n + \mathbf{E}_t \quad (6)$$

$$\text{At } O : \mathbf{E}' = (1/\epsilon) \mathbf{E}_n + \mathbf{E}_t \quad (7)$$

Equations (5) and (6) give

$$\mathbf{E}' = k \mathbf{E} = k \mathbf{E}_n + k \mathbf{E}_t \quad (8)$$

Comparing (7) and (8) we get (9) and (10)

$$(1/\epsilon) \mathbf{E}_n = k \mathbf{E}_n \rightarrow k = 1/\epsilon \neq 1 \text{ and} \quad (9)$$

$$\mathbf{E}_t = k \mathbf{E}_t \rightarrow (k - 1) \mathbf{E}_t = 0 \rightarrow \mathbf{E}_t = 0 \text{ since } k - 1 \neq 0 \quad (10)$$

$$\text{Since } \mathbf{E}_t = 0, \text{ Eq.(6) becomes } \mathbf{E} = \mathbf{E}_n \quad (11)$$

$$\text{and Eq.(7) becomes } \boxed{\mathbf{E}' = (1/\epsilon) \mathbf{E}_n = (1/\epsilon) \mathbf{E}} \quad (12)$$

So , the electric force \mathbf{F}' produced on the core ($-q_0$) is

$$\boxed{\mathbf{F}' = -q_0 \mathbf{E}' = - (1/\epsilon) q_0 \mathbf{E}} \quad (13)$$

\mathbf{F}' is thus always negative ; i.e., it always points in the opposite direction to \mathbf{E} (Fig.17) .

Fig.18 shows the variation of \mathbf{F}' vs ϵ according to Eq.(13) .

Note : The above results $\mathbf{E}t = 0$ (in Eq.10) and $\mathbf{E} = \mathbf{E}n$ (in Eq.11) imply that the point M must be located at either the north pole ($\alpha = 0$) or the south pole ($\alpha = \pi$) of the electron because $E_t = E \sin\alpha = 0$, hence $\sin\alpha = 0$. This result comes from the fact that the electron is spherically symmetric : \mathbf{E} and \mathbf{E}' are collinear (12) and normal (11) to the spherical surface only at the north and south poles of the electron . If M takes an arbitrary position on the surface of the electron , we cannot solve the equations of boundary conditions for the field \mathbf{E}' at the core O .

We have used two Eqs. (12) and (13) in the determination of \mathbf{F}' in the main text .

Appendix C : Radiation and absorption of light (or photons)

In part 2 we maintained that light is composed of particles called photons which are identified as electric dipoles that form the outer part of the electron ^{1(a)} . In the determination of the net electric force \mathbf{F}_e , we defined the parameter 'a' by the expression (16)

$$a \equiv (q/q_0) \sum_i^n \cos^2 \alpha_i , \quad (16)$$

which represents the structure of the extended electron , in which n is the number of surface dipoles . The parameter 'a' is thus a positive dimensionless number which increases monotonically with the

number n of surface dipoles because $\sum_i^n \cos^2 \alpha_i$ is the sum of n positive terms $\cos^2 \alpha_i$.

This expression suggests the idea that the radiation and the absorption of light of the extended electron are linked to n and 'a' in the following manner :

- * when the electron radiates , it emits its surface dipoles into space ; this means that the number n (of surface dipoles) decreases , and thus the parameter 'a' decreases accordingly ,
- * when the electron absorbs photons (e.g., by irradiation) , it collects dipoles onto its surface ; this means that n increases , and thus the parameter 'a' increases accordingly .

As mentioned on page 11 (section 6 , part 1) that the equation (20)

$$\frac{a-1}{\epsilon} - a = - (1 - v^2/c^2)^{N/2} \quad (20)$$

can suggest a way to relate the radiation and absorption of light of the extended electron to its velocity ; that is , when the electron radiates , it slows down ; and when it absorbs light , it speeds up . To prove this statement , let us extract v^2/c^2 from Eq.(20)

$$v^2/c^2 = 1 - [a - (a-1)/\epsilon]^{2/N}$$

As mentioned on page 10 that since $a \gg 1$, $\epsilon \approx 1 -$ (infinitesimally less than 1); so if ϵ is considered approximately constant, we can take the derivative of v^2/c^2 with respect to 'a':

$$d(v^2/c^2)/da = (2/N) [(1/\epsilon) - 1] [a - (a-1)/\epsilon]^{(2N)-1}$$

For $N > 0$; $a \gg 1$ and $\epsilon \approx 1 -$ or $(1-1/a) < \epsilon < 1$, we have

$(2/N) > 0$; $[(1/\epsilon) - 1] > 0$; $[a - (a-1)/\epsilon]^{(2N)-1} > 0$ for all values of the

exponent $(2/N) - 1$, therefore we get $\boxed{d(v^2/c^2)/da > 0}$

This means that v^2/c^2 and 'a' increase monotonically, that is

* when the electron radiates: 'a' decreases, and v^2/c^2 decreases as well; this means that the electron slows down; i.e., it loses its (kinetic) energy. (For example, in the synchrotron accelerator, electrons radiate and lose their energy).

* when the electron absorbs photons: 'a' increases, and v^2/c^2 increases as well; this means that the electron speeds up; i.e., it gains energy. (In the photoelectric effect, electrons absorb photons via irradiation and gain energy).

These results are not new findings, they are already known in the classical theory of radiation. What is new here is that they can be deduced from a few mathematical expressions which are derived from the extended model of the electron ^{1(a)}.

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