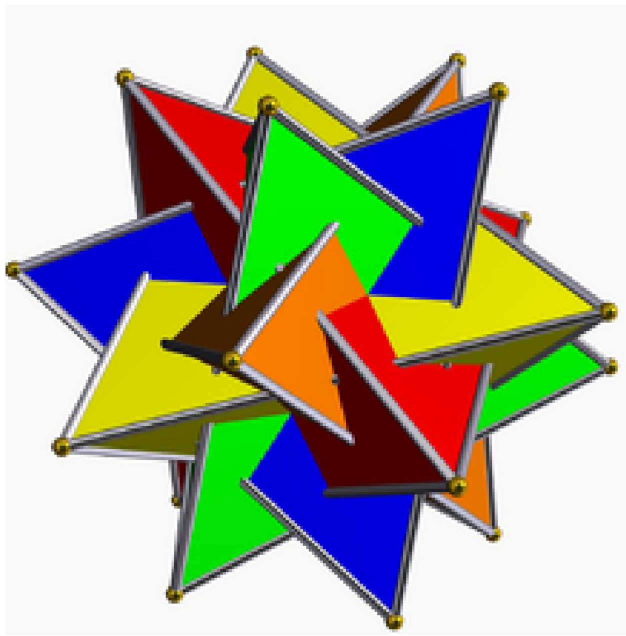
Exceptional Isomorphisms in the Qi Men Dun Jia Cosmic Board Model



By John Frederick Sweeney

Abstract

In math, especially within the Lie Algebra groups, there exist a small group of "exceptional isomorphisms" or "accidental isomorphisms." Following the lead of Swiss psychologist Carl Jung, the author does not accept the existence of "exceptional" or "accidental;" instead, these are merely phenomena which heretofore have not been explained satisfactorily by mathematical theory. However, articulation of the Qi Men Dun Jia Cosmic Board Model in a recent and forthcoming paper has helped to explain several of the exceptional isomorphisms.

Exceptional Isomorphisms

The great Swiss psychologist Carl Jung did not believe in accidents or coincidences, nor does the present author. There exists in mathematics a category of exceptional isomorphisms, primarily having to do with Lie Algebras.

In the view of the present author, they have been described as exceptional because no previous mathematician could provide an explanation for their existence: math theory lacked the tools and vision to account for these isomorphisms, and so described them as exceptional or accidental, like freaks of nature or an eccentric uncle in the family attic.

The present paper and its immediate predecessor go to some lengths to account for these exceptional isomorphisms. Specifically, the Qi Men Dun Jia Cosmic Board model helps to explain (from Wikipedia entry):

The exceptional isomorphisms between the series of finite simple groups mostly involve projective special linear groups and alternating groups, and are:[1]

- $L_2(4) \cong L_2(5) \cong A_5$, the smallest non-abelian simple group (order 60);
- $L_2(7) \cong L_3(2)$, the second-smallest non-abelian simple group (order 168) – PSL(2,7);

Groups of Lie type

In addition to the aforementioned, there are some isomorphisms involving SL, PSL, GL, PGL, and the natural maps between these. For example, the groups over $\mathbf{F}_5$ have a number of exceptional isomorphisms:

- $\mathrm{PSL}(2,5) \cong A_5 \cong I$, the alternating group on five elements, or equivalently the icosahedral group;
- $\mathrm{PGL}(2,5) \cong S_5$, the symmetric group on five elements;
- $\mathrm{SL}(2,5) \cong 2 \cdot A_5 \cong 2I$, the double cover of the alternating group A_5, or equivalently the binary icosahedral group.

As well as:

groups

Finite simple groups

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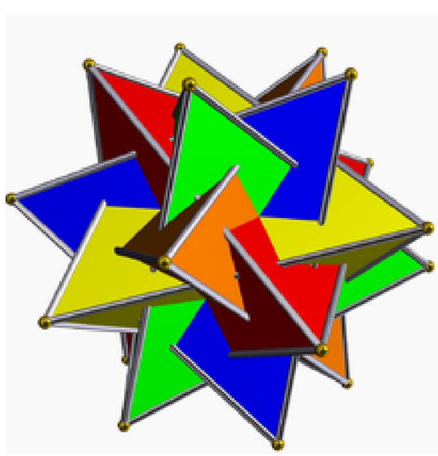
- $L_2(7) \cong L_3(2)$, the second-smallest non-abelian simple group (order 168) – PSL(2,7);
- $L_2(9) \cong A_6$,
- $L_4(2) \cong A_8$,
- $\mathrm{PSU}_4(2) \cong \mathrm{PSp}_4(3)$, between a projective special orthogonal group and a projective symplectic group.



The compound of five tetrahedra expresses the exceptional isomorphism between the icosahedral group and the alternating group on five letters.

The Qi Men Dun Jia Cosmic Board Model accomplishes these explanations by incorporating the icosahedron and its related concepts as a central feature of the model, in order to accomodate key aspects of Chinese metaphysics, namely, the 60 Jia Zi and the 60 Na Yin.

This series of papers has just begun, as the first paper charts the course of this expedition into mathematical physics. It remains highly probable that continued pursuit of this line of inquiry will lead to insights about the remaining "exceptional isomorphs."

Appendix I

The complete Wikipedia entry on these exceptional isomorphs follows, as of 30 June 2013.

Groups of Lie type[edit]

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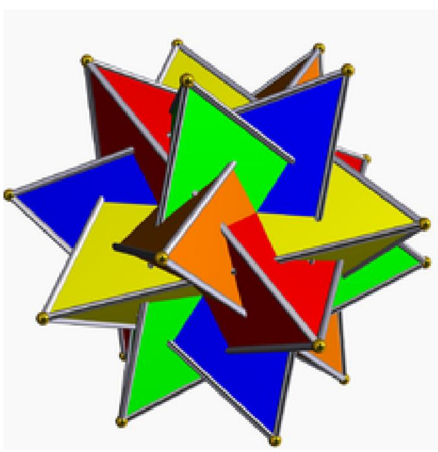
Alternating groups and symmetric groups[edit]



The compound of five tetrahedra expresses the exceptional isomorphism between the icosahedral group and the alternating group on five letters.

There are coincidences between alternating groups and small groups of Lie type:

- $L_2(4) \cong L_2(5) \cong A_5,$
- $L_2(9) \cong A_6,$
- $L_4(2) \cong A_8,$
- $O_6(2) \cong S_8.$

These can all be explained in a systematic way by using linear algebra (and the action of S_n on affine n-space) to define the isomorphism going from the right side to the left side. (The above isomorphisms for A_8 and S_8 are linked via the exceptional isomorphism $SL_4/\mu_2 \cong SO_6$.) There are also some coincidences with symmetries of regular polyhedra: the alternating group A5 agrees with the icosahedral group (itself an exceptional object), and the double cover of the alternating group A5 is the binary icosahedral group.