Fitting the ^{6}Li, ^{7}Li, ^{7}Be and ^{8}Be Binding Energies to High Precision based Exclusively on the Up and Down Quark Masses

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We extend the results of an earlier recent letter by expressing the ^{6}Li, ^{7}Li, ^{7}Be and ^{8}Be binding energies, each independently and each to about parts-per-million or small parts-per-100,000 accuracy, exclusively as a function of the up and down current quark masses.

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1. Introduction

In a recent letter [1] the author showed based on the Koide mass formula [2], [3] how a "Koide matrix" K defined as:

$$K_{AB} \equiv \begin{pmatrix} \sqrt{m_1} & 0 & 0 \\ 0 & \sqrt{m_2} & 0 \\ 0 & 0 & \sqrt{m_3} \end{pmatrix} \tag{1.1}$$

with the up and down quark mass assignments $m_1 = m_d$, $m_2 = m_3 = m_u$ for a proton $K_{P\,AB}$ and $m_1 = m_u$, $m_2 = m_3 = m_d$ for a neutron $K_{N\,AB}$ (see (3) in [1]) can be used to formulate relationships accurate to about parts-per-million AMU for the binding (and related fusion-release) energies of the ^{2}H, ^{3}H, ^{3}He and ^{4}He (1s shell) light nuclides as well as for the neutron minus proton mass difference. Specifically, for ^{4}He (observed $0.030376586499\ \mathrm{u}$), ^{3}He (observed $0.008285602824\ \mathrm{u}$), ^{3}H (observed $0.009105585412\ \mathrm{u}$) and ^{2}H (observed $0.002388170100\ \mathrm{u}$) respectively, it was reviewed in (25) through (28) of [1] that the following close retrodictions can be made using only up and down quark masses (1.12) and (1.13) infra:

$$^4He: 2\cdot\Delta E_P + 2\cdot\Delta E_N - 2\sqrt{m_u m_d} = 0.030373002032\ \mathrm{u}\ , \tag{1.2}$$

$$^3He: 2m_u + \sqrt{m_u m_d} = 0.008320783890\ \mathrm{u}\ , \tag{1.3}$$

$$^3H: 4m_u - 2\sqrt{m_\mu m_d} / (2\pi)^{\frac{3}{2}} = 0.009099047078\ \mathrm{u}\ , \tag{1.4}$$

$$^2H: m_u = 0.002387339327\ \mathrm{u}\ , \tag{1.5}$$

where $\Delta E_P \equiv \mathrm{Tr}K_P{}^2 - (\mathrm{Tr}K_P)^2/(2\pi)^{1.5}$ and $\Delta E_N \equiv \mathrm{Tr}K_N{}^2 - (\mathrm{Tr}K_N)^2/(2\pi)^{1.5}$ are given in eqs. (12) and (13) of [1] by (with numeric energy values below updated here to reflect the "recalibration" of the up and down quark masses in (22) of [1]):

$$\Delta E_P = m_d + 2m_u - \left(m_d + 4\sqrt{m_u m_d} + 4m_u\right)/\left(2\pi\right)^{1.5} = 0.008200606481 \text{ u}, \tag{1.6}$$

$$\Delta E_N = m_u + 2m_d - \left(m_u + 4\sqrt{m_u m_d} + 4m_d\right)/\left(2\pi\right)^{1.5} = 0.010531999770 \text{ u}. \tag{1.7}$$

Other (recalibrated) relationships reported in [1] are as follows: For energy E released during the fusion reaction $p + p \to {}^2_1H + e^+ + \nu + \text{Energy}$ (observed $0.000451141003\,\text{u}$), see (17) of [1]:

$$2\sqrt{m_\mu m_d} \,/\, \left(2\pi\right)^{\frac{3}{2}} = 0.000450310230\, u. \tag{1.8}$$

For the energy E released during the fusion reaction ${}^2_1H + p \to {}^3_1H + e^+ + \nu + \text{Energy}$ (observed $0.004780386215\,\text{u}$), see (18) of [1]:

$$2m_u = 0.004774678654\,\text{u}. \tag{1.9}$$

For the *neutron minus proton mass difference* $M_N - M_P$ (postulated exact), see (19) of [1]:

$$m_u - \left(3m_d + 2\sqrt{m_\mu m_d} - 3m_u\right)/\left(2\pi\right)^{\frac{3}{2}} \equiv 0.001388449188\,\text{u} = M_N - M_P. \tag{1.10}$$

Among the up and down quark and electron masses (postulated exact), see (9) of [1]:

$$3\left(m_d - m_u\right)/\left(2\pi\right)^{1.5} \equiv m_e = 0.000548579909 \text{ u}. \tag{1.11}$$

And, for recalibrated up and down masses, based on (1.10) and (1.11), see (23) and (24) of [1]:

$$m_u = 0.002387339327 \text{ u}, \tag{1.12}$$

$$m_d = 0.005267312526 \text{ u}. \tag{1.13}$$

While these results originated from theoretical rationales in four recent papers [4], [5], [6], [7], the more recent letter [1] strictly reported these objective numeric relationships among phenomenological masses and energies based solely on Koide matrices of the form (1.1). The author's forbearance from theoretical discussions was intended to enable others in the nuclear and particle physics communities to evaluate these results based on the data alone, and perhaps develop modified or alternative theories as to the physics which might be underlying these very accurate empirical retrodictions.

The author continues this work in this letter by developing similarly accurate relationships for the 2s shell nuclides ^{6}Li, ^{7}Li, ^{7}Be and ^{8}Be, on empirical grounds, with minimal if any theoretical discussions as to the meaning of these results except as is necessary for their immediate derivation. The results reported below are new; they have *not* been previously reported anywhere else.

2. Mass / Energy Relationships for the ^{6}Li, ^{7}Be and ^{7}Li Nuclides

The first nuclide we consider is ^{6}Li. In doing so, we observe, for example from [8], that there are no stable nuclides with $A=Z+N=5$. One $A=5$ candidate for possible stability, ^{5}He, has a half-life of 700(30)×10^{-24} s and immediately sheds the extra neutron decay into the ^{4}He alpha. The other candidate, ^{5}Li, has a half-life of 370(30)×10^{-24} s and sheds the extra proton to decay into the ^{4}He alpha. If we seek stability, the lightest stable nuclide in the 2s shell is ^{6}Li.

Let us therefore now consider the process ${}^4_2He+2p\rightarrow {}^6_3Li+e^{+}+\nu+\text{Energy}$ whereby one fuses an alpha particle with two protons in order to create a stable ^{6}Li nuclide plus a positron and neutrino. The energy released during this hypothetical fusion event is:

$$\text{Energy}={}^4_2M+2M_p-{}^6_3M-m_e=0.002033478\text{ u}, \tag{2.1}$$

where ${}^4_2M=4.001506179125\text{ u}$ is the observed nuclear weight of the ^{4}He alpha, $M_p=1.007276466812\text{ u}$ is the observed proton mass, ${}^6_3M=6.013477055\text{ u}$ is the observed ^{6}Li nuclear weight, and the electron mass is given in (1.11).

It was reported in [1] that m_u, m_d and $\sqrt{m_u m_d}$, which are the nine non-zero components of the outer products $K_P\otimes K_P=K_{P\,AB}K_{P\,CD}$ and $K_N\otimes K_N=K_{N\,AB}K_{N\,CD}$, as well as the foregoing divided by the natural number $(2\pi)^{1.5}$, are the "energy numbers" based exclusively on the up and down quarks masses that we need to look to, to try to fit the binding and fusion energy data. We again do the same here. It is readily determined that:

$$9\sqrt{m_u m_d}\,/\,(2\pi)^{1.5}=0.002026396\text{ u}, \tag{2.2}$$

is extremely close to (2.1), differing by a mere 7.08153×10^{-6} u, that is, about 7 parts per million AMU. Might this be a "significant" relationship, and not merely a close coincidence?

Here, we need to be cautious. The question is whether coefficient "9" in (2.2) has some physical significance in relation to the Koide matrix (1.1) and/ or the physical properties of the "target nuclide" ^{6}Li, which we are presently considering, and is not merely a fortuitous coincidence. Of course, (1.1) is a 9 component matrix, and its outer products have exactly 9 non-zero components. But the significance of the coefficient "9" is more physically-direct when we consider that ^{6}Li contains exactly 9 up quarks and 9 down quarks. That is, "9" is the number of *up/down quark pairs* contained in a ^{6}Li nuclide. So if (2.2) is in fact a theoretical expression to 7 parts per million for the energy released to fuse an alpha with two protons into a ^{6}Li, then this would mean that in order to bind together the ^{6}Li nuclide, each of the nine up/down quark pairs in the target ^{6}Li nuclide has to give up $1\cdot\sqrt{m_u m_d}\,/\,(2\pi)^{1.5}$ "dose" of energy. This suggests that perhaps "9" is not a random number but makes some physical sense.

So let us provisionally hypothesize that (2.2) correctly gives the fusion-release energy for the reaction (2.1), by writing:

$$\text{Energy}\left({}^{4}_{2}He+2p\rightarrow {}^{6}_{3}Li+e^{+}+\nu+\text{Energy}\right)={}^{4}_{2}M+2M_{p}-{}^{6}_{3}M-m_{e}\equiv 9\sqrt{m_{u}m_{d}}\,/\left(2\pi\right)^{1.5}. \qquad (2.3)$$

As noted, this is accurate to about 7 parts per million.

Now, having "built" a ^{6}Li nuclide, let us consider the hypothetical isomeric fusion process ${}^{6}_{3}Li+p\rightarrow {}^{7}_{4}Be+\text{Energy}$ whereby a ^{6}Li nuclide is fused with a proton to produce a ^{7}Be nuclide. For this event, the energy released is:

$$\text{Energy}={}^{6}_{3}M+M_{p}-{}^{7}_{4}M=0.006018011721\text{ u}. \qquad (2.4)$$

where we use the empirical values ${}^{6}_{3}M=6.013477055\text{ u}$, ${}^{7}_{4}M=7.014735510362\text{ u}$, and the proton mass ${}^{6}_{3}M=6.013477055\text{ u}$.

Comparing to our restricted set of ingredients m_u, m_d and $\sqrt{m_u m_d}$ and these divided by $\left(2\pi\right)^{1.5}$, we find that:

$$18m_{d}\,/\left(2\pi\right)^{1.5}=0.006019934830\text{ u}. \qquad (2.5)$$

This differs from (2.4) by $1.92310833848\times10^{-6}$ u, or just under 2 parts per million. What might be the significance of the coefficient "18," to be certain that these are not just coincidental integer multiples? Here, ^{6}Li, which is now the "source nuclide to" which we wish to add a proton, contains 18 quarks in total. So (2.5) may be explained on the basis that each of the 18 quarks inside of a ^{6}Li nuclide has to give up an energy "dosage" of exactly $1\cdot m_{d}\,/\left(2\pi\right)^{1.5}$ in order to bind with a proton and yield a ^{7}Be nuclide. That is, each quark in ^{6}Li has to give up some energy, precisely defined in relation to the down quark mass, in order to "motivate" the new proton to join the 2s shell and produce a ^{7}Be nuclide. This makes some physical sense as well, and especially so because a similar view was used to explain the energy released during the fusion event ${}^{4}_{2}He+2p\rightarrow {}^{6}_{3}Li+e^{+}+\nu+\text{Energy}$. In fact, the results in (2.2) and (2.5) appear to supplement one another and greatly reduce the probability of coincidence, because they each, independently, suggest that once we start building heavier nuclides on the stable "base" of an alpha ^{4}He nuclide, are prescribed "dosages" of energy which the existing quarks and / or nucleons need to contribute and which are precisely described (to parts per million) in terms of $\sqrt{m_u m_d}$ for ^{4}He→^{6}Li and in terms of m_d for ^{6}Li→^{7}Be.

Let us therefore also regard (2.5) to correctly specify the energy in (2.4) to parts per million, thus setting:

$$\text{Energy}\left({}_{3}^{6}Li+p\rightarrow{}_{4}^{7}Be+\text{Energy}\right)={}_{3}^{6}M+M_{p}-{}_{4}^{7}M\equiv 18m_{d}/\left(2\pi\right)^{1.5}. \tag{2.6}$$

Now that we have "built" the ^{7}Be nuclide, we take note that ^{7}Be is comparatively stable, with a half-life of 53.22(6) days after which it will decay into the completely stable ^{7}Li nuclide via electron capture. So let us now turn to this β-decay reaction, which is more formally stated as ${}_{4}^{7}Be+e\rightarrow{}_{3}^{7}Li+\nu+\text{Energy}$. Again, as in (2.1) and (2.4) we calculate the associated energy:

$$\text{Energy}={}_{4}^{7}M-{}_{3}^{7}M+m_{e}=0.000925280000\text{ u} \tag{2.7}$$

using the empirical values ${}_{4}^{7}M=7.014735510362\text{ u}$, ${}_{3}^{7}M=7.014358810272\text{ u}$ and the electron mass (1.11). Here, using our ingredients m_u, m_d and $\sqrt{m_u m_d}$ and $\left(2\pi\right)^{1.5}$ divisor, we find:

$$6m_{u}/\left(2\pi\right)^{1.5}=0.000909485124\text{ u}. \tag{2.8}$$

This differs from the empirical number (2.7) by $-1.579487551927\times10^{-5}\text{ u}$, or under two parts per 100,000. Previously we came up the numbers 9 (up/down pairs in ^{6}Li) and 18 (quarks in ^{6}Li). Now we come upon the number "6" which is the number of *nuclides* in ^{6}Li. So (2.8) would appear, if meaningful, to say that each nuclide in the underlying ^{6}Li nuclide gives up an energy dosage of $1\cdot m_{u}/\left(2\pi\right)^{1.5}$ to facilitate the isotopic beta decay of ^{7}Be→^{7}Li. This too makes sense in terms of this number not being random, but bearing a genuine physical meaning for the nuclide in question. Together with the result in (2.2) and (2.5), this seems to suggest that energies released to enable fusion or beta decay, at least in the 2s shell, come in discrete doses. For ^{4}He→^{6}Li the dose is $1\cdot\sqrt{m_u m_d}/\left(2\pi\right)^{1.5}$ for each *up/down quark pair* in ^{6}Li. For ^{6}Li→^{7}Be the dose is $1\cdot m_{d}/\left(2\pi\right)^{1.5}$ for each *quark* in ^{6}Li. Finally, for ^{7}Be→^{7}Li the dose is $1\cdot m_{u}/\left(2\pi\right)^{1.5}$ for each *nuclide* in ^{6}Li. Notably, these respectively utilize the three ingredients $\sqrt{m_u m_d}/\left(2\pi\right)^{1.5}$, $m_{d}/\left(2\pi\right)^{1.5}$ and $m_{u}/\left(2\pi\right)^{1.5}$. Taken all together, this suggests that the numbers "9," "18" and "6" which were emerged by comparing these ingredients to the empirical data are all meaningful numbers based on the physical properties of ^{6}Li itself.

So, we now take (2.8) to be a meaningful expression for the energy in (2.7) to under 2 parts per 100,000, and so write:

$$\text{Energy}\left({}_{4}^{7}Be+e\rightarrow{}_{3}^{7}Li+\nu+\text{Energy}\right)={}_{4}^{7}M-{}_{3}^{7}M+m_{e}\equiv 6m_{u}/\left(2\pi\right)^{1.5}. \tag{2.9}$$

3. Binding Energy for ^{8}Be

Next, to complete the 2s shell, we turn to ^{8}Be, which completes the 2s shell, providing 2 protons and 2 neutrons in addition to four nucleons which already subsist in the 1s shell. Despite having complete 1s and 2s shells and no extra nucleons, the ^{8}Be isotope has a half-life of

$6.7(17)\times 10^{-17}$ s, after which it alpha-decays via ${}^{8}_{4}Be \rightarrow {}^{4}_{2}He + {}^{4}_{2}He + \text{Energy}$ into two alpha particles. For ^{4}He, as noted in the introduction, the binding energy is observed to be $B\left({}^{4}_{2}He\right) = 0.030376586499\text{ u}$. The empirical value of the ^{8}Be binding energy is observed to be $B\left({}^{8}_{4}Be\right) = 0.060654750886\text{ u}$. And, the ^{4}He alpha binding energy is fitted to under four parts per million by $2\cdot\Delta E_P + 2\cdot\Delta E_N - 2\sqrt{m_u m_d}$ as reviewed in (1.2).

It is nothing new to note that the ^{8}Be binding energy is almost twice as large as the ^{4}He binding energy, and specifically, that the empirical ratio:

$$B\left({}^{8}_{4}Be\right)/B\left({}^{4}_{2}He\right) = 1.996759935. \tag{3.1}$$

So, we know at the outset that if we simply double the ^{4}He binding energy and write $B\left({}^{8}_{4}Be\right) \cong 2\times\left(2\cdot\Delta E_P + 2\cdot\Delta E_N - 2\sqrt{m_u m_d}\right)$, we will get a close approximation to under 1%. Certainly then, an expression of the form $4\cdot\Delta E_P + 4\cdot\Delta E_N - E_?$ should give us the result we want, that is, one would hope that $Z\cdot\Delta E_P + N\cdot\Delta E_N$ with Z=4 and N=4, minus some unknown energy $E_? \cong 4\sqrt{m_u m_d}$ will give us the ^{8}Be binding energy to within at least parts per 100,000, matching the accuracy for the other foregoing results. The question is, how do we determine $E_?$ using the same ingredients m_u, m_d and $\sqrt{m_u m_d}$ and the $(2\pi)^{1.5}$ divisor?

First, it is physically <u>very important</u> that (3.1) is *not* equal to 2 but is less than 2 by about 0.32%. Since it appears that physically, stable nuclides are those which tend toward higher rather than lower binding energies, the ratio (3.1) tells us that two ^{4}He will have more binding energy in total than one ^{8}Be, and for this reason, ^{8}Be will split into two ^{4}He in order to maximize this *total* binding energy. That is, there is more binding energy in two separate ^{4}He than in a single ^{8}Be and apparently nature prefers this. So the very existence of the alpha decay ${}^{8}_{4}Be \rightarrow 2\cdot{}^{4}_{2}He$ as a preferred transition over $2\cdot{}^{4}_{2}He \rightarrow {}^{8}_{4}Be$ appears to depend on the ratio (3.1) being slightly less than 2. Consequently, this small diminution from 2 needs to be understood and not simply neglected by approximating to $B\left({}^{8}_{4}Be\right)/B\left({}^{4}_{2}He\right) \cong 2$.

Next, as to "numbers" that would make sense in the same way as "9," "18" and "6" in the previous section, we note that ^{8}Be has A=8 nucleons. So certainly, "8" is a number that would be of interest. Now, we have used the 3-dimensional Gaussian integration number $(2\pi)^{1.5} = 15.7496099457$ throughout without elaboration simply to report close fits between empirical binding data and certain expressions built from of up and down quark masses via products of Koide-type matrices (1.1). But, if an expression like $2\sqrt{m_u m_d}$ was an ingredient in successfully matching the ^{4}He binding energy to parts per million and a $1\cdot\sqrt{m_u m_d}/(2\pi)^{1.5}$ energy dose per quark pair in ^{6}Li successfully reproduced the ${}^{4}_{2}He + 2p \rightarrow {}^{6}_{3}Li + e^+ + \nu + \text{Energy}$ reaction also to parts per million, we see that both $\sqrt{m_u m_d}/(2\pi)^{1.5}$ and $\sqrt{m_u m_d}$ are ingredients that provide suitable energy doses. So because $(2\pi)^{1.5}=15.7496099457\cong 16$, this means that $16\cdot\sqrt{m_u m_d}\,/\,(2\pi)^{1.5}\cong\sqrt{m_u m_d}$. For a nuclide ^{8}Be with 8 nucleons, a coefficient "16" which approximates $(2\pi)^{1.5}$ in fact becomes physically relevant and not just random.

With this in mind, given that $B\left({}^4_2He\right)=2\cdot\Delta E_P+2\cdot\Delta E_N-2\sqrt{m_u m_d}$ as reviewed in (1.3), and given that we need an $E_?\cong 4\sqrt{m_u m_d}$ for ^{8}Be, let us use the close approximation $16\cdot\sqrt{m_u m_d}\,/\,(2\pi)^{1.5}\cong\sqrt{m_u m_d}$ to form another energy number:

$$B'\equiv 2\cdot\Delta E_P+2\cdot\Delta E_N-32\cdot\sqrt{m_u m_d}\,/\,(2\pi)^{1.5}\cong B\left({}^4_2He\right). \tag{3.2}$$

that is close to $B\left({}^4_2He\right)$ of (1.2), but not exactly the same. Now, let us conduct the *gedanken* of fusing two ^{4}He into one ^{8}Be. Of course, this will split into two ^{4}He after $6.7(17)\times 10^{-17}$ s, but this is still useful to think about. One of the two ^{4}He will have to form the 1s shell. The other will need to overlay "around" the 1s shell and form the 2s shell. Let us suppose that the ^{4}He which forms the 1s shell retains the $B\left({}^4_2He\right)$ shown in (1.2). But let us suppose that the other ^{4}He which goes into the 2s shell instead carries with it the energy number (3.2) which is very close to, but not the same as, $B\left({}^4_2He\right)$. Accordingly, we now use (3.2) and (1.2) together to construct a hypothesized:

$$B\left({}^8_4Be\right)=4\cdot\Delta E_P+4\cdot\Delta E_N-2\sqrt{m_u m_d}-32\sqrt{m_u m_d}\,/\,(2\pi)^{1.5}=0.0606332509\text{ u}. \tag{3.3}$$

This differs from the empirical $B\left({}^8_4Be\right)=0.060654750886$ u by $-2.1500027391\times 10^{-5}$ u, just over two parts in 100,000. So the accuracy is in the desired range. But does this make sense in other ways, so it is not just a coincidental guess but has physical meaning? First, the ratio:

$$\frac{B\left({}^8_4Be\right)}{B\left({}^4_2He\right)}=\frac{4\cdot\Delta E_P+4\cdot\Delta E_N-2\sqrt{m_u m_d}-32\sqrt{m_u m_d}\,/\,(2\pi)^{1.5}}{2\cdot\Delta E_P+2\cdot\Delta E_N-2\sqrt{m_u m_d}}=1.9960521522, \tag{3.4}$$

compare (3.1), is less than 2 by 0.4%, versus the empirical 0.32% noted earlier, and so will also cause the reaction ${}^8_4Be\rightarrow 2\cdot{}^4_2He$ to be energetically favored rather than $2\cdot{}^4_2He\rightarrow{}^8_4Be$. This a very important prerequisite for (3.3) to be a valid candidate for the ^{8}Be binding energy.

Secondly, noting that ^{4}He contains 6 up and 6 down quarks and is fully symmetric under $u\leftrightarrow d$ quark interchange, we observe that ^{8}Be contains 12 up and 12 down quarks and that (3.3) is also fully symmetric under $u\leftrightarrow d$ quark interchange. Apparently, $u\leftrightarrow d$ invariance is a desirable binding energy symmetry at least for the full-shell nuclides ^{4}He and ^{8}Be with equal numbers of protons and neutron and hence of up and down quarks.

Third, the number 32=8x4 has a very natural meaning in terms of the energy dosage considerations uncovered in section 2. Referring to (1.6) and (1.7), we see that $4\sqrt{m_u m_d}\,/\,(2\pi)^{1.5}$

is an important "energy dose" arising from the Koide matrices applied to protons and neutrons. Given that ^{8}Be contains 8 nucleons, one can interpret the (3.3) as saying that each of the 8 nucleons in ^{8}Be "contributes" a $-4\sqrt{m_u m_d}/(2\pi)^{1.5}$ dose of energy to binding energy (3.4), to produce the term $-32\sqrt{m_u m_d}/(2\pi)^{1.5}$. And, because this contribution yields the ratio (3.4), our *gedanken* to fuse $2\cdot {}^{4}_{2}He \rightarrow {}^{8}_{4}Be$ will last all of 6.7(17)×10^{-17} s, after which we will witness the physically-preferred decay ${}^{8}_{4}Be \rightarrow 2\cdot {}^{4}_{2}He$. So (3.3) appears to touch all the bases required to be a credible relationship for ^{8}Be binding energy and we shall henceforth employ it as such.

With the foregoing, we now have an expression for ^{8}Be binding that is accurate to about 2 parts per 100,000, and we have expressions with similar accuracy for fusion / beta decay energies related to ^{6}Li (2.3), ^{7}Be (2.6) and ^{7}Li (2.9). These fusion / decay energies (2.3), (2.6) and (2.9) may be deductively be converted over into binding energies, as shown next.

4. Binding Energies for the ^{6}Li, ^{7}Be and ^{7}Li Nuclides

In general, for a nuclide with Z protons and N neutrons hence A=Z+N nucleons, the binding energy ${}^{A}_{Z}B$ is related to its atomic weight ${}^{A}_{Z}M$ according to:

$$ {}^{A}_{Z}B = Z\cdot M_P + N\cdot M_N - {}^{A}_{Z}M \ . \tag{4.1}$$

So for the ${}^{6}_{3}Li$, ${}^{7}_{4}Be$ and ${}^{7}_{3}Li$ binding energies respectively, we need to find:

$$\begin{aligned} {}^{6}_{3}B &= 3\cdot M_P + 3\cdot M_N - {}^{6}_{3}M \\ {}^{7}_{4}B &= 4\cdot M_P + 3\cdot M_N - {}^{7}_{4}M \ . \\ {}^{7}_{3}B &= 3\cdot M_P + 4\cdot M_N - {}^{7}_{3}M \end{aligned} \tag{4.2}$$

We first use the results in (2.3), (2.6) and (2.9) for ${}^{6}_{3}M$, ${}^{7}_{4}M$ and ${}^{7}_{3}M$ to rewrite the above equation set as:

$$\begin{aligned} {}^{6}_{3}B &= M_P + 3\cdot M_N + 9\sqrt{m_u m_d}/(2\pi)^{1.5} - {}^{4}_{2}M + m_e \\ {}^{7}_{4}B &= 3\cdot M_P + 3\cdot M_N + 18 m_d/(2\pi)^{1.5} - {}^{6}_{3}M \qquad . \\ {}^{7}_{3}B &= 3\cdot M_P + 4\cdot M_N + 6 m_u/(2\pi)^{1.5} - {}^{7}_{4}M - m_e \end{aligned} \tag{4.3}$$

We then use (2.3) and (2.6) again in the latter two expressions to obtain:

$$
\begin{aligned}
{}^{6}_{3}B &= M_P + 3\cdot M_N + 9\sqrt{m_u m_d}\,/\left(2\pi\right)^{1.5} - {}^{4}_{2}M + m_e \\
{}^{7}_{4}B &= M_P + 3\cdot M_N + 18m_d\,/\left(2\pi\right)^{1.5} + 9\sqrt{m_u m_d}\,/\left(2\pi\right)^{1.5} - {}^{4}_{2}M + m_e\,. \\
{}^{7}_{3}B &= 2\cdot M_P + 4\cdot M_N + 6m_u\,/\left(2\pi\right)^{1.5} + 18m_d\,/\left(2\pi\right)^{1.5} - {}^{6}_{3}M - m_e
\end{aligned}
\tag{4.4}
$$

And we then use (2.3) yet again in the final expression to obtain:

$$
\begin{aligned}
{}^{6}_{3}B &= M_P + 3\cdot M_N + 9\sqrt{m_u m_d}\,/\left(2\pi\right)^{1.5} - {}^{4}_{2}M + m_e \\
{}^{7}_{4}B &= M_P + 3\cdot M_N + 18m_d\,/\left(2\pi\right)^{1.5} + 9\sqrt{m_u m_d}\,/\left(2\pi\right)^{1.5} - {}^{4}_{2}M + m_e \qquad . \\
{}^{7}_{3}B &= 4\cdot M_N + 6m_u\,/\left(2\pi\right)^{1.5} + 18m_d\,/\left(2\pi\right)^{1.5} + 9\sqrt{m_u m_d}\,/\left(2\pi\right)^{1.5} - {}^{4}_{2}M
\end{aligned}
\tag{4.5}
$$

These expressions are now all reduced to contain the alpha nuclear weight ${}^{4}_{2}M$. For this we rewrite (4.1) for Z=2 and N=2 as:

$$
{}^{4}_{2}M = 2\cdot M_P + 2\cdot M_N - {}^{4}_{2}B\,.
\tag{4.6}
$$

Substituting (4.6) into all of (4.5), we next obtain:

$$
\begin{aligned}
{}^{6}_{3}B &= M_N - M_P + 9\sqrt{m_u m_d}\,/\left(2\pi\right)^{1.5} + {}^{4}_{2}B + m_e \\
{}^{7}_{4}B &= M_N - M_P + 18m_d\,/\left(2\pi\right)^{1.5} + 9\sqrt{m_u m_d}\,/\left(2\pi\right)^{1.5} + {}^{4}_{2}B + m_e \qquad . \\
{}^{7}_{3}B &= 2\cdot\left(M_N - M_P\right) + 6m_u\,/\left(2\pi\right)^{1.5} + 18m_d\,/\left(2\pi\right)^{1.5} + 9\sqrt{m_u m_d}\,/\left(2\pi\right)^{1.5} + {}^{4}_{2}B
\end{aligned}
\tag{4.7}
$$

Finally, we use the neutron minus proton mass difference (1.10), the up, down and electron relationship (1.11), and the ^{4}He binding energy (1.2) with (1.6) and (1.7), and reduce. We then use the quark masses (1.12), (1.13), directly, to obtain:

$$
\begin{aligned}
{}^{6}_{3}B &= 7m_u + 6m_d - 2\sqrt{m_u m_d} + \frac{-10m_u - 10m_d - 9\sqrt{m_u m_d}}{\left(2\pi\right)^{\frac{3}{2}}} = 0.0343364272\text{ u} \\
{}^{7}_{4}B &= 7m_u + 6m_d - 2\sqrt{m_u m_d} + \frac{-10m_u + 8m_d - 9\sqrt{m_u m_d}}{\left(2\pi\right)^{\frac{3}{2}}} = 0.0403563620\text{ u} \quad . \\
{}^{7}_{3}B &= 8m_u + 6m_d - 2\sqrt{m_u m_d} + \frac{2m_u + 2m_d - 11\sqrt{m_u m_d}}{\left(2\pi\right)^{\frac{3}{2}}} = 0.042105716\text{ u}
\end{aligned}
\tag{4.8}
$$

The respective *empirical* values are ${}^{6}_{3}B = 0.0343470932\text{ u}$ (difference of $-1.06660\times10^{-5}\text{ u}$), ${}^{7}_{4}B = 0.0403651049\text{ u}$ (difference of $-8.7429\times10^{-6}\text{ u}$), and ${}^{7}_{3}B = 0.0421302542\text{ u}$ (difference of $-2.45378\times10^{-5}\text{ u}$). So together with ^{8}Be from (3.3), we have now developed expressions for all of the 2s nuclide binding energies to small parts per 10^{5} or (for ^{7}Be) parts per million.

5. Conclusion

Figure 1 below summarizes the retrodicted expressions and calculated values for both the 1s and 2s nuclides in the form of the customary chart of binding energy per nucleon, converted from AMU into MeV via $1\text{ u} = 931.494061\text{ MeV}$, as such:

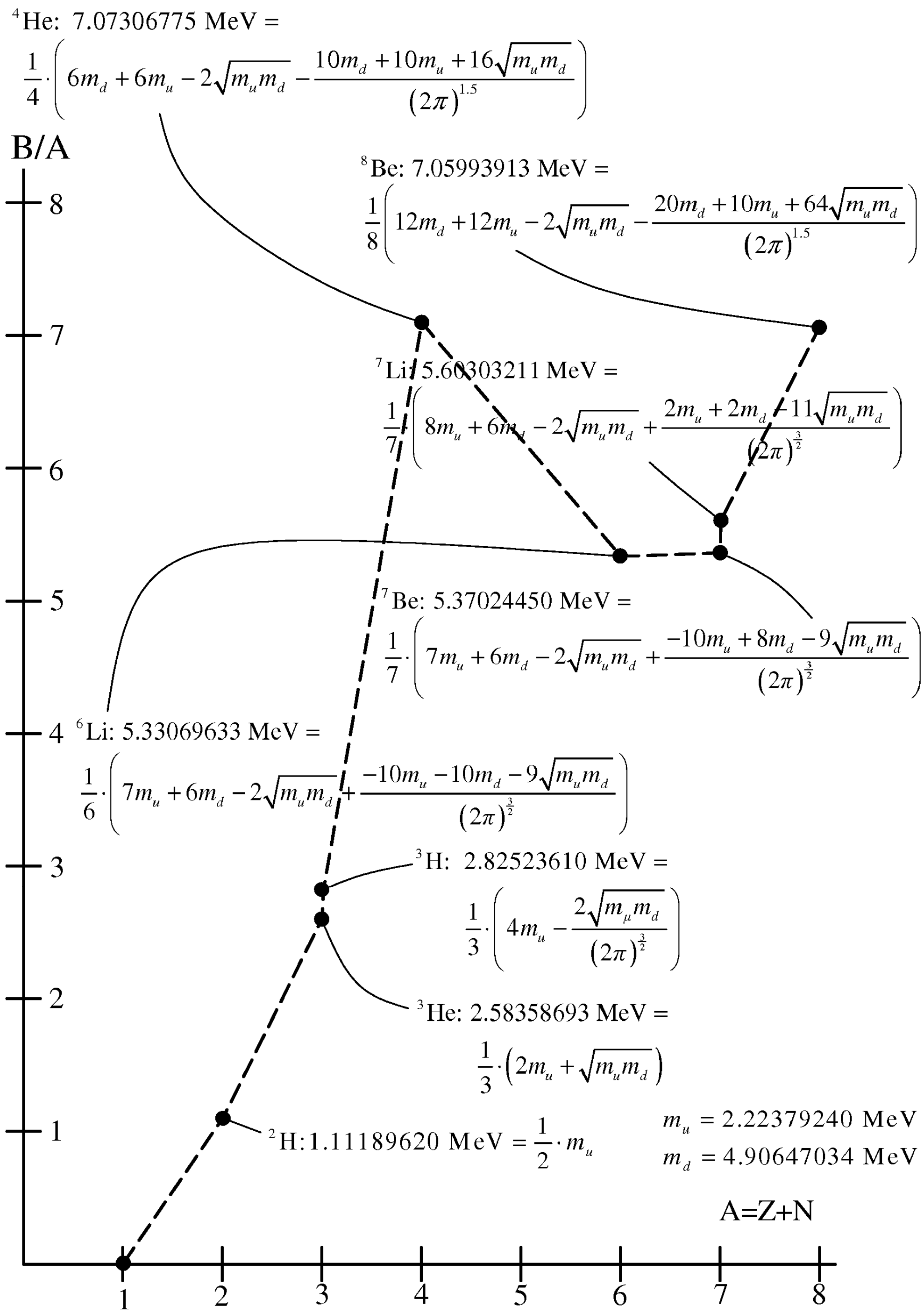

Figure 1: Retrodicted Binding Energies (B) per Nucleon (A=Z+N) for 1s and 2s Shells

This familiar curve shows eight of the very lightest elements in the well-known form of a per-nucleon binding energy graph. All of these energies, however, are no longer just empirical, but rather may be calculated strictly from the masses (1.12), (1.13) of the up and down quarks which, when the indicated calculations are performed, will enable a fit to the empirical data to parts per million or low parts per 100,000 in all cases. This provides strong validation that the foregoing approach taken together with what was separately reported in [1] enables nuclear binding energies to be fitted very precisely at a granular level, based solely as a function of the up and down quark masses. This fit in turn validates the values of masses (1.12), (1.13) via the observed nuclear binding energies which are known much more precisely than any quark mass values derived from deep inelastic scattering.

Also of interest is that the retrodicted binding energy per nucleon of ^{3}H exceeds that of its isobar ^{3}He by 0.24164918 MeV, while the retrodicted binding energy per nucleon of ^{7}Li exceeds that of its isobar ^{7}Be by the relatively similar 0.23278761 MeV. It is often assumed that separate consideration needs to be given to the electrostatic repulsion of an extra proton which *lowers* the binding energy of a proton-rich nuclide, e.g. ^{3}He and ^{7}Be. What the foregoing shows is that the binding energy difference owing to this electrostatic repulsion is already *inherently and integrally* built into both the quark masses, and the relationships in Figure 1 which combine these quark masses to arrive at nuclear binding energies.

Insofar as what we might learn from these results to progress in a granular way to even heavier nuclides, we see that we have essentially "woven" our way through the progression ^{4}He → ^{6}Li → ^{7}Be → ^{7}Li in (2.3), (2.6) and (2.9), which weaving was then deductively reflected in the binding energy calculations of section 4. Part of how we obtain confidence that our results are meaningful not randomly-coincidental, is that we progress carefully in this manner from one nuclide to the next along known fusion or decay routes, and make certain that the coefficients we use at each step to combine the m_u, m_d, $\sqrt{m_u m_d}$ and $(2\pi)^{1.5}$ ingredients make sense in relation to the nuclides in question. This way, as we build up heavier shells and nuclides, we know they are being constructed on a carefully-laid foundation.

Finally, the forgoing results do seem to validate that the European Muon Scattering Collaboration is indeed nothing less than a "paradigm shift" which must be recognized as such, sooner rather than later [9]. Certainly, one needs no more than the long-recognized evidence of nuclear mass defects to conclude that a free proton or neutron is different from a proton or neutron bound inside a nucleus. Here, we see in very clear fashion how quarks themselves reflect these mass defects by undergoing their own energetic changes as their nucleons are bound together to form composite nuclei. In fact, all of these results really just boil down to tracing mass defects in specific nuclides, down to the energy changes in the quark structure of individual protons and neutrons as those nucleons bind into composite nuclei.

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