The Bottom of Mathematics Education

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The International Association for the Evaluation of Educational Achievement (IEA) found in its 46 country 2003 study for 4th and 8th graders that "the ratio of Japanese students who said they have fun studying math and science hovered near the bottom of all participating nations and regions."[1] T. Yamanaka (50), an Osaka Prefecture Elementary school teacher states: "For children science has become a subject of rote memorization." M. Namiki (Takachiho University) notes that Japanese students were not good at solving questions close to daily life in the IEA study.

Trying to intervene in this alarming situation, the curriculum contents in Japanese schools has been cut by 30% and a 5 day school week has been introduced. The Japanese education minister Nariaki Nakayama says about these reforms with regard to the IEA results: "They were apparently aimed to enable children to think and act by themselves after teaching basics, but the goals are not necessarily cleared." The reforms have their vocal opponents. T. Sawada (Tokyo University of Science, Head of Japanese Society of Mathematical Education) even blames the very reduction in math lessons for falling achievements.

Some parents take a radical way out: home schooling, perhaps combined with selected private cram schools, but completely avoiding the demotivating public school system, with its overemphasize on rote learning. Some more moderate parents kick off additional home math education with card games or self programmed educational computer games, tunable to the interests of their children. Not everybody can do this, but Java based interactive, explorative dynamic geometry software like Cinderella[2] allows children to visually experiment and discover geometric facts by themselves. In Europe (and Korea) mathematics school classes visit a mathematical hands on traveling exhibition, which combines function rails, bubbles, mirrors, polygon games and much more into a fascinating trail of mathematical discovery. Two years ago it has been institutionalized into the unique and highly popular Mathematicum in Germany.[3]

I therefore conjecture, that conceptual learning, which is successfully applied to physics education[4] with its tenet that "conceptual learning is a creative act ... A student repeats the basic scientific discovery by rewriting the code book of his experience" is something mathematicians should be careful not to neglect.

But I think changing the way a subject is taught does not reform the mathematical paradigmas, which are the very guides of mathematical curriculum content. The discourse on the subject matter of mathematics should radically break with old conventions and simplify and unify the presentation to the learner. Today students start with counting to learn natural numbers, addition, subtraction, zero, and progress to negative, rational, real and complex numbers. They learn about polynomial, exponential and trigonometric functions, about vectors and matrices of numbers. At university (except for math majors) the study of mathematics extends to linear maps, to differentiation and integration with several variables and to complex functions. Subject specific, some students may learn (beyond synthetic and coordinate geometry, vector analysis and matrix algebra) from mathematical sub-disciplines like hypercomplex variable calculus, spinors, tensors, differential forms, etc.

Not all these systems are currently part of general mathematical education and so even scientists only know a few and cannot freely move between them. Often some information is represented in many different redundant ways and not well integrated, reducing information density and wasting energy and motivation of the learner. Relations of mathematical concepts in different symbolic systems are often difficult to recognize and use. This situation can be improved by restructuring mathematics according to the criteria of:

- Optimal algebraic encoding
- Coordinate free methodology
- Optimal uniformity across various domains
- Smooth articulation with traditional alternative systems
- Optimal computational efficiency

Geometric Algebra (GA) and the Geometric Calculus (based on GA) unify all the above mentioned sub-disciplines of mathematics and implement the optimization criteria for restructuring.[5] To work with scalar numbers (magnitudes), vectors (magnitudes with direction), oriented and directed areas and volumes (all basic entities of GA) comes straight from experience and observation. Ignorance of this approach lead to the current defects in mathematical knowledge representation and ultimately burdens mathematical education. To comprehensively teach and learn mathematics in this new way will make it easier and more meaningful to study mathematics and to master advanced mathematical tools faster, keeping closer to experience then with the current traditional mathematical curricula.

References (updated 21 June 2013)

[1] The Asahi Shinbun, p. 25, 2004-12-16.

[2] http://www.cinderella.de/ , http://erkenntnis.icu.ac.jp/cinderella1.4/index.html

[3] http://www.mathematikum.de/, http://erkenntnis.icu.ac.jp/teaching/homath/HoMeng.htm

[4] E. Hitzer, *New Ways for Physics Education*, University of Fukui, Faculty Development Forum, Vol. 5, p. 12 (August 2004). To be submitted to vixra.org .

[5] Proceedings of *Innovative Teaching of Mathematics with GA 2003*, Research Institute for Mathematical Sciences, Kyoto University. http://www.kurims.kyoto-u.ac.jp/~kyodo/kokyuroku/contents/1378.html .