What confinement really means in Quantum Chromodynamics

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Abstract

In spite of intense efforts it has not been possible to demonstrate that confinement of colour exists consistently in Quantum Chromodynamics. It is therefore one of the most puzzling issues in Quantum Chromodynamics. We study what antisymmetrization in colour space means fundamentally and how this is then matched with the conjugate symmetric state in the rest of the degrees of freedom of the quarks. It is shown that the present understanding, that confinement arises due to a single colour singlet state, is wrong. In this paper we prove that actually there are two independent colour singlet states, both of which are needed simultaneously to provide confinement in QCD. This in turn leads to a fundamental justification of the relativistic bag models and the non-relativistic quark models.

Keywords: Colour, Confinement, Quantum Chromodynamics, Bag Models, Quark Model

Quantum Chromodyanmics (QCD) is proving to be an extremely successful theory of the strong interaction. However in spite of intense efforts globally, the issue of confinemet of colour in QCD is still unproven [1]. The problem appears to be so puzzling that due to its intrinsic difficulty, it has been compared to the Fermat's Last Problem in mathematics [2]. This, we take to mean, that we must be missing some very basic and fundamental ingredient in this issue. What is it then?

QCD's successes are in the regime of high energies including that of the asymptotic freedom. It is at low energies that it seems to give problems - all of which are placed on the putative non-perturbative effects of QCD. It is in this context that modelling of QCD at these low energies plays a basic role. All kinds of models. which may have some resemblance to QCD in some respect, are invoked as motivations for these models. However these different models, very often, may be significantly different from each other. especially as to what goes as inputs in these models. For example, the bag models of the hadrons require the existence of almost massless relativistic quarks, while the potential models demand non-relativistic and heavy mass quarks. Surprisingly though, in spite of these fundamental differences, in turns out, that these models still match the experimental reality pretty well. Well, that is precisely the reason that we take them seriously. The question one should ask is whether the successes of these different models are merely accidental or it is pointing to some basic hidden reality. We take the latter as providing a more convincing physical picture.

Let us start with the SU(2) spin group. Let the two spins be denoted by $\uparrow$ and $\downarrow$ and let the the positions of the two fermions be denoted by '1' and '2'. Thus the antisymmetric wave function is

$$\Psi_A = \frac{1}{\sqrt{2}}(\uparrow(1)\downarrow(2) - \downarrow(1)\uparrow(2)) \tag{1}$$

It is antisymmetric under the exchange of the spin states $\uparrow \leftrightarrow \downarrow$. This exchange requires that the positions 1 and 2 be fixed in the order (12) while the state exchange is perfomed. The above wavefunction is also simultaneously antisymmetric under an independent exchange of the two positions '1' and '2'. Thus there is a duality here, the function Ψ_A being antisymetric under two independent exchanges, that of the two states and that of the two positions.

Let us use general notation as that of the states of α and β for the two states of the two fermions and then the above state is written as

$$\Psi_A = \frac{1}{\sqrt{2}}(\alpha(1)\beta(2) - \beta(1)\alpha(2)) \tag{2}$$

This can be written as a Slater Determinant

$$\Psi_A = \frac{1}{\sqrt{2}} \begin{vmatrix} \Psi_\alpha(1) & \Psi_\alpha(2) \\ \Psi_\beta(1) & \Psi_\beta(2) \end{vmatrix} \tag{3}$$

And the fact that the determinant can be expanded either along its rows or along its columns is related to the fact that eqns. (2) and (3) are antisymmetric under the exchange of either the states $\alpha \leftrightarrow \beta$ (while keeping the order of the positions (12) fixed) or that of the position $1 \leftrightarrow 2$. The above mentioned duality in antisymmetry is an inherent property of the Slater Determinant.

Generalize the above to three states α, β, γ of the group SU(3), then the totally antisymmetric state is given by the Slater Determinant

$$\Psi_A = \frac{1}{\sqrt{6}} \begin{vmatrix} \Psi_\alpha(1) & \Psi_\alpha(2) & \Psi_\alpha(3) \\ \Psi_\beta(1) & \Psi_\beta(2) & \Psi_\beta(3) \\ \Psi_\gamma(1) & \Psi_\gamma(2) & \Psi_\gamma(3) \end{vmatrix} \tag{4}$$

Again this state is antisymmetric under all the pairs of state exchanges ($\alpha \leftrightarrow \beta, \alpha \leftrightarrow \gamma, \beta \leftrightarrow \gamma$) (while keeping the location order (123) fixed) and also under the exchange of all the pairs of position ($1 \leftrightarrow 2, 1 \leftrightarrow 3, 2 \leftrightarrow 3$).

Note that the above antisymmetry makes it a genuine three body effect. To maintain its dual charachter of simulataneous exchange of all the pair of states and that of all the positions, that it is not possible to reduce it to any lower two body terms.

Now it is this colour antisymmetric state, which is also a singlet, is what is presently believed by all to lead to colour confinemet of quarks. Thus this is also what is exactly believed to be the the singlet state in the product $3 \times 3 \times 3 = 1 + 8 + 8 + 10$. The singlet state above arises from the product $\overline{3} \times 3 = 1 + 8$ as shown below:

$$3 \times 3 \times 3 = (\overline{3} + 6) \times 3 = (\overline{3} \times 3) + (6 \times 3) = (1 + 8) + (8 + 10) \tag{5}$$

So as per current understanding, the Slater Detrminant antisymmetruic state and the above singlet state in the tensor product, are exactly identical states which lead to colour confinement in QCD. However in this apper we show that this point of view misses a subtle and fundamental point. And that this has been the source of all the problems as to what confinement really means in QCD.

The mistake was not to notice the difference in how antisymmetrization is achieved in eqn (4) of the Slater Detreminant singlet state and what it means in eqn (5) in the singlet of the above tensor product $\overline{3} \times 3 = 1 + 8$. As we have discussed, in the former case, it is dually antisymmetric, independently both under all the state exchanges and also all the position exchanges. As we prove below, in the latter case, the singlet state is antisymmetric only under all the pairs of the state exchange while the other antisymmetry, under all the position exchanges, does not exist in all the cases. As we show below, there are two independent cases for the antisymmetric case in eqn. (5). One case gives the Slater determinant, but the other one is very different. And hence these two are NOT identical colour singlet states , but reveal two independent colour singlet states, both of which are needed for the confinement of colour in QCD.

The important point to note is that in SU(3) the correspondence between the fundamental representation between colour 3 and anti-colour $\overline{3}$ is not between a single lower and a single upper index (as it is actually true fo the group SU(2)). Rather it actually corresponds to an antisymmetrized state of two lower indices to an upper index. Thus [3]

$$q^{\alpha}(3) = \epsilon^{\alpha\beta\gamma}(q_{\beta}(1)q_{\gamma}(2) - q_{\gamma}(1)q_{\beta}(2)) \tag{6}$$

Here we have placed the anticolour at the position '3' while the antisymmetric pair sits at the new positions '1' and '2'. Here the $\beta\gamma$ pair is demanded to be an antisymmetric state. As we discuss below this antisymmetry is dual, i.e. under the state exchange $\beta \leftrightarrow \gamma$ as well as the position exchange $1 \leftrightarrow 2$.

Now the colour singlet state in $3 \times 3 = 1 + 8$ is given by

$$q^{\alpha}(3)q_{\alpha}(3) = \epsilon^{\alpha\beta\gamma}(q_{\beta}(1)q_{\gamma}(2) - q_{\gamma}(1)q_{\beta}(2))q_{\alpha}(3) \tag{7}$$

The totally antisymmetric tensor $\epsilon^{\alpha\beta\gamma}$ in the colour space ensures antisymmetry with respect to the γ state also as ($\alpha \leftrightarrow \beta, \alpha \leftrightarrow \gamma, \beta \leftrightarrow \gamma$) (while keeping the location order (123) fixed). As to position exchanges, only the original exchange ($1 \leftrightarrow 2$) is there while no exchange is guaranteed with respect to the third position, and shall be shown to hold in only one special situation. There exists another independent situation where this singlet gets reduced to a two body term. Hence a complete duality in exchange of the Slater Dererminant is missing here. Thus this colour singlet state is fundamentally different from the previous one. Hence the previous understanding of these two always being identical was wrong. As such these two colour singlet states both should be required to play their roles in providing confinement in QCD.

How come there are two colour singlet states? This is because in the colour singlet state $q^\alpha q_\alpha$ the locations where the colour state **3** and the anti-colour state $\bar{3}$ may sit be at the same position or at different positions, say as

$$(a)..q^\alpha(3)q_\alpha(3)...OR...(b)..q^\alpha(4)q_\alpha(3) \quad (8)$$

First the case (a) above. Now as per eqn. (6) the anti-colour at position '3' creates two colour states at independent positions '1' and '2'. Because of the identity, we expect that the position '3' should be the Centre of Mass position of the two new positions '1' and '2'. Hence it is because of this that in eqn. (7) the position '3' cannot be exchanged with either positions '1' or '2'. The Centre of mass position '3' is inert. Thus fixing the order of positions (123), we obtain antisymmetry of the state in eqn. (7) only under the three state exchanges.

Next the case (b) above. Here the locations '4' and '3' are distinct. Now let the anti-colour at position '4' create colour states at two new and distinct positions '1' and '2'. Now as the original position '3' is independent of the postions '1' and '2', thus all the three exchanges for any pair of positions are permitted here. This is obvioulsy in addition to the antisymmetry under the three state exchanges. Thus this situation corresponds to the dual antisymmetry generated by the Slater Determinant eqn. (4). Thus we obtain the two independent colour singlet states which should both be included to understand confinement of colour in QCD.

Note that the correspondence of the anti-colour state with the antisymmetric state of two colours is an intrinsically non-local effect. Here it arises entirely as a property of the fundamental Irreducible Representations of the group SU(3).

Now, if there are two independent colour singlet states with antisymmetry arising in the above two manner, then what are the corresponding conjugate and symmetric states in the rest of the degrees of freedom of the quarks in the $SU(3)_F \times SU(2)_S \times SO(3)$ space?

There is a plethora of phenomenological models of QCD at low energies [4]. However these may be broadly and quite generically classified in two categories: (a) the bag models (which include the MIT and the SLAC bag models) and (b) the potential models (the harmonic oscillator model being the best example of it). We are treating these two as fundamentally distinct as the former ones require relativistic quarks which are almost massless and the latter demands non-relativistic and heavy quarks. Note that we are ignoring all the hybrid models which include arbitrary mixtures of both of these models in some measure or the other, and hence they lack any fundamentality in their choice.

First let us look at the bag models. We look at the generic bag model which in the limit of a certain parameter describes all the bag models, in particular the MIT bag model and the SLAC bag model. Without tying ourselves to a particular bag model, let us follow Lee [5]. Assume that the vacuum in QCD is a perfect (or nearly perfect) colour dia-electric medium. Whenever quarks and antiquarks are present, there arises an inhomogeneity in the surrounding space around the particle. We call this a "bag". Inside the bag $\kappa = 1$, while outside κ_∞ is zero or $\ll 1$. With colour singlet state inside the bag, $\kappa_\infty \to 0$. Thus bag supports colour singlet states [5], .

$$mesons.. \sim q^\alpha q_\alpha .and.baryons.. \sim \epsilon^{\alpha\beta\gamma} q_\alpha(1) q_\beta(2) q_\gamma(3) \tag{9}$$

Clearly for baryons one might as well write

$$\epsilon^{123} q_\alpha(1) q_\beta(2) q_\gamma(3) \tag{10}$$

Thus the baryons are colour singlet with the Slater Detrminant state eqn. (4). As the bag boundary is a macroscopic size global surface property, it makes sense that it does not distinguish between the three colours individually but treats it as a three body entity [4,5]. This idea finds confirmation and support for consistency in terms of the conjugate symmetric state in the other degrees of freedom, as we see below.

Lee also shows [5] how these bags may be generic with the MIT bag and the SLAC bags arising as the limitimg cases of a particular parameter of the model. On general grounds the hadron masses for light (or zero mass) quarks is

$$M = N\frac{\phi}{R} + \frac{4\pi}{3}R^3 p + 4\pi R^2 s \qquad (11)$$

where N is the number of quark or anti-quarks (N=2 for mesons and N=3 for baryons), R is the radius of the bag, ϕ is a constant. The second term is volume energy with 'p' as pressure and the third term is surface energy with 's' as a constant. So there are three prameters ϕ, p and s. This is a bag (gas bubble) immersed in a medium, the vacuum. The medium exerts a bag pressure 'p' on the boundary giving the volume energy. Surface tension 's' provides the surface energy, while the first term gives the thermodynamic energy of the gas inside the bag [5].

If we neglect vector exchange in zeroth order, the energy inside is entirely the kinetic energy (first term) which is independent of the quark spins. Thus this is providing for the SU(6) spin-flavour symmetry. For zero mass quarks in the baryon ground state, the wave function of the three quarks differs only in the z-component of their angular momenta. The three quarks are in relativistic angular momentum 1/2 orbit where the z-component can be 1/2 and -1/2. Let us denote this by $\uparrow and \downarrow$. For simplicity we shall call these as the spin components, even though these really are the z-component of the total - spin plus orbital, angular momemtum of the individual quarks [5]. With 3 flavours and two spins (actually j , j_3) it has six degrees of freedom which is taken as fundamental representation of $SU(6)_{SF}$. Though conventionally we use the subscript SF here for spin-flavour, in the relativistic case, the orbial angular momentum part is already intrinsically there.

This is the correct conjugate state to go with the colours antisymmetric part in eqns. (9) and (10). This fully symmetric SU(6) wave finction, for example for proton with spin up, is

$$\Phi(p^{\uparrow}) = \frac{1}{3}[u^{\uparrow}(1)u^{\uparrow}(2)d^{\downarrow}(3) + u^{\uparrow}(1)d^{\uparrow}(2)u^{\downarrow}(3) + d^{\uparrow}(1)u^{\uparrow}(2)u^{\downarrow}(3)$$

$$+u^{\uparrow}(1)u^{\downarrow}(2)d^{\uparrow}(3) + u^{\uparrow}(1)d^{\downarrow}(2)u^{\uparrow}(3) + d^{\uparrow}(1)u^{\downarrow}(2)u^{\uparrow}(3)$$

$$+u^{\downarrow}(1)u^{\uparrow}(2)d^{\uparrow}(3) + u^{\downarrow}(1)d^{\uparrow}(2)u^{\uparrow}(3) + d^{\downarrow}(1)u^{\uparrow}(2)u^{\uparrow}(3)] \qquad (12)$$

Note that this SU(6) wave function is fully symmetric under the three state exchanges (with positions order (123) fixed) and under independent all the position exchanges $(1 \leftrightarrow 2, 1 \leftrightarrow 3, 2 \leftrightarrow 3)$. Thus is the genuine conjugate symmetric state to go with the colour antisymetric state eqn. (9) in the bag model. Thus the bag model provides the corrrect conjugate symmetric state to go with the Slater determinant colour singlet state eqn (4).

Next what is the correct symmetric conjugate state to go with the colour singlet state eqn. (7)? Remember in that case the colour antisymmetry was due to all the state exchanges $(\alpha \leftrightarrow \beta, \alpha \leftrightarrow \gamma, \beta \leftrightarrow \gamma)$ while the location order (123) is fixed. As to the exchange of positions, there was only one exchange allowed $1 \leftrightarrow 2$ with position '3' inert and no exchanges with respect to it. Hence the conjugate state in the rest of the space $SU(3)_F \times SU(2)_S \times SO(3)$ should be symmetric under identical exchanges. Below we show that the other category of the hadron models, i.e. the potential model, fulfills this condition, and hence providing an order and completeness amongst the models of the hadrons at low energies [3,4,6].

We look at the non-relativistic harmonic oscillator model [3,4,6]. To build up the proton ground state with spin up, we need

$$\phi_p^\lambda = -\frac{1}{\sqrt{6}} \left(udu + duu - 2uud \right) \tag{13}$$

$$\phi_p^\rho = \frac{1}{\sqrt{2}} \left(udu - duu \right) \tag{14}$$

and

$$\chi_{\frac{1}{2}}^\lambda = -\frac{1}{\sqrt{6}} \left(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2 \uparrow\uparrow\downarrow \right) \tag{15}$$

$$\chi_{\frac{1}{2}}^\rho = \frac{1}{\sqrt{2}} \left(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow \right) \tag{16}$$

Here the state ρ is antisymmetric, and the state λ is symmetric independently under the exchange of both the first two states and the first two positions '1' and '2'. Both of these have no symmety with respect to the exchange of the third state or postion '3'. Now these are combined as follows, to get the proper wave function for the ground state with l=0,

$$\mid p_\uparrow \rangle = \frac{1}{\sqrt{2}} \left(\chi^\rho \phi^\rho + \chi^\lambda \phi^\lambda \right) \psi_{00S}^0 \tag{17}$$

However certain subtleties here should not be ignored. So let us combine the individual flavour-spin states above as

$$\phi^\rho\chi^\rho = \frac{1}{2}[u^\uparrow(1)d^\downarrow(2)u^\uparrow(3) - u^\downarrow(1)d^\uparrow(2)u^\uparrow(3)$$

$$-d^\uparrow(1)u^\downarrow(2)u^\uparrow(3) + d^\downarrow(1)u^\uparrow(2)u^\uparrow(3)]$$

$$\phi^\lambda\chi^\lambda = \frac{1}{6}[u^\uparrow(1)d^\downarrow(2)u^\uparrow(3) + u^\downarrow(1)d^\uparrow(2)u^\uparrow(3) - 2u^\uparrow(1)d^\uparrow(2)u^\downarrow(3)$$

$$+d^\uparrow(1)u^\downarrow(2)d^\uparrow(3) + d^\downarrow(1)u^\uparrow(2)u^\uparrow(3) - 2d^\uparrow(1)u^\uparrow(2)u^\downarrow(3)$$

$$-2u^\uparrow(1)u^\downarrow(2)d^\uparrow(3) - 2u^\downarrow(1)u^\uparrow(2)d^\uparrow(3) + 4u^\uparrow(1)u^\uparrow(2)d^\downarrow(3)] \qquad (18)$$

Note that both $\phi^\rho\chi^\rho$ and $\phi^\lambda\chi^\lambda$ are seperately symmetric under only the exchanges of the first two states or the first two positions '1' and '2'. Individually there is no exchange symmetry under the third state or the third position. Now to add up these two as done above to get the full spin-flavour-orbital wave function as per eqn. (17), certain specific conditions have to be satisfied. Let us fix the order of locations as (123) then only the corresponding terms in the above two terms in eqn. (18) may add up. Then only we may add up say $\frac{1}{2}u^\uparrow(1)d^\downarrow(2)u^\downarrow(3)$ from the first state above to $\frac{1}{6}u^\uparrow(1)d^\downarrow(2)u^\downarrow(3)$ from the second term. It is important to note that if the order of position (123) is not fixed the corresponding terms cannot be combined.

$$\phi^\rho\chi^\rho + \phi^\lambda\chi^\lambda = \frac{1}{3}[2u^\uparrow(1)d^\downarrow(2)u^\uparrow(3) - u^\downarrow(1)d^\uparrow(2)u^\uparrow(3) - u^\uparrow(1)d^\uparrow(2)u^\downarrow(3)$$

$$-d^\uparrow(1)u^\downarrow(2)d^\uparrow(3) + 2d^\downarrow(1)u^\uparrow(2)u^\uparrow(3) - d^\uparrow(1)u^\uparrow(2)u^\downarrow(3)$$

$$-u^\uparrow(1)u^\downarrow(2)d^\uparrow(3) - u^\downarrow(1)u^\uparrow(2)d^\uparrow(3) + 2u^\uparrow(1)u^\uparrow(2)d^\downarrow(3)] \qquad (19)$$

Now this sum develops the additional symmetry with respect to the state in position '3'. Thus this sum is fully symmetric under the exchange of all the pair of states in the three positions. Note how this conjugate state develops full symmetry under all the three state exchanges only after addition of the above two terms just as the Levi-Civita antisymmetric tensor imposed antisymmetry for all the three colours in SU(3). Note that because of the separate symmetry under space exchange for the positions '1' and '2' we can add the terms above also as long as we leave position '3' as inert. Thus this is clearly the correct symmetric state to go along with the other colour singlet state. Note also how different this harmonic oscillator SU(6) spin-flavour state eqn. (19) is with respect to the bag model SU(6) wave function eqn. (12). they are definitely not representing the same mathematical and physical reality. It is also heartening to note that this justfies as to why and how the quark model actually works.

As we said there are two independent colour singlet states needed to obtain confinemet in QCD. We have shown that these two then get linked up separately to the bag model and the harmonic oscillator potential model of the hadrons. Thus both of these are needed simulatneously to get a more complete understanding of hadrons. Hence there is thus a duality between the bag models and the potential models. One has to first understand this duality in terms of the hadronic structure. Note that this duality of description of a single nucleon is reminiscient of the duality between the Liquid Drop Model and the Independent Particle Shell Model of the nucleus [7]. This has to be studied more carefully in the future. Note, as two different colour singlet states are required to provide confinenment and one of which in particular is linked to three-bodiness of the baryons, it may be ruling out a large number of exotics, which for example [8] were expected on the basis of the earlier erroneos understanding of a single colour singlet state. One important question that has to be resolved in future is how does the recent successes of the topological Skyrme model [9] relate it to the quark models and the bag models?

To put things in proper perspective, we quote Gross below [10]

"Quantum field theory is today at a pinnacle of success. .. quantum field theory works from the Planck length to the edge of the universe - over 60 orders of magnitute. No other theory has been so universally successful."

"Today we believe that global symmetries are unnatural. They smell of action at a distance."

"QCD as a perfect QFT" - a full section devoted to showing this.

On the basis of what we have shown, the quantum field theory - QCD, gives only half of the answer for confinement. As such quantum field theory provides only part of the theoretical framework. Non-relativistic quantum mechanics is not just some kind of an approximation to quantum field theory. But it comes out on its own here as being as fundamental as quantum field theory. One has to adjust to this new paradigm as shown in this paper.

Next we are used to accepting spin entanglement as a non-local effect in say a state of two spin halves from a spin 0+ state (as in eqn (2)). Here a similar non-local entanglement appears in the antisymmetric colour state of the two colours in eqn (6). This implicit non-locality shows that for confinement the global aspect of SU(3) is fundamentally required here. Global SU(3), not as some approximation of QCD, but as a basic entity which is required as fundamentally as the local aspect of QCD is. Therefore, we have to look at the Newtonian dynamics, the Galilean transformation and the absolute space from a different angle now.

So many prejudices of our present theoretical understanding have to be given up to undertstand what confinement really means in QCD.

References

1. R. Alkofer and J. Greensite, J. Phys. **G34** (2007) S21

2. A. Smilga, "Lectures on Quantum Chromodynamics", World Scientific, Singapore, (2001).

3. Fayyazuddin and Riazuddin, "An introduction to particle physics", World Scientific, Singapore (2011)

4. A. Hosaka and H. Toki, "Quarks, baryons and chiral symmetry", World Scientific, Singapore, (2001)

5. T. D. Lee, "Particle physics and introduction to field theory", Harwood Academic Pub. , London, (1981)

6. F. E. Close, "An Introduction to Quarks and Partons", Academic Press, London, (1979).

7. W. Greiner and J. A. Maruhn, "Nuclear Models", Springer-Verlag, Berlin, (2008)

8. A. Th. M. Aerts, P. J. G. Mulders and J. J. de Swart, Phys. Rev. **D17** (1978) 260

9. S. A. Abbas, "What has been discovered at 125 GeV by the CMS‘ and the ATLAS experiemnts?", Eur. Phys. J. A. **49** (2013) 72 ; DOI 10.1140/epja/12013-13072-1

10. D. J. Gross, "Triumphs and limitations of quantum field theory", in Ed. T. Y. Cao, "Conceptual foundations of quantum field theory", Cambridge University Press, Cambridge (1999), p. 56-67