Investigation of the Formalism of Particle Dynamics under the Framework of Classical Mechanics

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In this paper we reconstruct the formulism of particle dynamics under the framework of classical mechanics according to the causal consistency principle, and obtain a new particle dynamical equation. In this derivation there is a most natural and simple assumption that an absolute background of space exists. Because in essence, the absolute background of space must be distinguished from the relative scales of space. The existence of an absolute background can not only be mostly compatible with the physical logic in special theory of relativity, but also retains the most fundamental elements in our intuitional experience. Certainly, the absolute background of space is also the underlying part of Newton's absolute view of space-time. In the application of the new dynamical equation, inertial reference frames are no longer required and inertial forces are no longer introduced by hand. This new dynamical equation can be directly applied in any reference frame which is irrotational with respect to the absolute background of space, namely a moderate general principle of relativity is realized on particle dynamics. The nature of the inertial force is nothing but the real forces exerted on the reference object. Further analysis illustrates that the new particle dynamical equation is more in line with the empirical laws of classical mechanics experiments, than the traditional theoretical formula of Newton's second law.
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1 INTRODUCTION

In the framework of classical mechanics, the fundamental dynamics equation is Newton's second law. But as is well known, Newton's second law is only valid in inertial reference frames. Provided that we apply the equation form of Newton's second law in a non-inertial reference frame, we need to introduce a fictitious force—inertial force additionally. The magnitude of the inertial force is usually determined by the relative acceleration between the non-inertial reference frame in question and a certain inertial reference frame[1–7]. Therefore, the classical particle dynamics is totally based on the concept of inertial reference frame. However, as well-known to us, we are never able to find a real inertial reference frame in practice. This situation is surely not satisfactory[8, 13]. On the other hand, there is a pending question which exists in astronomical observation, namely why the orbital velocities of stars in the Milky Way galaxy is greater than the prediction calculated from the "luminous matter" it contains? In a most popular approach it is explained from the point of view of dark matter[9]. Besides, a theory of modified Newtonian dynamics($MOND$) was proposed by Mordehai Milgrom in 1983. In this theory, the problem of flat rotation curve was explained by modifying the dynamics of classical mechanics, or modifying Newton's gravity theory. Milgrom found that the flat rotation curve problem can be elegantly solved if the Newtonian dynamics is modified only when the acceleration of reference frames is very small[10]. However, from recent results of laboratory tests, such an approach as $MOND$ theory to modify the Newtonian dynamics by artificially introducing a scaling factor is not so promising[11, 12]. Therefore, in this paper we reinvestigate the formal logic problem in classical dynamics starting from the most fundamental requirment—the principle of causal consistency.

In fact, every reference frame must be established on a real reference object. Otherwise, there would be no reference value in measuring any object's motion in the natural world. Therefore, a physical reference frame must be the real reference frame. As for the relationship between the reference object and the reference frame, the reference frame can be naturally established by identifying the reference object as its origin point if we assign a reference object first in practical cases. Otherwise, in principle any real object which is fixed in the reference frame can be identified as its reference object if the reference frame is assigned first. For example, as a most usual reference frame, the ground-based reference frame actually selects an arbitrary object fixed on the ground as the reference object.

In this paper, we study the formulism of particle dynamics under the framework of classical mechanics. The paper is organized as follows. In Sec.2, we present the traditional formulism of particle dynamics in the framework of Newtonian mechanics. We illustrate that we have to adopt a series of approximations when the current theoretical formula of Newton's second law is applied in practice, and point out that there is a causal inconsistency problem which exists in the theoretical formula of Newton's second law. In Sec.3, we reconstruct a new formulism of particle dynamics under the guidance of causal consistency principle. In Sec.4, the new particle dynamical equation is examined by making a comparison with the empirical laws from classical mechanics experiments. We show that the new dynamical equation is able to present a more concise and natural picture to explain all these empirical laws than Newton's second law. At the same time, the inertial reference frame is no longer required and the inertial force is no longer introduced by hand. In Sec.5, we discuss the relationship between the new dynamical equation and Newton's second law from the historical point of view. It is fully shown that the new particle dynamical equation is essentially an indispensable development of Newton's second law when the latter is restored to be an empirical law. In Sec.6, we discuss some potential physical significance of this new dynamical equation.

2 FORMULISM OF NEWTONIAN PARTICLE DYNAMICS AND EXISTING PROBLEMS

In Newtonian mechanics, the main equation of particle dynamics is given by Newton's second law[2–7]. The mathematical form can be generally expressed as

$$\mathbf{F}|_p = m_p \mathbf{a}|_{p-O}. \tag{1}$$

In theory, the left hand side of this equation must denote the total force from the whole universe exerted on the particle p. Otherwise, when the equation is applied into concrete cases, we will not be able to make it clear what forces should be included in the count, and what forces should not be counted. The m_p of the right hand side of this equation denotes the mass of the particle p; $\mathbf{a}|_{p-O}$ denotes the acceleration of particle p with respect to the reference frame O. Here the real reference object which is assigned to define this reference frame O may as well be denoted by O. Since the reference object is definitely fixed in the reference frame, the acceleration ($\mathbf{a}|_{p-O}$) of the particle p with respect to the reference frame O, is equivalently measured relative to the reference object O.

It is well known to us that Newton's second law (1) is valid only for inertial reference frames. But by now we have not found even one exact inertial reference frame in practice. Actually, it is also not possible for us to find an exact inertial reference frame, since all real matter in the universe are acted on by the gravitational forces from other matter and so are in a never-ceasing motion and change. Therefore, any real reference frame must be a non-inertial reference frame. According to Newtonian mechanics, if we want to make a mechanical application of Newton's second law in a non-inertial reference frame, in theory we still need to find out an inertial reference frame firstly. After that, we can introduce a fictitious inertial force according to the relative acceleration between the inertial and non-inertial reference frames[14],

$$\mathbf{f}|_{inertial} = -m_p \mathbf{a}|_{O'-O}. \tag{2}$$

Therefore, in the application of Newton's second law, it is always required to approximate a real reference frame to be a inertial reference frame. It should be noted that such an approximation is actually made at the stage of theory application, but not at the stage of the practical measurement of physical quantities. In this sense, Newtonian mechanics is still not a perfect theory.

In fact, most of classical mechanics experiments are conducted and analyzed under the ground-based laboratory reference frame. If we assume the object under study is denoted by A, and the ground-based laboratory reference frame is denoted by B, the empirical law which is satisfied by all these ground-based classical mechanics experiments can be written down as,

$$(\mathbf{f}_A)_{NonGrav} + m_A \mathbf{g} = m_A \mathbf{a}|_{A-B}. \tag{3}$$

Here $\mathbf{g}$ is the gravitational acceleration on the ground. It should be noted that we don't need to count the gravitational forces from the outside of the earth system. This is far from being negligible since the orbit acceleration of the earth around the sun reaches to $6\times10^{-3} m/s^2$. Besides, another empirical law well known to us is that at the planetary scale. Here the sun-centered reference frame is usually favored, and the empirical law which is satisfied by the mechanical experiments at the solar system is given by

$$(\mathbf{f_1})_{SolarGrav} = m_1 \mathbf{a}|_{1-2}. \tag{4}$$

Here 1 denotes the celestial body under study, and 2 denotes the sun-centered reference frame. $(\mathbf{f_1})_{SolarGrav}$ denotes the gravitational forces exerted on the particle 1 from the matter in solar system, rather than the total forces exerted from the whole universe.

In the practical application, both the equations (3) and (4) have achieved a remarkable success in precision. But if we make a comparison between equations (3-4) and theoretical formula of Newton's second law (1), we can find that the reference frames actually used in (3) and (4) are not the inertial reference frame which is compulsory required by (1). At the same time, their forces being counted are not the total forces which is also theoretically required by Newton's second law (1). In fact, applying Newton's second law to explain above successful empirical laws is a complex and subtle thing[15]. Roughly, it can be understood according to the following steps. First, a gravitationally bounding system at a more large scale should be adopted as an approximated inertial reference frame, meanwhile the count of the forces should also be limited to that exerted only by the matter inside of this more large scale system. According to the relative acceleration between this approximated inertial reference frame and the actually used reference frame under study, the inertial force in this case can be calculated. Second, since this actually used reference frame is not an exact inertial reference frame, this inertial force should be added into the equation (1) by hand so we can approximately obtain empirical laws like (3) and (4). However, if we want to further explain why the count of the forces in the reference frame of above more large scale system can be limited only from this current system, we must invoke above steps once again[15]. Therefore, the Newton's second law, as a theoretical formula, fails to give a concise and elegant physical picture in its interpretation of empirical laws of classical mechanics experiments.

3 CAUSAL CONSISTENCY PRINCIPLE AND NEW FORMULISM OF PARTICLE DYNAMICS

Since for Newton's second law, neither the current theoretical formula nor the physical picture is satisfactory, we consider whether it is possible to reconstruct the physical logic of particle dynamics. In this process, the only one most fundamental principle which can be resorted to is the causal consistency principle.

Essentially, Newton's second law should be a causal law of particle dynamics. Here the forces exerted on the particle under study should be the cause and the resulting acceleration should be the effect. In the traditional theoretical formula of Newton's second law : $\mathbf{F}|_p = m_p\mathbf{a}|_{p-O}$, the left hand side $\mathbf{F}|_p$ is the total force exerted on the particle p from the whole universe, which only depends on p. Yet the right hand side $\mathbf{a}|_{p-O}$ is the acceleration of the particle p with respect to the reference frame O, equivalently measured relative to the reference object O. Therefore in fact, the effect (namely the result) $\mathbf{a}|_{p-O}$ depends not only on the particle p, but also on the reference object O which corresponds to the origin point of the reference frame. In this sense, the causality of Newton's second law is not symmetric and consistent in the form. In other words, if the current theoretical formula of Newton's second law is desired to be directly applicable into any practical reference frames, since any practical reference frames must be defined according to real reference objects, there is only the motion of the reference object gets involved into the current theoretical formula of Newton's second law, but the corresponding force exerted on the reference object itself is never under consideration. This is the very point to account for why Newton's second law is theoretically valid only in inertial reference frames, but none of them can be found in practice.

Then how to solve the problem of causal inconsistency? The key point is how to describe the corresponding effect according to the causal consistency principle, if the total force from the whole universe exerted on the particle is considered. We still suppose that the particle dynamics is certainly to be a theory with causal principle. We regard forces as the cause, and regard accelerations as the effect(result). The total force exerted on a single particle should be objective, namely it will not change with the variation of observers. Therefore, the corresponding effect should also be objective, and not relevant with any reference frame. In this way, a completely objective acceleration can only be expressed as the acceleration with respect to the absolute background of the whole universe,

$$\mathbf{F}|_p = m_p \frac{d^2}{dt^2} \bigodot |_p. \tag{5}$$

Here the objective position of the particle p in the absolute background of space is particularly denoted by $\bigodot |_p$.

The conjecture that an absolute background exists for the space of the universe is mainly originated from our intuitional experiences. Therefore, the absolute background of space (may also be called cosmic spatial background) can be intuitively understood. As the name implies, the spatial background is just what still exist in a space region after all objects inside it were moved away. And the cosmic spatial background is just what still exist in the whole universe after all concrete objects in the universe were moved away. Here "absolute" means that the background of space exists homogeneously and infinitely, and is independent of the motion and distribution of any matter which exists in the universe. There is no concept of scales for the background of space-time since the background itself does not contain specific objects. On the contrary, only the scales of space-time is defined by proper events occurred in real objects. In principle, the unit scales of space will change with the spatial intervals of these unit proper events. For this reason, we must make a physical distinguish between the absolute background and the relative scales of space. For instance, the background of space just like a blank sheet of paper. Originally, there is no coordinates on it. It is the observation that requires the introduction of coordinate scales. Usually, we define the unit coordinate scales by resorting to the proper events occurred in specific objects, so the coordinate system is established.

We believe that the motion of all objects in the universe must be performed over this common absolute three-dimensional space background, because only on this basis then the existence of an objective dynamical law is possible. On the other hand, from the logical point of view, we also suppose that every natural principle describing pure relative law must have an absolute basis. There is one point should be emphasized. Here the existence is proposed for absolute background, instead of absolute scales or absolute space-time. Only in this way can the absolute background of space be mostly compatible with the solidest part of modern physics. Newton's absolute view of space-time mainly claims that the measurement of space-time intervals is absolute. In contrast, Einstein's special and general theory of relativity can be naturally interpreted as the change rules of space-time scales[17, 18]. All of them are irrelevant to the background of space-time. Moreover, in the logical deduction of Einstein's special theory of relativity, it is easy to see that every event is assumed to have an objective position[17]. In essence, this is the reflection of the existence of absolute background.

Logically, the background should exist for the space of our universe although it is not concrete like general objects. For example, we have known that the dimensions of cosmic spatial background should be of three. But for the absoluteness of cosmic spatial background, strictly speaking, it is an assumption at the level of natural philosophy. Only based on the existence of an absolute background of space, it is possible for us to assume that any particle at any time has its objective position in the universe. We also conjecture that both the number of types of interactions in the universe and their calculation rules are objective. In principle, all kinds of interactions can be recognized and understood by people, and their calculation rules can be ultimately obtained. The reason is that all these interactions must be able to interpret the motions of all objects in the universe, simultaneously and self-consistently.

Above all, to say at least, the existence of the absolute background of space itself is the underlying part of Newton's absolute view of space-time, under the framework of classical mechanics. Therefore, in this paper it always be rational for us to discuss the formulism of particle dynamics based on the absolute background of space. Although every particle has its objective position in the absolute background of space, there is still a problem that the objective position in cosmic spatial background can not be directly measured. What we can really measure is the difference between any two objective positions, which substantially constructs a mathematical vector,

$$\mathbf{r}|_{p-O} = \bigodot|_p - \bigodot|_O. \tag{6}$$

After that, we are able to construct a particle dynamical equation which is really available to any observers. In fact, every reference frame must be established on a real reference object. Otherwise, there would be no reference value in measuring any object's motion in the natural world. Therefore, a physical reference frame must be the real reference frame. As for the relationship between the reference object and the reference frame, the reference frame can be naturally established by identifying the reference object as its origin point if we assign a reference object first in practical cases. Otherwise, in principle any real object which is fixed in the reference frame can be identified as its reference object if the reference frame is assigned first. All objects in the universe, including objects under study (p) and reference objects (O), should be of equal status in the most fundamental law of dynamics. For example, the dynamics of any real reference object should also satisfy

$$\mathbf{F}|_O = m_O \frac{d^2}{dt^2} \bigodot|_O. \tag{7}$$

Here the reference object O naturally corresponds to the origin point of a reference frame, so we can establish a reference frame which is irrotational with respect to the absolute background of space. The introduction of reference frames is just to make relative measurements on kinematical quantities. As a causal correspondence, the forces should also be relatively counted in nature.

$$m_O\mathbf{F}|_p - m_p\mathbf{F}|_O = m_p m_O \frac{d^2}{dt^2}(\bigodot|_p - \bigodot|_O) = m_p m_O \frac{d^2\mathbf{r}|_{p-O}}{dt^2}. \tag{8}$$

Finally, we obtain

$$\frac{\mathbf{F}|_p}{m_p} - \frac{\mathbf{F}|_O}{m_O} = \mathbf{a}|_{p-O}. \tag{9}$$

In this equation, the definition of the force and the acceleration are just the same as that in the traditional theoretical formula of Newton's second law (1). $\mathbf{F}|_p$ and $\mathbf{F}|_O$ are the total forces from the whole universe exerted on the particle

p and the reference object O respectively. m_p and m_O denote the mass of the particle p and the reference object O respectively. Since the reference object is definitely fixed in the reference frame, the acceleration $(\mathbf{a}|_{p-O})$ of the particle p is measured relative to the reference object, equivalently with respect to the origin point of the reference frame. Keep in mind, any physical reference frame must be related to a certain real reference object. Therefore, the equation (9) is just the dynamical equation of the particle p with respect to the reference frame O.

In the equation (9), the object under study and the reference object are now placed on an equal status. So the special status for the reference object in the universe is removed. It reflects that all objects in the universe have the equal status in dynamics. That is obvious as well, since what object should be selected as the particle under study, and what particle should be selected as the reference object are essentially assigned by people. In fact, there is no essential division of them. More importantly, the nature of the inertial force is nothing but the real force exerted on the reference object, and which is supposed to appear in the new dynamical equation (9) according to the principle of causal consistency. To demonstrate the difference between the equation (9) and the theoretical formula of Newton's second law (1), we may rewrite (9) to be,

$$\mathbf{F}|_p - \frac{m_p}{m_O}\mathbf{F}|_O = m_p\mathbf{a}|_{p-O}. \tag{10}$$

Here the left hand side of this equation can be called as a relative counting of forces. Obviously, the equation (10) has a net term $(-\frac{m_p}{m_O}\mathbf{F}|_O)$ more than Newton's second law, while the other terms are identical. The equations (9) and (10) are just the new dynamical equation which is proposed to replace the current theoretical formula of Newton's second law under the framework of classical mechanics, since the new dynamical law (9) or (10) presents a more natural and concise physical picture based on reinterpreting all empirical laws from classical mechanics experiments. In the application of (9) or (10), it is easy to find that the inertial reference frame is no longer required and the inertial force is no longer introduced by hand.

4 PRACTICAL EXAMINATION OF THE NEW PARTICLE DYNAMICAL EQUATION

4.1 the realization of general principle of relativity and the nature of inertial force

The equation (10) can be directly applied in any reference frame which is irrotational with respect to the absolute background of space, and naturally interprets the inertial force. In other words, a quasi-general principle of relativity is realized. In the last paragraph of Sec.5, it will be illustrated that such a general principle of relativity is more realistic than Einstein's general principle of relativity. Obviously, $\mathbf{F}|_p$ and $\mathbf{F}|_O$ in the equation (10) can be calculated respectively according to existing knowledge of all types of interactions. In principle any real object or arbitrary part of an existing object can be identified as a reference object. Therefore, the equation (10) can be applied in all irrotational reference frames. Moreover, according to the equation (10), the inertial force is essentially the real force exerted on the reference object, which must be deducted in a relative counting of forces. To illustrate this point, we assume that there are two relatively accelerated reference frames. Their reference objects are respectively denoted as O and O'. For above two reference frames, it is the difference in their forces being exerted that results in a relative acceleration between these two reference frames. In Newtonian mechanics, such a relative acceleration is depicted by a so-called inertial force. But now the relative acceleration between reference frames is naturally interpreted by the difference in their forces being exerted,

$$\frac{m_p}{m_{O'}}\mathbf{F}|_{O'} - \frac{m_p}{m_O}\mathbf{F}|_O = m_p\mathbf{a}|_{p-O} - m_p\mathbf{a}|_{p-O'} = m_p\mathbf{a}|_{O'-O} = -\mathbf{f}|_{inertial}. \tag{11}$$

The equation (11) self-consistently goes back to the new dynamical equation (9) when it is applied on reference objects O and O'. Above analysis illustrates that the new dynamical equation (9)(or (10)), compared with Newton's second law, has been endowed with a more complete and symmetric theoretical structure and so possesses the real universality. More importantly, in the application of (9)(or (10)), the inertial force is no longer introduced by hand.

4.2 The validity of the new dynamical equation

It can be proved that all successful empirical laws from classical mechanics experiments can also be naturally accounted for by the new dynamical equation (10). In general, we assume two arbitrary objects from the solar system denoted by 1 and 2(such as the Moon and Venus). The total force from the whole universe exerted on the particle i can be written down as (Here we assume that non-gravitational forces are negligible)

$$\mathbf{F}|_i = (\mathbf{f}_i)_{NonGrav} + (\mathbf{f}_i)_{SolarGrav} + (\mathbf{f}_i)_{OutSolarGrav} \approx (\mathbf{f}_i)_{SolarGrav} + (\mathbf{f}_i)_{OutSolarGrav}. \tag{12}$$

Here $(\mathbf{f}_i)_{SolarGrav}$ means the gravitational forces exerted on the particle i from the matter in solar system, and $(\mathbf{f}_i)_{OutSolarGrav}$ the gravitational forces exerted on the particle i from the matter outside of the solar system. According to the equations (10) and (12), the relative dynamics between 1 and 2 can be expanded as

$$\mathbf{F}|_1 - \frac{m_1}{m_2}\mathbf{F}|_2 \simeq [(\mathbf{f_1})_{SolarGrav} - \frac{m_1}{m_2}(\mathbf{f_2})_{SolarGrav}] + [(\mathbf{f_1})_{OutSolarGrav} - \frac{m_1}{m_2}(\mathbf{f_2})_{OutSolarGrav}] = m_1\mathbf{a}|_{1-2}. \tag{13}$$

Owing to both 1 and 2 being bound in the same solar system, the gravitational forces exerted from the matter outside of the solar system satisfy,

$$\frac{(\mathbf{f_1})_{OutSolarGrav}}{(\mathbf{f_2})_{OutSolarGrav}} \approx \frac{m_1}{m_2}. \tag{14}$$

So the relative dynamics (13) between objects 1 and 2 can be approximately rewritten as

$$\mathbf{F}|_1 - \frac{m_1}{m_2}\mathbf{F}|_2 \approx [(\mathbf{f_1})_{SolarGrav} - \frac{m_1}{m_2}(\mathbf{f_2})_{SolarGrav}] = m_1\mathbf{a}|_{1-2}. \tag{15}$$

Specifically, if we select the center of sun as the origin point (the location of particle 2) of the reference frame in the above equation, at that time the reference object 2 can be the whole sun, or can be any real object located at the center of sun. For instance, if we choose the whole sun as the reference object, the second term in the left hand side of the above equation would be suppressed since $m_1 \ll m_2$. Otherwise, we may choose a real object at the center of sun as the reference object 2, which has a comparable mass as the object 1. Since for the reference object 2, the distribution of the matter in the solar system has a high approximated spherical symmetry, the forces exerted on the reference object 2 from the solar system may usually be approximated to be zero: $(\mathbf{f_2})_{SolarGrav} \approx 0$. Therefore, if we don't desire a high precision, the dynamics of object 1 can always be expressed as

$$\mathbf{F}|_1 - \frac{m_1}{m_2}\mathbf{F}|_2 \approx (\mathbf{f_1})_{SolarGrav} = m_1\mathbf{a}|_{1-2}. \tag{16}$$

That is the empirical law well satisfied by celestial bodies in the solar system, namely the equation (4) is recovered. Obviously, above empirical law is substantially an approximation of the new dynamical equation (10) when it is applied in gravitational bound systems (It undergoes about two steps of approximations). The approximation such as (16) is valid for usual sun-centered reference frame, earth-centered reference frame and innumerable other gravitational bound systems in our universe. Even for the most common ground-based reference frame, it substantially comes down to the earth-centered reference frame. Therefore, all successful empirical laws from classical mechanics experiments can be naturally accounted for again by the new dynamical equation (10). Besides, why the gravitational forces exerted from the outside of the solar system do not have to be taken into account? The direct reason is not because the magnitude of these forces is so small that all of them can be ignored, but owing to the fact that these forces are deducted in the relative counting of forces.

Now we consider the case that the new dynamical equation (10) is applied in ground-based reference frame. We assume that A is the particle under consideration which is moving on the ground and B denotes the reference object which is at rest with respect to to the earth's surface. According to the equation (10), the relative dynamics between A and B can be directly expressed as

$$\mathbf{F}|_A - \frac{m_A}{m_B}\mathbf{F}|_B = m_A\mathbf{a}|_{A-B}. \tag{17}$$

In theory, the total forces exerted on the A and B can be expanded as

$$\begin{aligned} \mathbf{F}|_A &= (\mathbf{f}_A)_{OutEarthGrav}(\propto m_A) + (\mathbf{f}_A)_{EarthGrav}(\propto m_A) + (\mathbf{f}_A)_{NonGrav}, \\ \mathbf{F}|_B &= (\mathbf{f}_B)_{OutEarthGrav}(\propto m_B) + (\mathbf{f}_B)_{EarthGrav}(\propto m_B) + (\mathbf{f}_B)_{NonGrav}. \end{aligned} \tag{18}$$

The non-gravitational forces exerted on B can be equivalently calculated from the case that B is at rest with respect to a smooth and horizontal surface for the force balance. In that case, the non-gravitational force must be numerically equal to the support force from the ground,

$$(\mathbf{f}_B)_{NonGrav} = -m_B\mathbf{g}. \tag{19}$$

Substituting it into the equation (17), at the same time we adopting an approximation that the gravitational forces from the earth and outside of the earth can all be counterbalanced between A and B, we finally obtain,

$$\begin{aligned}\mathbf{F}|_A - \frac{m_A}{m_B}\mathbf{F}|_B &= (\mathbf{f}_A)_{NonGrav} - \frac{m_A}{m_B}((\mathbf{f}_B)_{NonGrav}) = (\mathbf{f}_A)_{NonGrav} - \frac{m_A}{m_B}(-m_B\mathbf{g}) \\ &= (\mathbf{f}_A)_{NonGrav} + m_A\mathbf{g} = m_A\mathbf{a}|_{A-B}.\end{aligned} \tag{20}$$

This is the empirical law well satisfied by the classical mechanics experiment when it is conducted and analyzed under ground-based reference frames, namely the equation (3) is recovered. There is one point which should be emphasized. The change from $(\mathbf{f}_A)_{EarthGrav}$ in (18) to $m_A\mathbf{g}$ in (20), is usually interpreted by a fictitious inertial force owing to the rotation of the earth. But now we can see that it actually should be attributed to the real force from the ground exerted on the reference object B. Broadly speaking, the surface of the earth is rigid in spatial distance relative to the center of the earth, so the measurement of kinematical effect in ground-based reference frames is equivalent to that relative to the center of earth if the rotation of the earth is not taken into account. Accordingly, for the part of counted force, the gravitational forces exerted from the outside of the earth system should not be considered.

Furthermore, if we assume the reference object B an arbitrary object moving over the ground(including the case of acceleration), at that time $(\mathbf{f}_B)_{NonGrav} \neq -m_B\mathbf{g}$, so the formula of particle dynamical equation (20) should be changed into,

$$(\mathbf{f}_A)_{NonGrav} - \frac{m_A}{m_B}(\mathbf{f}_B)_{NonGrav} = m_A\mathbf{a}|_{A-B} = [(\mathbf{f}_A)_{NonGrav} + m_A\mathbf{g}] - \frac{m_A}{m_B}[(\mathbf{f}_B)_{NonGrav} + m_B\mathbf{g}]. \tag{21}$$

Therefore, in all senses, the empirical laws from classical mechanics experiments can always be regarded as the approximation of the new dynamical equation (9) (or (10)) under some special conditions.

4.3 the universality of the new dynamical equation

The equation (10) is directly applicable in the relative dynamics between galaxy clusters. For instance, we assume that there are only two particles (such as 1 and 2) existing in the whole universe, and there is only gravitational interaction between them. In the framework of Newtonian mechanics, none of real particles can be approximated as an inertial reference frame, so Newton's second law can not be directly applied in this situation. Even someone resorts to the center of mass for this system, but essentially the center of mass method can be finally attributed to an assumption that the center of mass is at rest with respect to a certain inertial reference frame. However, according to the equation (10) the relative dynamics between above two particles can be directly written down as

$$\mathbf{F}|_1 - \frac{m_1}{m_2}\mathbf{F}|_2 = (\frac{Gm_1m_2}{r^3}\mathbf{r}_{1\to 2}) - \frac{m_1}{m_2}(\frac{Gm_1m_2}{r^3}\mathbf{r}_{2\to 1}) = m_1\mathbf{a}|_{1-2} = m_1\frac{d^2\mathbf{r}_{2\to 1}}{dt^2}. \tag{22}$$

This case also shows that in the application of the new dynamical equation (9) (or (10)), the inertial reference frame is no longer required.

5 the Relationship between the New Dynamical Equation and Newton's Second Law

By now we have seen that, the new dynamical equation (9) naturally accords with existing all classical mechanics experiments. In the meantime, the new dynamical equation (9) is more universal than Newton's second law. Especially, the theoretical necessity of the existence of inertial reference frame and inertial force is logically removed from the formulism of particle dynamics. But more importantly, the new dynamical equation (9) is more in line with the causal consistency principle. Even so, there is one thing should still be pointed out that the new dynamical equation (9) can be exactly derived under the framework of Newtonian mechanics. Since in all concepts of reference frame, besides the property of space-time scales, there is only the reference object is physical. Therefore under the framework of classical mechanics, any object's dynamical problem in any reference frame can be substantially regarded as a two-body problem, if the space-time scales have been assumed to be constant for all reference frames. Now we derive the dynamical equation based on Newtonian mechanics to solve such a two-body problem. Firstly, we assume there really exists an inertial reference frame which is denoted by Ω. According to the theoretical formula of Newton's second law, an arbitrary object p satisfies,

$$\mathbf{F}|_p = m_p\mathbf{a}|_{p-\Omega}. \tag{23}$$

Here, the forces $\mathbf{F}|_p$ is defined to include all the forces exerted on the particle p from the whole universe. Similarly, to an arbitrary reference object O, which must also obey the same natural law,

$$\mathbf{F}|_O = m_O \mathbf{a}|_{O-\Omega}. \tag{24}$$

Performing a simple algebraic manipulation, we obtain

$$\mathbf{F}|_p - \frac{m_p}{m_O}\mathbf{F}|_O = m_p \mathbf{a}|_{p-\Omega} - \frac{m_p}{m_O}(m_O \mathbf{a}|_{O-\Omega}) = m_p[\mathbf{a}|_{p-\Omega} - \mathbf{a}|_{O-\Omega}] = m_p \mathbf{a}|_{p-O}. \tag{25}$$

Therefore, under the framework of Newtonian mechanics, the new dynamical equation (10) is recovered. Why the new form of particle dynamical equation was not explored and not paid enough attention in past several centuries, I think a possible reason is that people have never recognized that the dynamical dependence on reference frames can be finally attributed to their real reference objects[15].

From above analysis, the new dynamical equation (9)(or (10)) is more universal than traditional Newton's second law, but now it is proved that this new dynamical equation is able to be derived totally under the framework of Newtonian mechanics. If both of them are correct, there must be some conflicts between them. In essence, this logical problem can be avoided. But the premise is that the Newton's second law can not be regarded as a theoretical formula just like in traditional Newtonian mechanics. More exactly, Newton's second law can only be regarded as an empirical law. The reason is that an empirical law don't need to deal with the total force exerted on the particle p under study, but only need to calculate the new additionally exerted force on the basis of previous mechanical state. The quantitative relation between this new additionally exerted force and the resulting relative acceleration is just the empirical law obtained by Newton in his time from ground-based laboratory mechanical experiments. If we still regard Newton's second law as a theoretical formula, at first the forces exerted on the particle p must be defined to count all the forces which are exerted on the particle p from the whole universe. Otherwise, when the equation is applied into concrete cases, we will not be able to make it clear what forces should be included in the count, and what forces should not be counted. In other words, the classical mechanics experiments can not directly tell us the theoretical formula of particle dynamical equation. Inversely, we also find that the theoretical formula (1) of Newton's second law can not be exactly applied in any practical cases, and putting an inertial force by hand is always needed.

How to correctly view the role played by Newton's second law. In history, Newton's second law does truly come from a lot of classical mechanics experiments. However, due to the constraints from contemporary historical circumstances and the limited knowledge of the universe at that time, most of these classical mechanics experiments were conducted and analyzed under the condition of the ground-based laboratory reference frame being selected. The resulting empirical law can only depict the quantitative relation between the new additionally exerted force and the resulting relative acceleration under the premise of reference frame being fixed. In this sense, Newton's second law is essentially an empirical law, a phenomenological law, and can not be directly promoted as a theoretical formula without any changes. Therefore, we must further distinguish such an empirical law from the traditional theoretical formula of Newton's second law. If we want to promote such an empirical law to be a theoretical formula which can be universally applied in practice, a further reformulation according to the causal consistency principle and general principle of relativity must be required. Under this guidance, we reconstruct the physical logic of particle dynamics to obtain a new equation (9) which can actually be regarded as an indispensable development of Newton's second law when the latter is restored to be an empirical law.

Looking back, why the new dynamical equation (9)(or (10)) is more universal than Newton's second law, but it seems to be able to be derived from Newton's second law? The real reason is that the absolute background of space is exactly not a reference frame. But in the derivation of (25) the role of the absolute background of space can be equivalently played in mathematics by an inertial reference frame, which is just required by the theoretical formula of Newton's second law. Of course, the true evidence supporting the new dynamical equation (9) can only come from the natural coincidence between the new dynamical equation and existing experiments of classical mechanics, which has been demonstrated in the forth part of this paper.

Finally, a moderate general principle of relativity is essentially a practical requirement. On one hand, we are never able to know about the actual state of motion of our terrestrial reference frame where our observers exist. On the other hand, we are always able to determine the rotation of any reference frame with respect to cosmic spatial background by resorting to the galaxies far enough away since the cosmic spatial background is objective and motionless. In principle, the exact rotation of any practical reference frame with respect to the absolute background of space can be mathematically solved because the reference frame given in practice must interpret the dynamics for all objects in the universe, simultaneously and self-consistently. Therefore, what the practical observation really requires is that dynamical laws must keep form invariant to any reference frame which is irrotational with respect to the absolute background of space. Furthermore, in particle dynamics, the rotation phenomena can always be attributed to the relative motion between different particles. But for single particle, there is no concept of rotation. In other words, once any reference object is regarded as a particle, no problem of rotation exists for reference particle. Just as the problem of variable mass system under the framework of Newtonian mechanics, the variable mass phenomena should be attributed to the relative motion between different particles in the system of particles, so the fundamental particle dynamical equation is still $\mathbf{F}|_p = m_p \mathbf{a}|_{p-O}$. But for $\mathbf{F}|_p = \frac{d(\mathbf{p}|_{p-O})}{dt}$, it actually can be generalized from the former equation when a system of particles is considered. In this sense, the problem of reference frames' rotation is essentially a mathematical problem. Ultimately, the problem of the rotation of reference frames can be separated from the problem of dynamical relativity.

6 PHYSICAL SIGNIFICANCE OF THE NEW DYNAMICAL EQUATION

First, the new dynamical equation can somewhat improve the precision in the physical application. In principle, we should always make an approximation on the inertial reference frame before Newton's second law can be applied. Actually, such an approximation is made in theory, rather than in practical measurement. Therefore, if the contribution from the forces exerted on the reference object can not be ignored, the error would be significant. By contrast, if we adopt the equation (9) as the new particle dynamical law, it at least has entirely solved the problem of the approximation on inertial reference frames. Since some research has indicated that the flat problem of galaxy rotation curve may be elegantly solved if the Newtonian dynamics is modified on the level of very small acceleration[10–12], it deserves to further investigate what kind of impact might be brought by the application of the new dynamical equation (9).

Second, the new dynamical equation (9) illustrates that a more realistic general principle of relativity for particle dynamics can be realized in a very simple and natural approach which is obviously different from Einstein's view[13, 16, 18]. Because the concept of inertial force still exists in Einstein's theory of relativity, and Einstein's (strong) equivalence principle further claims that the inertial force is equivalent to the gravitational force on all their physical effects. However, we can see from the equation (9), which obviously is more natural and simple in physical picture. The nature of the inertial force is the real force exerted on the reference object. Therefore, the so-called inertial force can actually be all kinds of interactions such as the gravitational interaction, electromagnetic interaction and so on. It will bring some influence on the physics of space-time[19] since so far we know that only the gravitational interaction has the time dilation effect. In this sense, we may have found a counterexample of Einstein's (strong) equivalence principle.

Third, the new particle dynamical equation (9) can be proved to be compatible with the existing theoretical structure of classical mechanics. On the one hand, if we adopt the new dynamical equation (9) as the fundamental particle dynamical equation in classical mechanics, there emerges a new question immediately. How should we deal with many theorems and deductions based on Newton's second law in classical mechanics? In fact, this problem can be easily solved without any unnaturalness. Because it doesn't matter what is the real counting range of $\mathbf{f}$ which gets involved in the derivation of these theorems and deductions. In other words, whether the force term $\mathbf{f}$ should be totally counted $\mathbf{f} = \mathbf{F}|_p$, or relatively counted $\mathbf{f} = \mathbf{F}|_p - \frac{m_p}{m_O}\mathbf{F}|_O$, does not affect the validity of the formulas of these theorems and deductions. Therefore, the new dynamical equation can be naturally and easily incorporated into existing theoretical structure of classical mechanics, and the only thing need to do is that all force terms appeared in these theorems and deductions should be understood as relatively counted forces between the particle under study and the reference object. On the other hand, as we have known, another formulism of classical mechanics is analytical mechanics. Under the least action principle of analytical mechanics, forces are depicted by potential energies. However, there is a premise that under the condition of reference frames being fixed, any work which is done by forces should be path independent, and completely determined by particles' initial and final states. Therefore, the real meaning of potential energy is a relatively counted energy which is defined directly based on the relative position between the particle under study and the reference object. In this sense, The spirit to relatively count forces in our new dynamical equation (9) is naturally in harmony with analytical mechanics.

7 CONCLUSION

In this paper we have discussed the formulism of particle dynamics under the framework of classical mechanics. The main motivation comes from that a theoretical problem of depending inertial reference frame still exists in the theoretical formula of Newton's second law. Objectively speaking, Newton's second law is essentially an empirical law from ground based experiments which depicts the quantitative relation between the new additionally exerted force and the resulting relative acceleration. We must thereby distinguish such an empirical law from the traditional theoretical formula of Newton's second law. Consequently, a further reconstruction of particle dynamics based on this empirical law is required according to a moderate general principle of relativity. In precision, the new dynamical equation (9) is at the same level as traditional Newton's second law when the latter approaches to its theoretical limits. In formulism, the new dynamical equation (9) can be directly applied in any reference frame without assuming an inertial reference frame and introducing a fictitious inertial force by hand. In concept, the new dynamical equation (9) removes the special status of inertial reference frames and inertial forces from its application, and presents a more concise physical picture based on reinterpreting all existing experiments for classical mechanics. Therefore, we suggest that the current theoretical formula of Newton's second law (1) should be replaced by the new dynamical equation (9) in the actual mechanics analysis.

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[1] Chen Shouzhu, Jiang Zhiyong. General Physics, Higher Education Press, Sixth Edition, 2006.
[2] Douglas M J. Leibniz on the Foundations of the Calculus: The Question of the Reality of Infinitesimal Magnitudes. Perspectives on Science, 1998, 6, 6-40.
[3] Landau, L.D., Lifshitz, E.M, Mechanics Course of Theoretical Physics, [Volume 1](Franklin Book Company, 1972). ISBN 0-08-016739-X.
[4] Feynman, R., Lectures on Physics,(Perseus Publishing, 1999). ISBN 0-7382-0092-1.
[5] Daniel, K., Robert J. K., An Introduction to Mechanics, (McGraw-Hill, 1973). ISBN 0-07-035048-5.
[6] Herbert, G., Charles, P.P., Stewart, J.L., Classical Mechanics, (Addison Wesley, 3rd ed, 2002). ISBN 0-201-65702-3.
[7] Alexander, L F., John D W., Theoretical Mechanics of Particles and Continua, [33C39] (Courier Dover Publications, 2003). ISBN 0-486-43261-0.
[8] Essen H. Mechanics, cosmology and Mach's principle, Eur.J.Phys. 2013,34,139-145.
[9] Sofue Y., Rubin V. rotation curves of spiral galaxies, Annu. Rev. Astron. Astrophys. 2001, 39, 137.
[10] Milgrom M. A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis. Astrophys. J. 1983, 270, 365.
[11] Ignatiev A. Y. Is violation of Newton's second law possible. Phys.Rev.Lett. 2007, 98:101101.
[12] Gundlach J. H., Schlamminger S., Spitzer C. D. et al. Laboratory Test of Newton's Second Law for Small Accelerations. Phys.Rev.Lett. 2007, 98, 150801.
[13] Liao, L., General Relativity, [26-30; 188-190](High Education Press, Shanghai, 1987).
[14] Harald I., A Modern Approach to Classical Mechanics, [180](World Scientific, 2002). ISBN 981-238-213-5.
[15] Hanno Essen, Mechanics, cosmology and Mach's principle, Eur.J.Phys. 34 (2013) 139-145
[16] Bernard, F.S., A first course in general relativity, (Cambridge Univers press, 1985).
[17] Zhang Sanhui, University Fundamental Physics, Tsinghua University Publishing House, 2009. Li Kang, Yang Jiansong. Introduction to Modern Physics, Tsinghua University Publishing House, 2011:8.
[18] Weinberg, S., Gravitational and Cosmology: Principles and Applications of the General Theory of Relativity, [Part 3](John Wiley & Sons, Inc, New York. 1972).
[19] Chen, C.Y., A Modification of the Standard Cosmological Metric, hep-th/0411047v10; Chen, C.Y., The Realization of General Principle of Relativity in Particle Dynamics, hep-th/0312225v9.