

A possible unimportant but sure interesting conjecture about primes

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Abstract. Studying, related to Fermat pseudoprimes, my main object of study, the concatenation and the primes of the form $n \cdot p - n + 1$, where p is also prime, I found incidentally an interesting possible fact about primes. Because the proof or disproof of the conjecture, and also the consequences in the case that is true, are beyond me, I shall limit myself to enunciate the conjecture and give few examples.

Conjecture:

Every prime p that ends in a group of digits that form a prime can be written at least in one way as $p = n \cdot q - (n - 1) \cdot r$, where n is positive integer, $n > 1$, and q, r another primes that ends in the same group of digits.

Examples:

The numbers 11, 211, 311, 811, 911, 1511, 1811, 2011, 2111, 2311 are the first ten primes that end in the digits 11, that form a prime.

These primes can be written as:

: $11 = 3 \cdot 211 - 2 \cdot 311$;
: $211 = 7 \cdot 811 - 6 \cdot 911$;
: $311 = 6 \cdot 811 - 5 \cdot 911$;
: $811 = 4 \cdot 211 - 3 \cdot 11$;
: $911 = 3 \cdot 311 - 2 \cdot 11$;
: $1511 = 5 \cdot 311 - 4 \cdot 11$;
: $1811 = 9 \cdot 211 - 8 \cdot 11 = 6 \cdot 311 - 5 \cdot 11 = 2 \cdot 911 - 1 \cdot 11$;
: $2011 = 10 \cdot 211 - 9 \cdot 11 = 3 \cdot 811 - 2 \cdot 211$;
: $2111 = 7 \cdot 311 - 6 \cdot 11 = 2 \cdot 1511 - 1 \cdot 911 = 2 \cdot 1811 - 1 \cdot 1511$;
: $2311 = 3 \cdot 911 - 2 \cdot 211$.

The numbers 29, 229, 829, 929, 1129, 1229, 1429, 2029 are the first eight primes that end in the digits 29, that form a prime.

These primes can be written as:

: $29 = 9 \cdot 829 - 8 \cdot 929 = 3 \cdot 829 - 2 \cdot 1229 = 4 \cdot 929 - 3 \cdot 1229$;
: $229 = 7 \cdot 829 - 6 \cdot 929 = 6 \cdot 1229 - 5 \cdot 1429 = 3 \cdot 1429 - 2 \cdot 2029$;
: $829 = 4 \cdot 229 - 3 \cdot 29 = 3 \cdot 1229 - 4 \cdot 1429 = 2 \cdot 1429 - 1 \cdot 2029$;
: $929 = 3 \cdot 1129 - 2 \cdot 1229$;

: $1129 = 3 \cdot 929 - 2 \cdot 829;$
 : $1229 = 6 \cdot 229 - 5 \cdot 29;$
 : $1429 = 7 \cdot 229 - 6 \cdot 29;$
 : $2029 = 10 \cdot 229 - 9 \cdot 29 = 4 \cdot 1429 - 3 \cdot 1229 = 3 \cdot 1429 - 2 \cdot 1129.$

Observation:

Seems that the conjecture above can be extended for primes that end in a group of digits that form not a prime but a square of prime.

Example:

The numbers 3529, 6529, 10529, 21529, 27529, 30529, 33529, 36529 are eight from the first ten primes that end in the digits 529, that form a square of a prime. These primes can be written as:

: $3529 = 4 \cdot 21529 - 3 \cdot 27529 = 5 \cdot 27529 - 4 \cdot 33529;$
 : $6529 = 2 \cdot 3529 - 1 \cdot 529 = 5 \cdot 30529 - 4 \cdot 36529;$
 : $10529 = 4 \cdot 7529 - 3 \cdot 6529;$
 : $21529 = 7 \cdot 3529 - 6 \cdot 529 = 6 \cdot 6529 - 5 \cdot 3529;$
 : $27529 = 9 \cdot 3529 - 8 \cdot 529 = 8 \cdot 6529 - 7 \cdot 3529;$
 : $30529 = 10 \cdot 3529 - 9 \cdot 529 = 5 \cdot 6529 - 4 \cdot 529;$
 : $33529 = 10 \cdot 6529 - 9 \cdot 3529 = 2 \cdot 27529 - 1 \cdot 21529;$
 : $36529 = 6 \cdot 6529 - 5 \cdot 529 = 11 \cdot 6529 - 10 \cdot 3529.$

Note: Obviously, in this case is admitted for r from the expression $p = n \cdot q - (n - 1) \cdot r$ to be not just a prime but the ending group of digits itself, that form a square of prime.

Observation:

The conjecture above can also be extended for Fermat pseudoprimes that end in a group of digits that form a prime. For instance, the number 1729 is the first absolute pseudoprime that ends in the digits 29, and can be written as $1729 = 2 \cdot 1429 - 1 \cdot 1129$, where 1429 and 1129 are primes.