Asymmetry Due to Quantum Collapse

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Abstract

This paper points out an internal tension between quantum collapse and expressions which set eigenstates equal to superposition states in a different basis and thereby imply that pre-measurement and immediate post-measurement states are of the same kind. Its resolution appears to be either to discard the collapse postulate or to consider such states to be of distinct kinds with respect to their association with a superposition of properties.

Keywords: Quantum collapse, Measurement problem

1 Introduction

Quantum collapse is a postulate in orthodox quantum mechanics, but it is poorly understood, though much has been written about it [1]. Here we point out that it also leads to a subtle internal tension in the theory when it is considered in conjunction with expressions that equate eigenstates with superposition states in a different basis because such expressions are consistent with a symmetry between states and observables under quantum superposition (to be elaborated below) that is broken by quantum collapse. The tension can be relieved either by discarding the collapse postulate or by keeping it and considering pre-measurement and immediate post-measurement states to be of distinct kinds, in which case quantum collapse must be considered a transformation between two kinds of states.

The basic strategy employed here is to convert a standard quantum mechanical relation into a logical equivalence, and then show that the imposition of the collapse postulate makes it possible to formulate an argument such that the truth value of the conclusion depends on which of the implications that are part of the equivalence is used, from which it is then possible to derive a contradiction.

2 An Internal Tension

From the fact that a quantum state $|\Psi\rangle$ is a vector in Hilbert space follows that any eigenstate $|\psi_i\rangle$ that is part of the superposition that constitutes $|\Psi\rangle$ in some basis can be re-expressed itself as a superposition state in a different basis of measurement outcomes. Considering 2-level spin states for simplicity and without loss of generality, we can for example express the basis vector $|{+\mathbf{x}}\rangle$ as

$$|{+\mathbf{x}}\rangle = \frac{1}{\sqrt{2}}|{+\mathbf{z}}\rangle + \frac{1}{\sqrt{2}}|{-\mathbf{z}}\rangle \tag{1}$$

where the coefficients are of course the amplitudes for the states $|{\pm\mathbf{z}}\rangle$ to be in $|{+\mathbf{x}}\rangle$. To make the asymmetry obvious, consider (1) as a logical equivalence and re-express it in terms of the conjunction of two implications. Then we have

$$|\Psi\rangle = |{+\mathbf{x}}\rangle \Leftarrow |\Psi\rangle = \frac{1}{\sqrt{2}}|{+\mathbf{z}}\rangle + \frac{1}{\sqrt{2}}|{-\mathbf{z}}\rangle \qquad \wedge \qquad |\Psi\rangle = |{+\mathbf{x}}\rangle \Rightarrow |\Psi\rangle = \frac{1}{\sqrt{2}}|{+\mathbf{z}}\rangle + \frac{1}{\sqrt{2}}|{-\mathbf{z}}\rangle \tag{2}$$

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Argument	Truth Values of Premises	Truth Value of Conclusion
$M \wedge N \wedge (Q \Rightarrow P) : R$	True, True, True	True
$M \wedge N \wedge (P \Rightarrow Q) : R$	True, True, True	False

Table 1: A truth table for the arguments for which the axioms of QM, the collapse postulate and each of the two conjoined implications serve as premises. The symbols M, N, P, Q and R are defined as follows:
M : The standard axioms of quantum mechanics, as found in any introductory textbook e.g. [3], *sans* collapse postulate
N : "Upon a measurement, a state $|\Psi\rangle$ reduces to an eigenstate $|\psi_i\rangle$ in the basis of measurement outcomes."
$P : |\Psi\rangle = |+\mathbf{x}\rangle$
$Q : |\Psi\rangle = \frac{1}{\sqrt{2}}|+\mathbf{z}\rangle + \frac{1}{\sqrt{2}}|-\mathbf{z}\rangle$
R :"It is possible to prepare a state $|\Psi\rangle$ such that the operator $\hat{s}_\Psi$ produces all the eigenvalues associated with the eigenstates of $|\Psi\rangle$ in the implied basis (i.e. the basis that appears after the implication sign) in a single measurement."
The problem, in a nutshell, is that if the truth value assignments in the above arguments are correct and the logical equivalence in (2) is true, then it is possible to construct the argument $M \wedge N \wedge (Q \equiv P) : R \wedge (\sim R)$, where $\sim R$ is the negation of R

where the equality to $|\Psi\rangle$ was added to convert each side of equation (1) into a logical statement. Let $\hat{s}_{+x}$ be the operator that produces eigenvalue $\hbar/2$ when it acts on $|+\mathbf{x}\rangle$. The implication on the left side of the conjunction entails that when $\hat{s}_{+x}$ acts on the state $\frac{1}{\sqrt{2}}|+\mathbf{z}\rangle + \frac{1}{\sqrt{2}}|-\mathbf{z}\rangle$ it is certain to produce $\hbar/2$ along the x-axis in a single measurement, since we can use that implication to rewrite this state as $|+\mathbf{x}\rangle$.
Now, if the quantum superposition principle were invariant under an interchange between quantum states and quantum observables, then this would constitute a symmetry of the theory, and by that symmetry, when we consider the right side of the conjunction we might have expected there to be an operator $\hat{s}_{(+z)+(-z)}$ (which is nothing other than $\hat{s}_{+x}$ in the $\mathbf{z}-$basis) such that when it acts on $|+\mathbf{x}\rangle$ it produces a superposition of the eigenvalues $\hbar/2$ and $-\hbar/2$ along the $z-$axis in a single measurement. For instance, if in a Stern-Gerlach (SG) experiment a beam of spin 1/2 particles in state $|+\mathbf{x}\rangle$ was carefully controlled so that only one particle at a time passed through magnetic fields inhomogeneous along the z-direction, and it was consistently found that each particle contributed both to the band that was associated with spin up *and* the band that was associated with spin down along the z-direction, then this would be an experimental realization of such a symmetry. Actually, it is quite possible that such a result would at first be interpreted as a situation in which a particle in state $|+\mathbf{x}\rangle$ has decayed into two others of opposite spin along the z-direction (the conservation of angular momentum issues that this raises could, for example, be addressed by positing undetected particles that balance everything out), but an experiment involving a modified SG device, first introduced by Feynman [2], could be used to support the symmetry interpretation. The modified SG device consists of an ordinary SG magnet followed by a magnet which has opposite polarity and is twice as long, and is followed by another ordinary SG magnet with the same polarity as the first. Under this set-up, a beam of particles separates out along the first half of the device according to whether they are spin up or spin down, and then recombines in the second half. In the case of a single-particle beam, it would seem more contrived to interpret this as a decay followed by a recombination in just the right way so that the particle exiting the set-up is exactly in the same state as it was when it entered it every single time, than to simply posit that quantum mechanics is symmetric between observables and quantum states with respect to quantum superposition.
The key observation here that there is nothing in (2) *per se* to prohibit the existence of such a symmetry, yet as best as we can tell it is not a feature of nature, and hence it is absent in quantum mechanics. One might argue that the definition of a quantum state as a probability amplitude conflicts with it, but recall that a quantum state can be conceptualized as a probability amplitude only because of the Born Rule, which, at least under the orthodox interpretation, incorporates the collapse postulate, a part of quantum mechanics external to (2). But if one accepts that the collapse postulate breaks this symmetry, then the implications can be separately written as part of the premises of two arguments which are otherwise identical, yet produce different conclusions (see table 1.)

This poses the following dilemma: The conclusion R has different truth values in each case but if, as equation (1) implies, $P \equiv Q$, then the truth value of R in both arguments should be the same, since the only difference in the premises is the implication used, and permitting a different truth value for the conclusion when the implication is changed allows one to derive a contradiction just from the axioms of quantum mechanics including the collapse postulate and any relation like (2).

The dilemma could be avoided if it were allowed that under certain circumstances $Q \Rightarrow P$ has a different truth value from $P \Rightarrow Q$ (e.g. by positing that at the level of predicate logic there are subtle differences between P and Q), for then the truth value of the premise could change when the implication is changed and so could, correspondingly, the truth value of R. But that would mean that $P \equiv Q$ is false. On the other hand, if it is assumed that $P \equiv Q$ is true and that R should really be either true or false in both arguments, then this would also solve the dilemma. But if R is false in both cases, then the negation of the conclusion of the first argument would contradict the rules of quantum mechanics applied to the case in which a quantum state happens to be an eigenstate in the measurement basis, and if R is true in both cases then that would mean that it is possible to prepare a quantum state such that operators like $\hat{s}_{(+z)+(-z)}$ produce superpositions of eigenvalues, and this conflicts plainly with the collapse postulate.

Ultimately, this dilemma is due to an internal tension between two parts of the theory, one part which permits (and in the stricter logical sense could even be said to *require*) immediate post-measurement superposition states and thus the observation of superpositions of eigenvalues, and another which prohibits them. Put differently, the internal tension arises because one part of the theory is consistent with a symmetry between quantum states and observables under quantum superposition, while another part is not.

The standard against which any possible interpretation of quantum mechanics is held is that if it permits an experimental outcome that is prohibited by the mathematical formalism, it is ruled out (see, for instance, a recent proof for which this was a key feature [4]). If (2) is considered a logical equivalence and held to this standard, then the internal tension between it and the collapse postulate is a problem.

3 Possible Resolutions

One way to resolve the internal tension is to discard the collapse postulate, which renders R true in both cases. Indeed, there exists a class of interpretations of quantum mechanics, called the Everettian or sometimes the Many-Worlds interpretations of quantum mechanics, which has historically been characterized at the most fundamental level by the omission of the collapse postulate [5]. As this seems to result in fewer assumptions than even standard quantum mechanics is based on, this is sometimes advertised as a theoretical virtue of these interpretations [6].

Omitting the collapse postulate renders a statement like R admissible when a quantum state is a superposition state in the implied basis, but because R is compatible with situations in which an operator like $\hat{s}_{(+z)+(-z)}$ produces superpositions of eigenvalues in the same observation (as in the example given earlier), this may pose a problem. In Everettian interpretations, each eigenvalue is taken to be realized in a separate branch of the wavefunction, and, based on the above considerations, just discarding the collapse postulate by itself may not be sufficient to ensure that this happens, as it appears that the possibility is still left open that in some branches an observer could observe a superposition of observables. This may be easily fixed by requiring each post-measurement eigenstate to be associated with a distinct Hilbert space, but then the question arises whether this does not re-introduce the above dilemma in a different guise. Incidentally, more modern treatments tend to favor presentations of the formalism in terms of positive operator valued measures (POVM's), rather than in terms of eigenvalues and quantum collapse [7]

The opposite way the tension can be resolved is by retaining the collapse postulate and considering pre-measurement and immediate post-measurement or "collapsed" states to be of intrinsically distinct kinds. This can be achieved by defining the latter explicitly to be associated with the absence of a superposition of properties, or, more formally, by considering them as states which, after an eigenvalue has been produced, are no longer vectors in Hilbert space so that e.g. relations like P and Q no longer apply to them. Then, the symmetry breaking of the collapse postulate manifests itself directly in the characteristics of a state. With the introduction of this distinction, the domain of applicability of equations like (1) shrinks to pre-measurement kinds of states only, the tension with the collapse postulate is averted, and R is rendered false in both cases because it is a statement about pre-measurement kinds of states which does not apply to them but to immediate post-measurement kinds of states only. Under this distinction, R is a statement about pre-measurement kinds because of its reference to the implications between P and Q, but applies to immediate post-measurement kinds because only these are directly associated with eigenvalues. In essence, because R implies that two intrinsically distinct kinds of states are of the same kind, it is internally inconsistent and therefore always false.

To symbolically distinguish between the two kinds of states, let us introduce the convention of underlining immediate post-measurement or collapsed kinds of states. Then, equations which involve both kinds must be thought of as *transformations* from one kind of state to another kind, from a vector in Hilbert Space to an object that is no longer such a vector (but rather more like a classical state). For instance, we would write

$$\frac{1}{\sqrt{2}}\left|+\mathbf{z}\right\rangle + \frac{1}{\sqrt{2}}\left|-\mathbf{z}\right\rangle \longrightarrow \left|\underline{\pm\mathbf{x}}\right\rangle \tag{3}$$

to denote the transformation from one kind to another upon a measurement, in contrast to standard quantum mechanics, which recognizes no intrinsic difference between them. Note that due to the spread of the state according to Schrödinger's equation, immediate post-measurement kinds of states must be considered to eventually transform back to pre-measurement kinds of states unless they are continually 'measured' at a sufficiently rapid rate.

4 Conclusion

This paper pointed out a tension between the collapse postulate and the assumption that pre-and immediate post-measurement states are of the same kind, implicit in relations that equate superposition states to eigenstates in a different basis. The tension can be resolved either by discarding that postulate or by considering such states to be of distinct kinds with respect to the superposition of properties.
The arguments presented here suggest that any interpretation of quantum mechanics which contains the collapse postulate but fails to make an intrinsic distinction between pre-measurement and immediate post-measurement states with respect to the quantum superposition is ruled out.
While such a distinction will seem unfamiliar, it may open an approach to better understand the physical process behind quantum collapse. In particular, one possibility is that underlying this distinction is a corresponding distinction between the concepts of mass in quantum physics and in classical physics, and in particular general relativity [8]. A framework meant to help 'make sense' out of quantum mechanics in which these distinctions are key features can be found in [9].

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