Six polynomials in one and two variables that generate Poulet numbers

Marius Coman
Bucuresti, Romania
email: mariuscoman13@gmail.com

Abstract. Fermat pseudoprimes were for me, and they still are, a class of numbers as fascinating as that of prime numbers; over time I discovered few polynomials that generate Poulet numbers (but not only Poulet numbers). I submitted all of them on OEIS; in this paper I get them together.

(1) Poulet numbers of the form
7200*n^2 + 8820*n + 2701.

First 8 terms: 2701, 18721, 49141, 93961, 226801, 314821, 534061, 665281 (sequence A214016 in OEIS).

Note: The Poulet numbers above were obtained for the following values of n: 0, 1, 2, 3, 5, 6, 8, 9.

(2) Poulet numbers of the form
144*n^2 + 222*n + 85.

First 8 terms: 1105, 2047, 3277, 6601, 13747, 16705, 19951, 31417 (sequence A214017 in OEIS).

Note: The Poulet numbers above were obtained for the following values of n: 2, 3, 4, 6, 9, 10, 11, 14.

(3) Poulet numbers of the form
3*(2*n + 1)*(18*n + 11)*(36*n + 17).

First 4 terms: 561, 62745, 656601, 11921001 (sequence A213071 in OEIS).

Note: The Poulet numbers above were obtained for the following values of n: 0, 2, 5, 14.

Note: All 4 terms from above are Carmichael numbers.

(4) Poulet numbers of the form
(6*m - 1)*((6*m - 2)*n + 1).

First 11 terms: 341, 561, 645, 1105, 1905, 2047, 2465, 3277, 4369, 4371, 6601 (sequence A210993 in OEIS).

Notes:

For m = 1 the formula becomes 20*n + 5 and generates all the Poulet numbers divisible by 5 from the sequence above (beside 645, all of them have another solutions beside n = 1).
For m = 2 the formula becomes 110*n + 11 and generates the Poulet numbers: 341, 561 etc.
For m = 3 the formula becomes 272*n + 17 and generates the Poulet numbers: 561, 1105, 2465, 4369 etc.
For m = 4 the formula becomes 506*n + 23 and generates the Poulet numbers: 2047, 6601 etc.

For n = 1 the formula generates a perfect square.
For n = 2 the formula becomes 3*(6*m - 1)*(4*m - 1) and were found the following Poulet numbers: 561 etc.
For n = 3 the formula becomes (6*m - 1)*(18*m - 5) and were found the following Poulet numbers: 341, 2465 etc.
For n = 4 the formula becomes (6*m - 1)*(24*m - 7) and were found the following Poulet numbers: 1105, 2047, 3277, 6601 etc.

Note: The formula is equivalent to Poulet numbers of the form p*(n*p - n + 1), where p is of the form 6*m - 1. From the first 68 Poulet numbers just 26 of them (1387, 2701, 2821, 4033, 4681, 5461, 7957, 8911, 10261, 13741, 14491, 18721, 23377, 29341, 31609, 31621, 33153, 35333, 42799, 46657, 49141, 49981, 57421, 60787, 63973, 65281) can’t be written as p*(n*p - n + 1), where p is of the form 6*m – 1.

(5) Poulet numbers of the form
(6*m + 1)*(6*m*n + 1).

First 10 terms: 1105, 1387, 1729, 2701, 2821, 4033, 4681, 5461, 6601, 8911 (sequence A214607 in OEIS).

Notes:

For m = 1 the formula becomes 42*n + 7.
For m = 2 the formula becomes 156*n + 13.
For m = 3 the formula becomes 342*n + 19.
For m = 4 the formula becomes 600*n + 25.

For n = 1 the formula generates a perfect square.
For n = 2 the formula becomes (6*m + 1)*(12*m + 1) and were found the following Poulet numbers: 2701, 8911 etc.
For n = 3 the formula becomes (6*m + 1)*(18*m + 1) and were found the following Poulet numbers: 2821, 4033, 5461 etc.
For n = 4 the formula becomes (6*m + 1)*(24*m + 1) and were found the following Poulet numbers: 1387, 83665 etc. (see the sequence A182123 in OEIS).

Note: The formula is equivalent to Poulet numbers of the form p*(n*p - n + 1), where p is of the form 6*m + 1. From the first 68 Poulet numbers just 7 of them (7957, 23377, 33153, 35333, 42799, 49981, 60787) can't be written as p*(n*p - n + 1), where p is of the form 6*m ± 1.

(6) Poulet numbers of the form
m*n^2 + (11*m - 23)*n + 19*m - 49.

First 10 terms: 341, 645, 1105, 1387, 2047, 2465, 2821, 3277, 4033, 5461 (sequence A215326 in OEIS).

Note: The solutions (m,n) for the Poulet numbers from the sequence above are: (3,9); (3,13); (4,14); (4,16); (9,11) and (4,20); (3,27); (3,29); (4,26); (3,35); (290,0).