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Smarandache Half-Groups

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Abstract: In this paper we introduce the concept of *half-groups*. This is a totally new concept and demands considerable attention. R.H.Bruck [1] has defined a half groupoid. We have imposed a group structure on a half groupoid wherein we have an identity element and each element has a unique inverse. Further, we have defined a new structure called Smarandache half-group. We have derived some important properties of Smarandache half-groups. Some suitable examples are also given.

Key Words: half-group, subhalf-group, Smarandache half-group, Smarandache subhalf-Group, Smarandache hyper subhalf-group.

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§1. Introduction

Definition 1.1 *Let $(S, *)$ be a half groupoid (a partially closed set with respect to $*$) such that*

(1) *There exists an element $e \in S$ such that $a * e = e * a = a, \forall a \in S$. e is called identity element of S;*

(2) *For every $a \in S$ there exists $b \in S$ such that $a * b = b * a = e$ (identity) b is called the inverse of a.*

Then (S,) is called a half-group.*

Remark It is easy to verify that

(a) identity element in S is unique;

(b) each element in S has a unique inverse;

(c) associativity does not hold in S as there is at least one product that is not defined in S.

Note In all composition tables in the following examples the blank entries show that the corresponding products are not defined.

Example 1.1 Let $S = \{1, -i, i\}$. Then S is a half-group w.r.t. multiplication. We write this multiplication table in the following.

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$*$	1	-i	i
1	1	-i	i
-i	-i		1
i	i	1	

Example 1.2 Let $S = \{e, a, b, c\}$. Then $(S, *)$ is a half subgroup defined by

$*$	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	

Here the product $c * c$ is not defined.

Definition 1.2 it Let $(S, *)$ be a half-group and H a subset of S. If H itself is a half-group w.r.t. $*$, then H is called a subhalf-group of S.

Example 1.3 Let $S = \{e, a, b, c, d\}$ be a half-group defined by the following table.

$*$	e	a	b	c	d
e	e	a	b	c	d
a	a	c	e	b	a
b	b	e	c	a	d
c	c	d	a	e	b
d	d	b	c		e

Then , $H = \{e, a, b\}$ is a subhalf-group of S.

Definition 1.3 *A half-group $(S, *)$ is called a Smarandache half-group if S contains a proper subset G such that G is a nontrivial group w.r.t. $*$.*

Definition 1.4 *If S is Smarandache half-group such that every group contained properly in S is commutative, then S is called Smarandache commutative half-group.*

Definition 1.5 *If S is a Smarandache half-group such that every group contained properly in S is cyclic, then S is called a Smarandache cyclic half-group.*

Example 1.4 Let S be a half-group defined by the following table.

*	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	e		e

Then $G = \{e, a\}$ is a nontrivial group contained in S. So, S is a Smarandache half-group. Also, $\{e, a, b\}$ is a Smarandache half-group. S is also a Smarandache commutative half-group. Also S is a Smarandache cyclic half-group.

Example 1.5 $S = \{1, -i, i\}$ is not a Smarandache half-group.

Example 1.6 Let L be the Half-Group given by the following table.

*	e	f	g	h	i	j	k	l
e	e	f	g	h	i	j	k	l
f	f	e	j	g	k	h	l	i
g	g	j	e	k	h	l	i	f
h	h	g	k	e	l	i	f	j
i	i	k	h	l	e	f	j	g
j	j	h	l	i	f	e	g	k
k	k	l	i	f	j	g	e	
l	l	i	f	j	g	k		e

Then L is a half-group which contains a group $G = \{e, g\}$. So, L is a Smarandache Half-Group.

There are many Smarandache half-groups in this structure. Results following are obtained immediately by definition

(1) *The smallest half-group is of order 3.*

This follows from the very definition of half-groups.

(2) *The smallest Smarandache half-group is of order 3.*

As a nontrivial group has order at least 2, the half-group which will contain this group properly will have order at least 3.

§2. Substructures of Smarandache Half-Groups

In this section we introduce Smarandache substructure.

Definition 2.1 *Let S be a half-group w.r.t. $*$. A nonempty subset T of S is said to be Smarandache subhalf-group of S if T contains a proper subset G such that G is a nontrivial group under the operation of S.*

Theorem 2.1 *If S is a half-group and T is a Smarandache subhalf-group of S then S is a Smarandache half-group.*

Proof As T is a Smarandache subhalf-group of S , S contains T properly. Also, T properly contains a non trivial group. As a result S is a hlf-group which properly contains a nontrivial group. Therefore S is a Smarandache half-group. □

We also note facts following on Smarandache half-groups.

(1)If R is a Smarandache half-group then every subhalf-group of R need not be a Smarandache subhalf-group.

We give an example to justify our claim.

Example 2.1 Consider a half-group S defined by the following table.

*	e	f	g	h	i	j
e	e	f	g	h	i	j
f	f	h	e	g	j	i
g	g	e	h	f	i	i
h	h	g	f	e	e	j
i	i	j	i	j	e	
j	j	i	f	i		e

Then $S \supset H = \{e, f, g, h\}$ and H is a group. Therefore S is a Smarandache half-group. Consider a half-group $R = \{e, f, g\}$. Then R is not a Smarandache subhalf-group of S as there does not exist a non trivial group contained in R.

We give a typical example of a half-group following whose subhalf-groups are Smarandache subhalf-group.

Example 2.2 Consider the following table.

*	e	f	g	h	i	j	k	l
e	e	f	g	h	i	j	k	l
f	f	e	j	g	k	h	l	i
g	g	j	e	k	h	l	i	f
h	h	g	k	e	l	i	f	j
i	i	k	h	l	e	f	j	g
j	j	h	l	i	f	e	g	k
k	k	l	i	f	j	g	e	
l	l	i	f	j	g	k		e

One can easily verify that every subhalf-group is a Smarandache subhalf-group.

Definition 2.2 *If S is a Smarandache half-group such that a subhalf-group A of S contains the largest group in S then A is called a Smarandache hyper subhalf-group.*

In the example above, the largest non-trivial group in S is of order 2 and every Smarandache subhalf-Group of S contains the largest group in S. Thus, every Smarandache subhalf-Group in S is a Smarandache hyper subhalf-Group.

References

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