# Estimates of the Enclosed Mass and its Distribution for several Spiral Galaxies

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## Abstract

Recently, high quality rotation curves for several spiral galaxies have become available from THINGS: **T**he **HI N**earby **G**alaxy **S**urvey (Reference 1). This paper analyzes 15 of those galaxy rotation curves using the Newtonian Disk Model (Reference 2). The Model assumes that virtually all the mass of stars, gas, dust and dark-matter is contained as a “flattened” mass within the galactic disk, which extends out to the edge of the observable rotation. Estimates of the enclosed mass and its radial distribution are provided.

## Introduction

The high quality HI rotation curves from THINGS (Reference 1) span a wide range of spiral galaxy type, mass, and basic characteristics. These galaxies have been extensively analyzed in Reference 1, using the traditional separate mass components of stellar disk, stellar nuclear bulge, HI gas disk and an assumed spheroidal dark-matter halo. These component velocities are added in quadrature to match the rotation curve velocities.

In contrast to the traditional approach, this paper utilizes the Newtonian disk model described in Reference 2. This model assumes that virtually all the galaxy mass is contained within a flattened disk; that includes the stellar disk, stellar nuclear bulge, gas disk and dark-matter components. There is no spheroidal dark-matter halo in this model. By adjusting the 12 ring masses, the model is able to produce a rotation curve that matches the individual rotation curve from the THINGS galaxy data. It is then possible to calculate the enclosed galaxy mass, display its radial distribution, and calculate the surface mass density and disk radius of gyration.

## Notation

This listing provides symbol notation as used in the Newtonian model of Reference 2.

| | |
|---|---|
| **D** | Distance to galaxy ~ Mpc |
| $\mathbf{F}_d$ | Gravitational Force in disk, toward galaxy center |
| $\mathbf{F}_p$ | Gravitational Force in disk if $\mathbf{M}_m$ is a point mass at galaxy center |
| $(\mathbf{F}_{\boldsymbol{d}}/\mathbf{F}_{\boldsymbol{p}})_m$ | Gravitational Force ratio at $\mathbf{R}_m$ |
| **G** | Gravitational Constant ~ 4.3 x $10^{-6}$ kpc $(\frac{\mathrm{km}}{\mathrm{s}})^2$ $\mathbf{M}_{\odot}^{-\mathbf{1}}$ |
| $\mathbf{I}_m$ | Moment of Inertia of galaxy disk =$\sum \mathrm{m}_i \mathrm{r}_i^2$ |
| $\mathbf{m}_i$ | Mass of ring *i* |
| $\mathbf{M}_m$ | Enclosed Mass of galaxy disk = $\sum \mathrm{m}_i$ = $(\mathbf{R}_{\boldsymbol{m}}$ x $\mathbf{V}_m^2)/($**G** x $(\mathbf{F}_{\boldsymbol{d}}/\boldsymbol{F}_{\boldsymbol{p}})_m)$ |
| **M**sun, $\mathbf{M}_{\odot}$ | Solar Mass |
| $\mathbf{R}_g$ | Radius of Gyration of galaxy disk $\mathbf{R}_{\boldsymbol{g}}^{\mathbf{2}} = \mathbf{I}_{\boldsymbol{m}}/\mathbf{M}_{\boldsymbol{m}}$ |
| $\mathbf{R}_m$ | Outermost radius of observed galaxy rotation ~ kpc |
| **SMD** | Surface Mass Density at given disk location ~ $\mathrm{m_i/pc^2}$ |
| $\mathbf{SMD}_{av}$ | Average SMD in galaxy disk = $\mathbf{M}_m/(\pi\, \mathbf{R}_{\boldsymbol{m}}^{\mathbf{2}})$ |
| $\mathbf{V}_{\boldsymbol{m}}$ | Circular Velocity in galaxy disk at $\mathbf{R}_{\boldsymbol{m}}$ ~ km/s |

## The Galaxy Sample

Rotation curves of fifteen spiral galaxies from the THINGS data have been chosen for analysis and are listed in Table 1.

| Galaxy | Type | D Mpc | $\mathrm{R_m}$ kpc | $\mathrm{V_m}$ km/s | $\mathrm{(F_d/F_p)_m}$ | $\mathrm{M_m}$ $\mathrm{10^{10}M_\odot}$ | $\mathrm{SMD_{av}}$ $\mathrm{M_\odot/pc^2}$ | $\mathrm{R_g/R_m}$ |
|---|---|---|---|---|---|---|---|---|
| **NGC 925** | **SBd** | **9.2** | **13** | **120** | **1.82** | **2.39** | **45** | **0.58** |
| **NGC 2403** | **Sc** | **3.2** | **18** | **139** | **1.60** | **5.06** | **50** | **0.51** |
| **NGC 2841** | **Sb** | **14.1** | **34.8** | **288** | **1.54** | **43.6** | **115** | **0.47** |
| **NGC 2903** | **SBd** | **8.9** | **29** | **183** | **1.46** | **15.5** | **59** | **0.45** |
| **NGC 2976** | **Sc** | **3.6** | **2.55** | **86.5** | **1.92** | **0.231** | **113** | **0.59** |
| **NGC 3031** | **Sab** | **3.6** | **14.8** | **196** | **1.32** | **10.0** | **146** | **0.41** |
| **NGC 3198** | **SBc** | **13.8** | **38** | **154** | **1.55** | **13.5** | **30** | **0.49** |
| **NGC 3521** | **SBc** | **10.7** | **31.2** | **204** | **1.53** | **19.7** | **65** | **0.45** |
| **NGC 4736** | **Sab** | **4.7** | **10.3** | **115** | **1.21** | **2.63** | **79** | **0.35** |
| **DDO 154** | **Sc** | **4.3** | **8.3** | **53.2** | **1.81** | **0.302** | **14** | **0.55** |
| **NGC 4826** | **Sab** | **7.5** | **22** | **155** | **1.52** | **8.10** | **53** | **0.48** |
| **NGC 5055** | **Sbc** | **10.1** | **48.8** | **170** | **1.34** | **24.4** | **33** | **0.43** |
| **NGC 6946** | **Scd** | **5.9** | **19.1** | **199** | **1.54** | **11.4** | **100** | **0.50** |
| **NGC 7331** | **Sb** | **14.7** | **25.4** | **256** | **1.61** | **24.1** | **119** | **0.49** |
| **NGC 7793** | **Sd** | **3.9** | **7.7** | **100** | **1.40** | **1.28** | **69** | **0.48** |

**Table 1. Spiral Galaxy Characteristics**

The galaxies in Table 1 are at distances between 3 and 15 Mpc, and have typical linear resolutions between ~100 and ~500 pc. The radio synthesis measurements of the velocities across the entire HI-filled disks, provide an unusual sample of galaxies analyzed and modeled in a uniform way at sub-kpc resolution. Ref. 1 shows the rotation curve data for the individual galaxies. A “faired” line is drawn through the data, and then approximately 20 points are extracted to provide the necessary values for the Newtonian model.

The individual masses of the fifteen spiral galaxies cover a wide range from 2.3 to 440 billion solar masses. The maximum circular velocity is about 320 km/s (for NGC 2841) , while the average circular velocity (Vm) at the disk rim (Rm) is 160 km/s for these fifteen galaxies.

The average Surface Mass Density (SMDav) for this set of galaxies spans a wide range from 14 to 146 $\mathrm{M_{\odot}/pc^2}$. (In the solar neighborhood of the Galaxy, the SMD is estimated at 75 $\mathrm{M_{\odot}/pc^2. as\ noted\ in\ Reference\ 6.)}$

The gyration ratio (Rg/Rm) of this set of galaxies spans a limited range between 0.35 and 0.59, with most values clustered around the mean of 0.48

For each of the galaxies in Table 1, Figures 1 through 15 shows the estimated galaxy mass and its radial distribution. These Figures have been grouped together to provide easy cross-reference and comparison. The upper chart for each galaxy shows the rotation curve extracted from THINGS data (Reference 1) as a solid-black curve. The dashed-red curve shows how well the Newtonian model matches the galaxy data. This upper chart also notes the estimated galaxy mass, and the distance D associated with the galaxy disk radial dimensions. The galaxy mass varies directly with the disk size, Rm, and therefore varies directly with the assumed distance D. When comparing results of this paper with other studies, it is necessary to adjust values based on the assumed distance in those studies.

The lower chart for each galaxy model shows the radial distribution of the Ring Masses that produce the model rotation curve. Also included in the lower chart is the Rg/Rm gyration ratio of the mass distribution.

## Discussion

Reviewing Figures 1 through 15, it can be seen that the model achieves a reasonable match with the galaxy rotation curves. With only 12 ring masses, the current model does not match all the minor undulations in some of the observed rotation curves. Given the approximations involved in defining the observed rotation curves, the current model is considered to provide satisfactory rotation curve matching.

The ring mass distribution for the individual galaxies and the Rg/Rm ratios show that the galaxy mass is well dispersed radially in the galaxy disk. (This dispersion is in marked contrast with the solar system, where over 99% of the system mass is contained within the sun, and the system gyration radius lies deep below the surface of the sun.)

Most of these galaxies have relatively straightforward rotation curves and many have been the subject of earlier analyses, as noted in Reference 1. However, NGC 4826 has the unusual feature that HI gas in the outer disk rotates in the opposite direction to the HI gas in the inner disk. This feature is considered evidence of a collision and merging of two galaxies millions of years ago. There is a transition phase (4 kpc<R<12 kpc) in the rotation curve between the inner and outer disks, which shows some confused and non-circular motions. It is difficult to define the rotation curve in this transition phase, so the model estimates of galaxy mass for NGC 4826 should be regarded as tentative only.

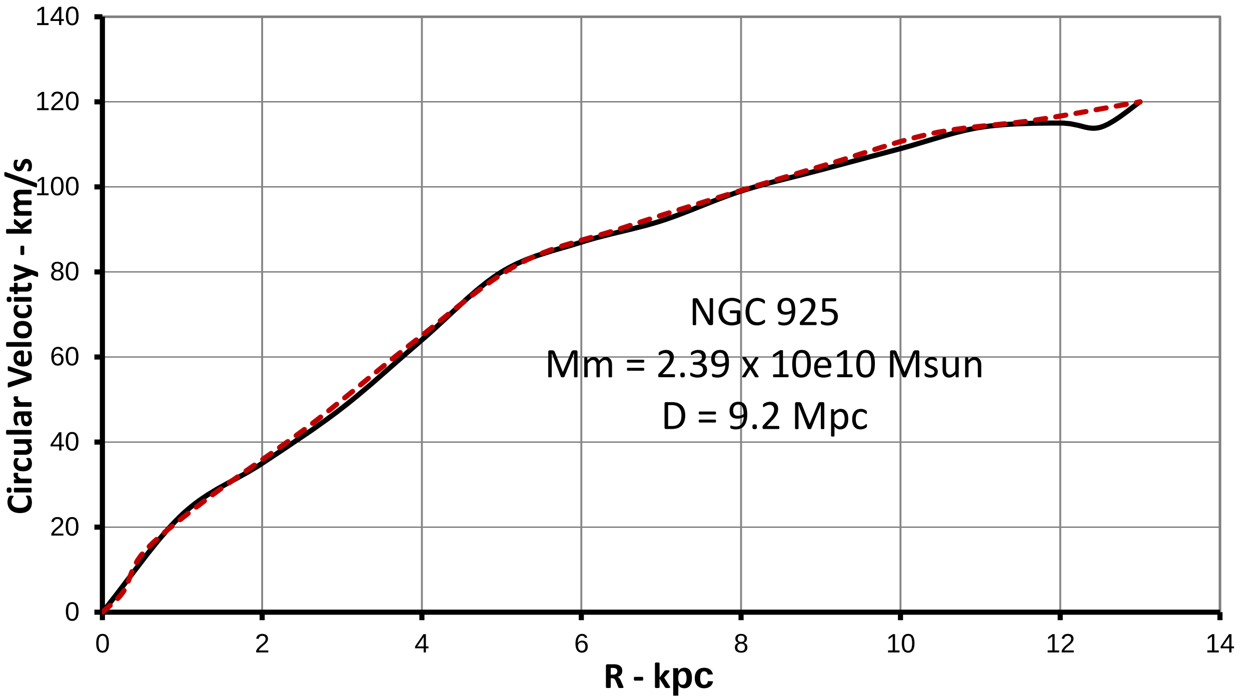

Figure 1a. Velocity Profiles for NGC 925 and Model

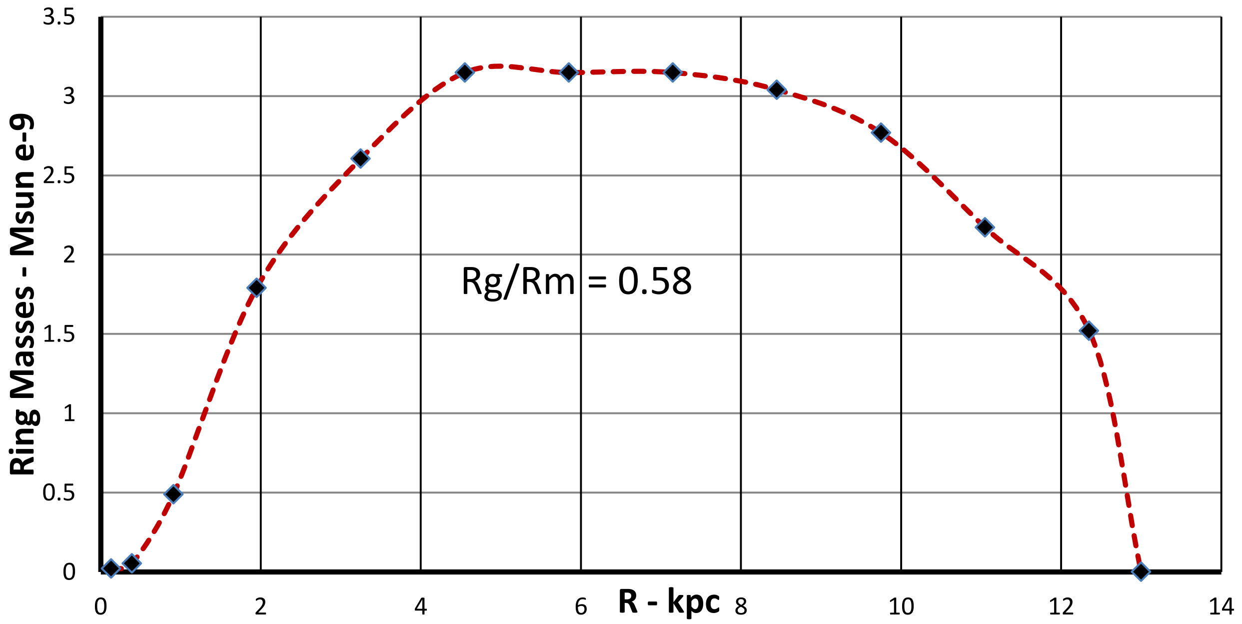

Figure 1b. Ring Mass Distribution for NGC 925 Model

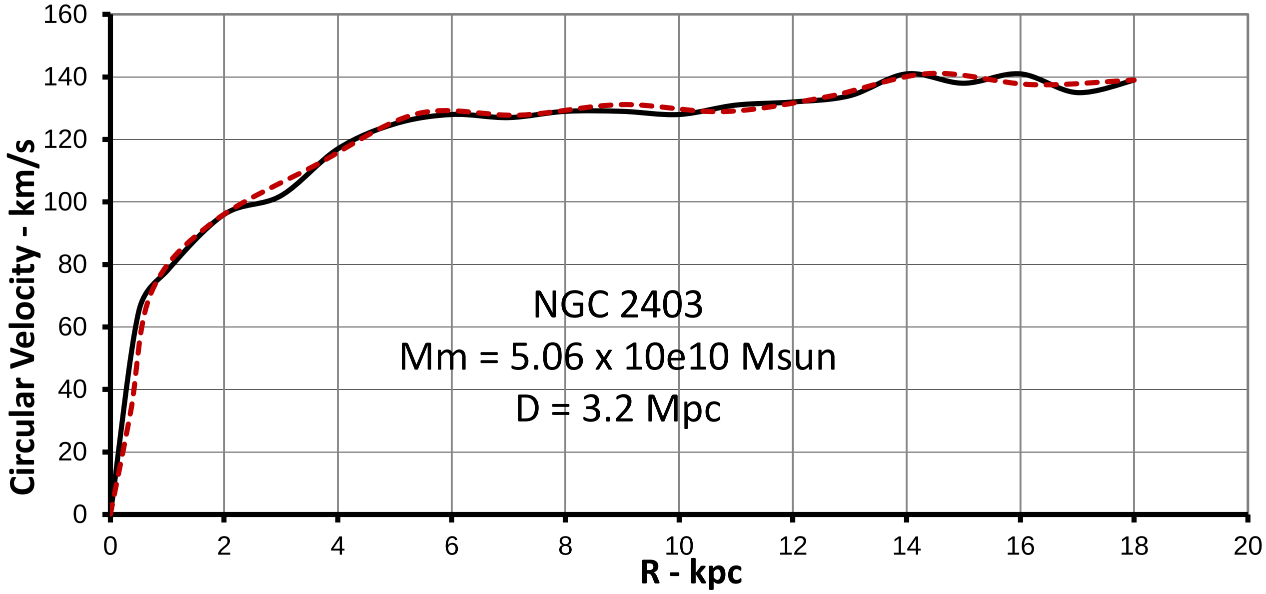

**Figure 2a. Velocity Profiles of NGC 2403 and Model**

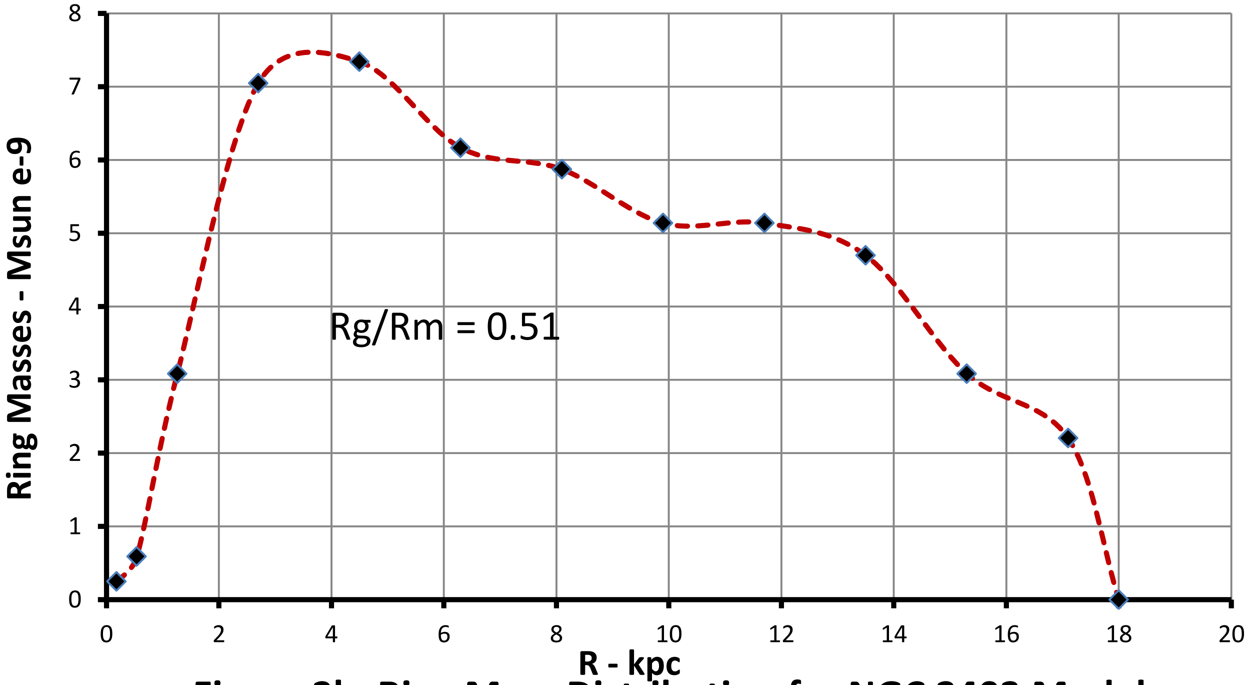

**Figure 2b. Ring Mass Distribution for NGC 2403 Model**

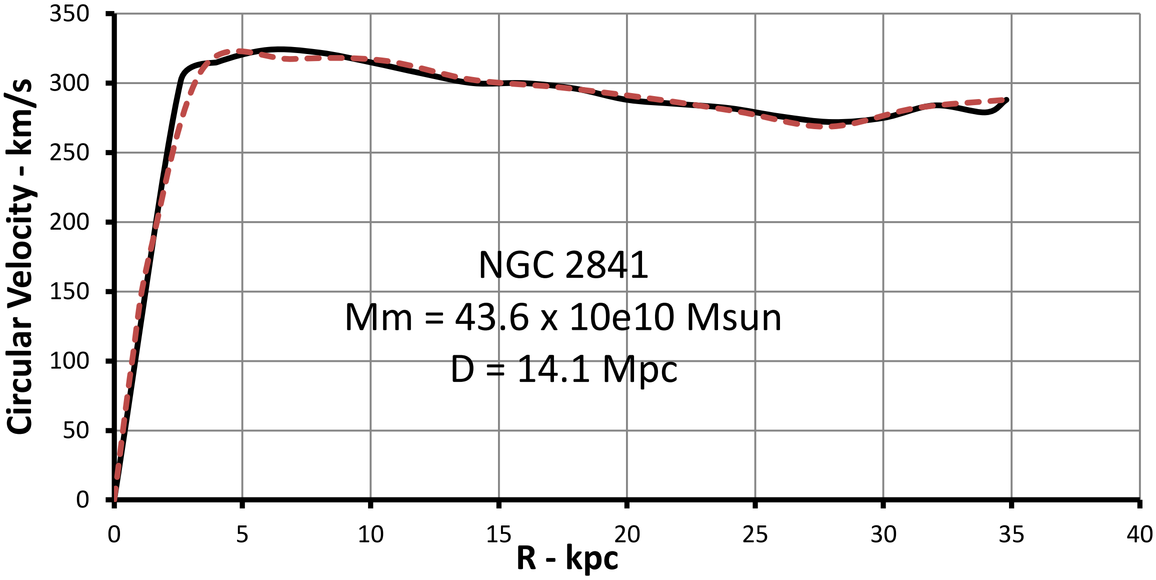

Figure 3a. Velocity Profiles of NGC 2841 and Model

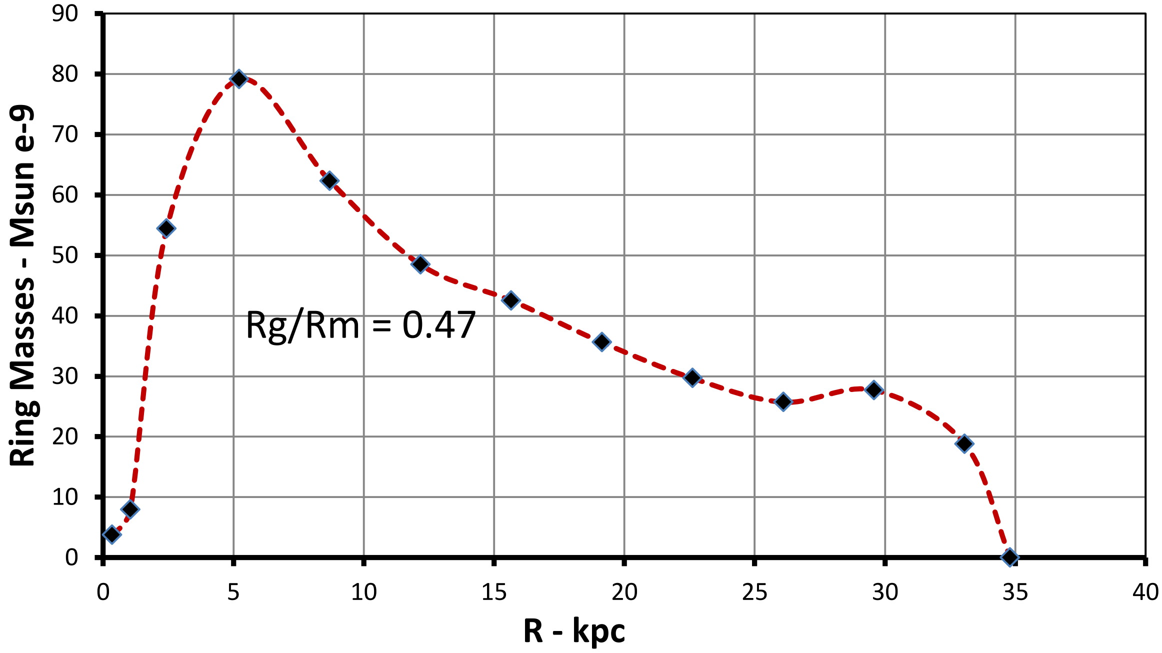

Figure 3b. Ring Mass Distribution for NGC 2841 Model

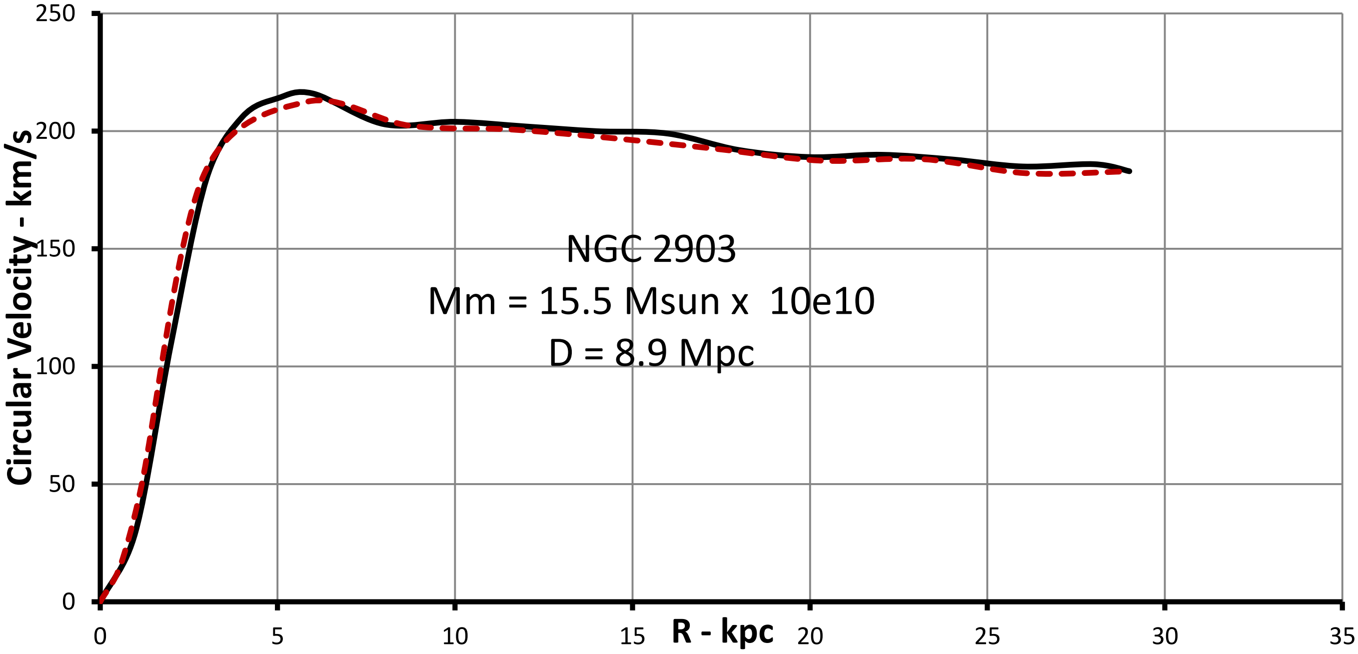

**Figure 4a. Velocity Profiles of NGC 2903 and Model**

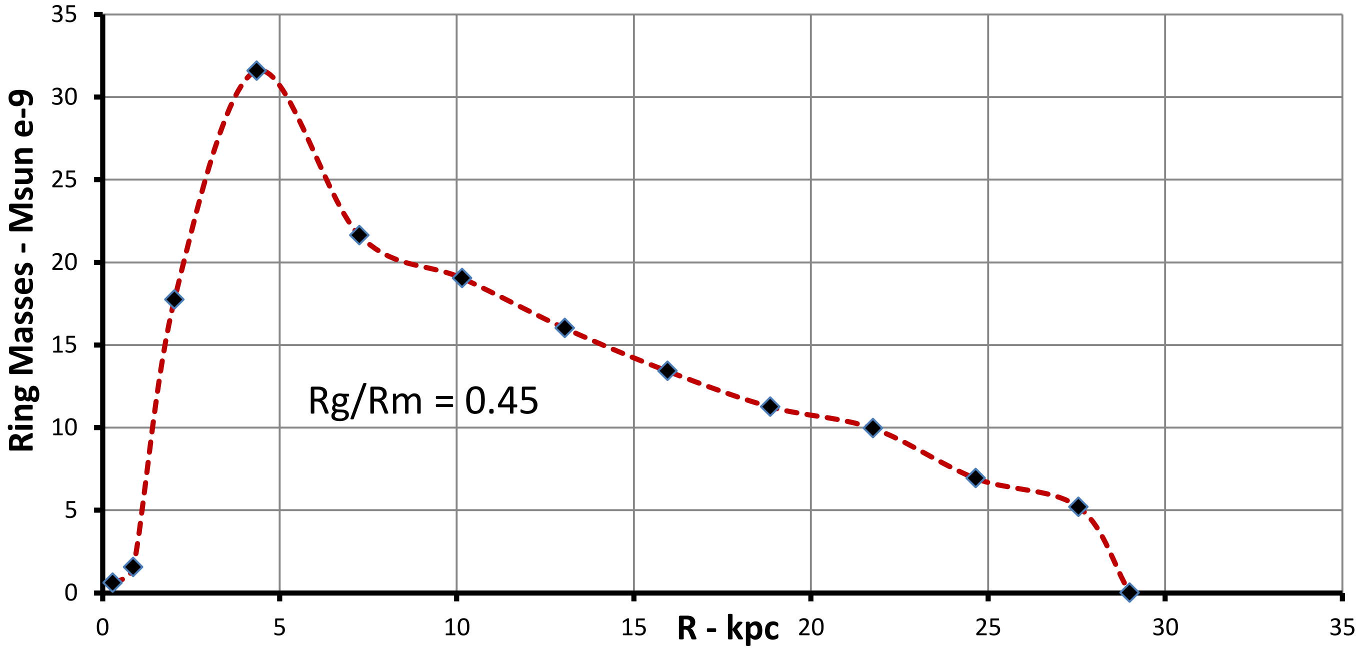

**Figure 4b. Ring Mass DIstribution for NGC 2903 Model**

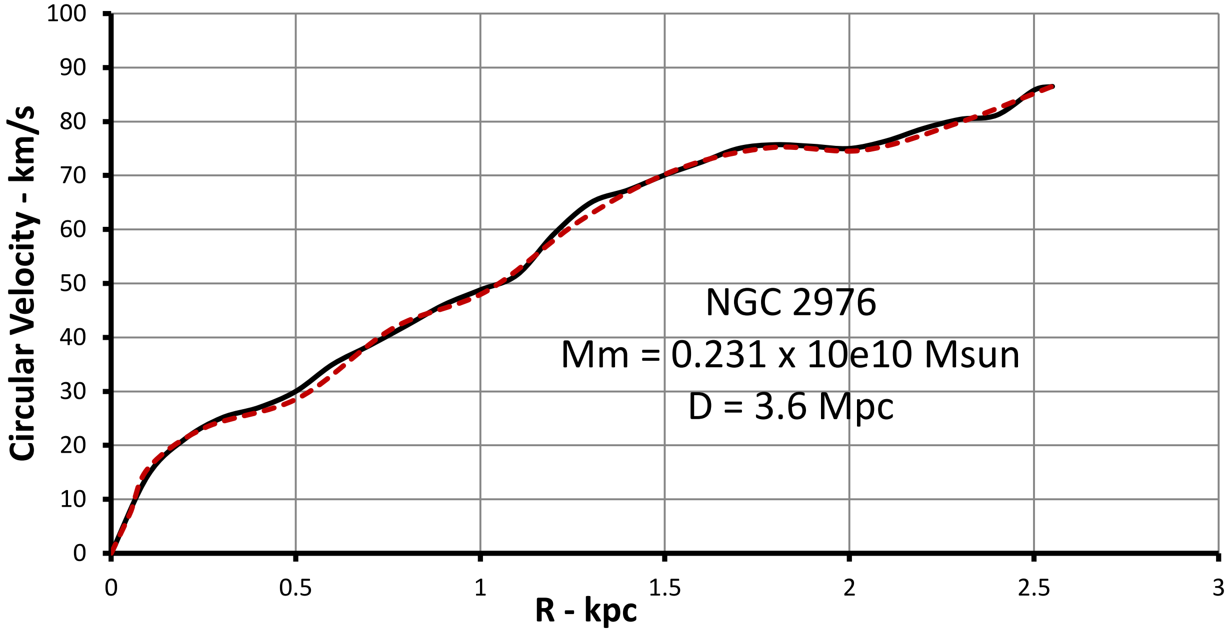

Figure 5a. Velocity Profiles of NGC 2976 and Model

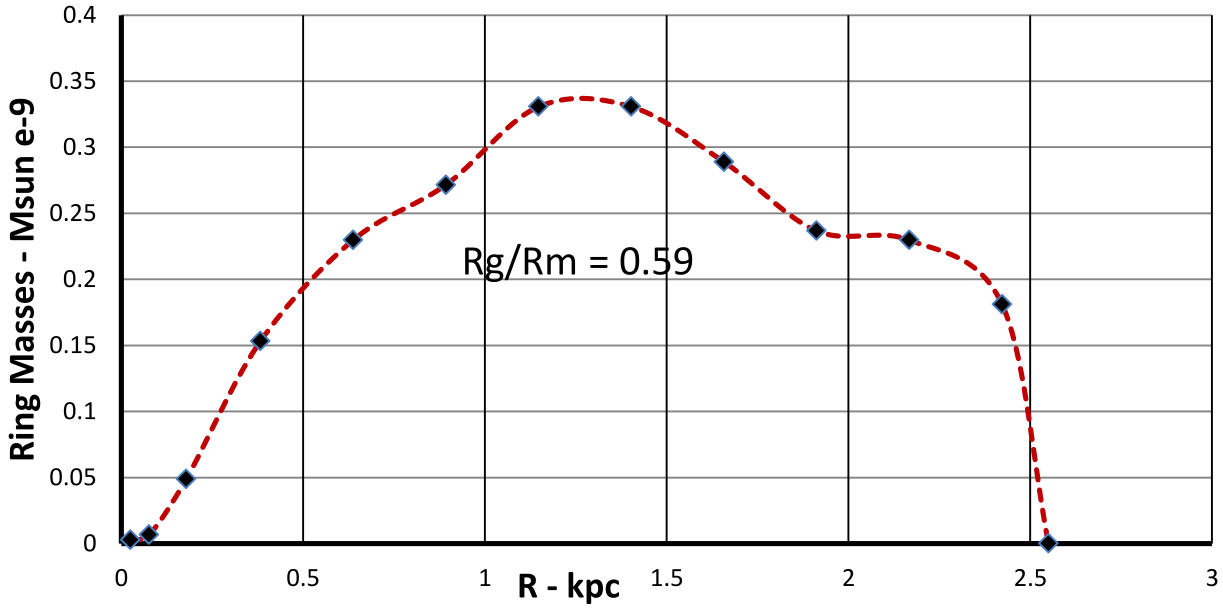

Figure 5b. Ring Mass Distribution for NGC 2976 Model

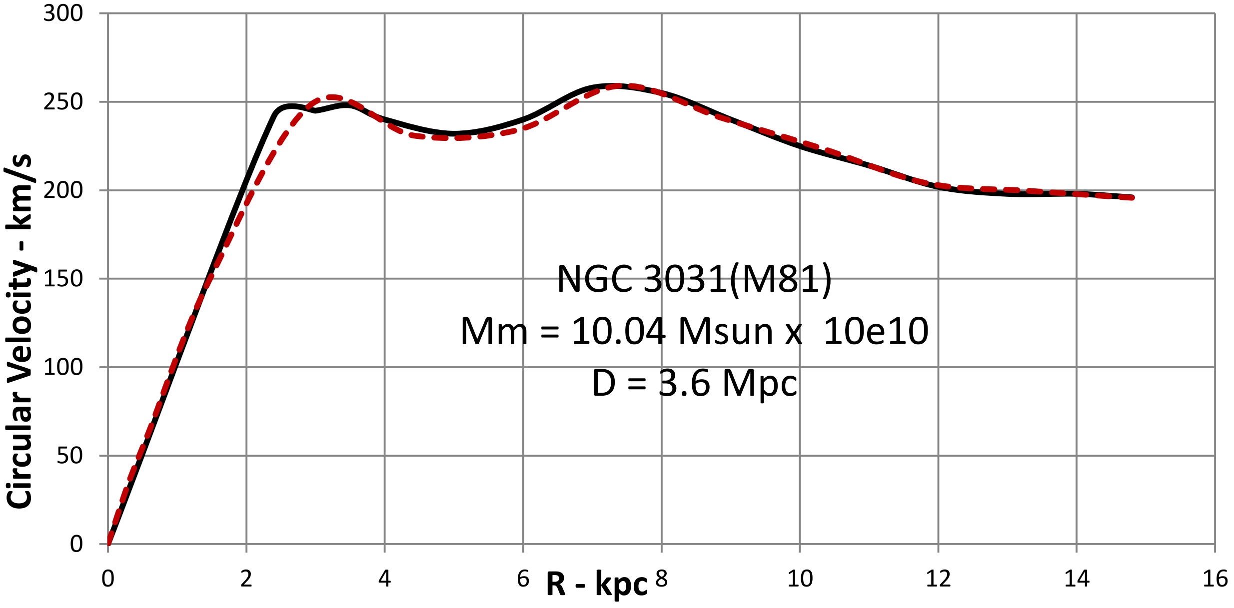

Figure 6a. Velocity Profile of NGC 3031 and Model

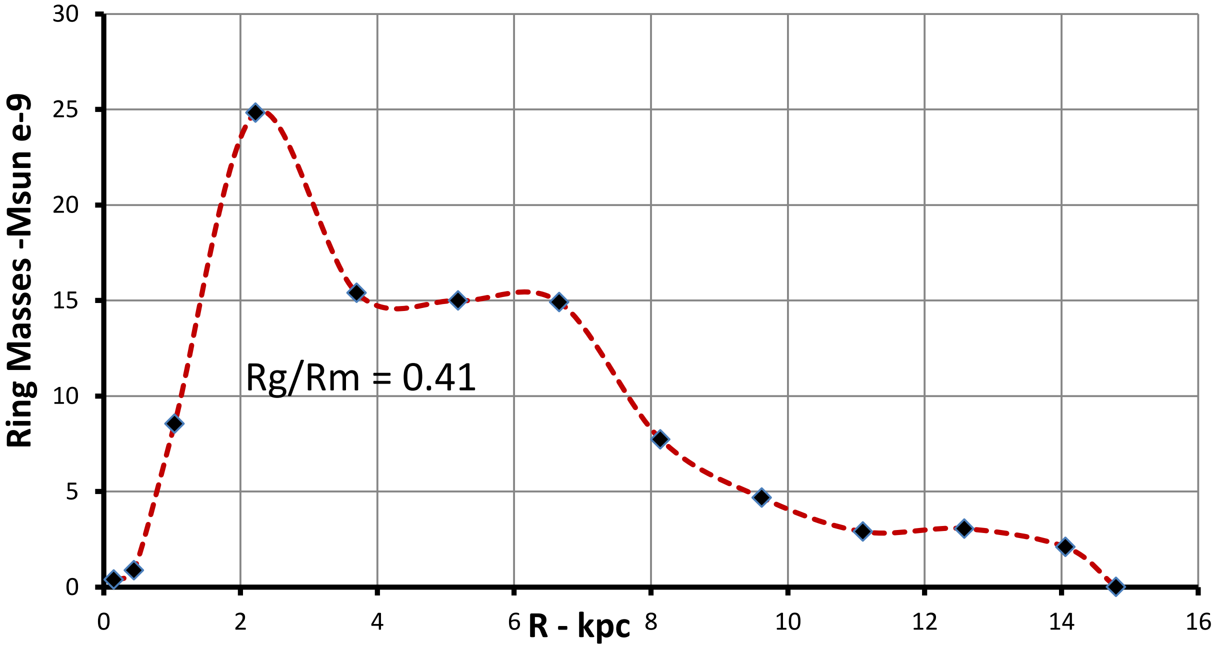

Figure 6b. Ring Mass Distribution for NGC 3031 Model

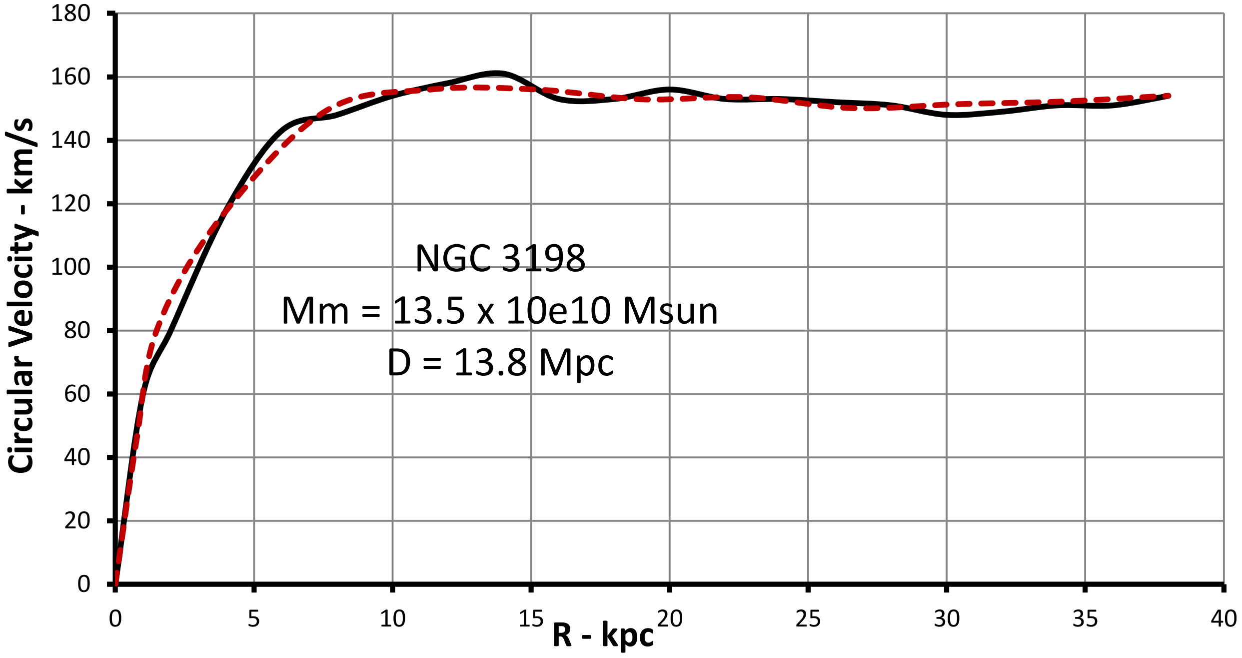

**Figure 7a. Velocity Profiles for NGC 3198 and Model**

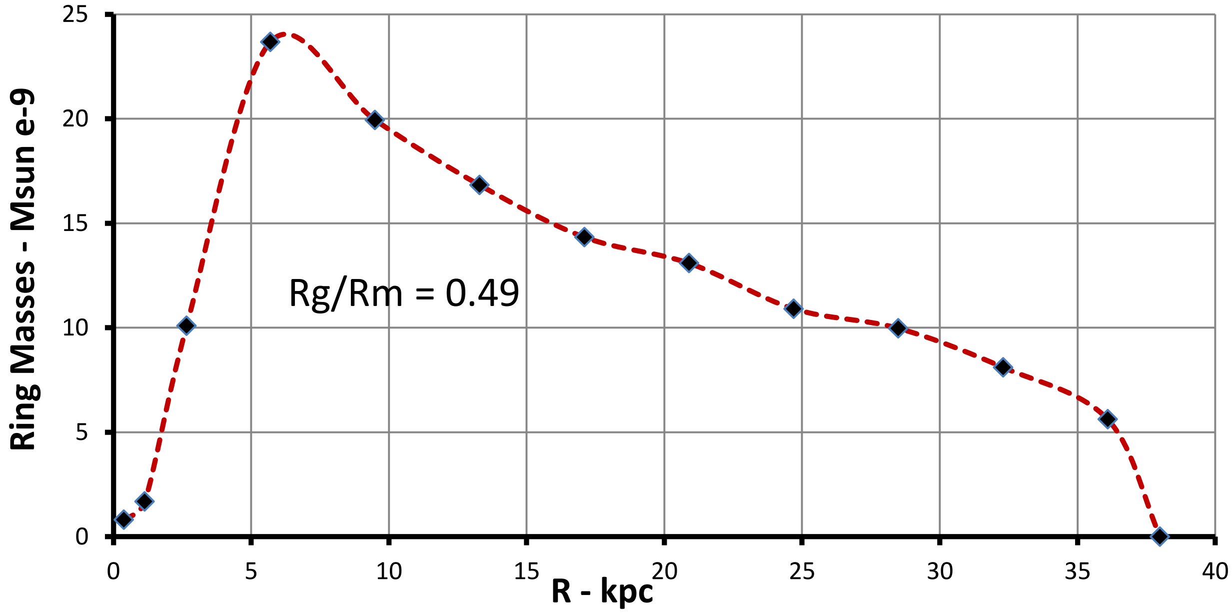

**Figure 7b. Ring Mass Distribution for NGC 3198 Model**

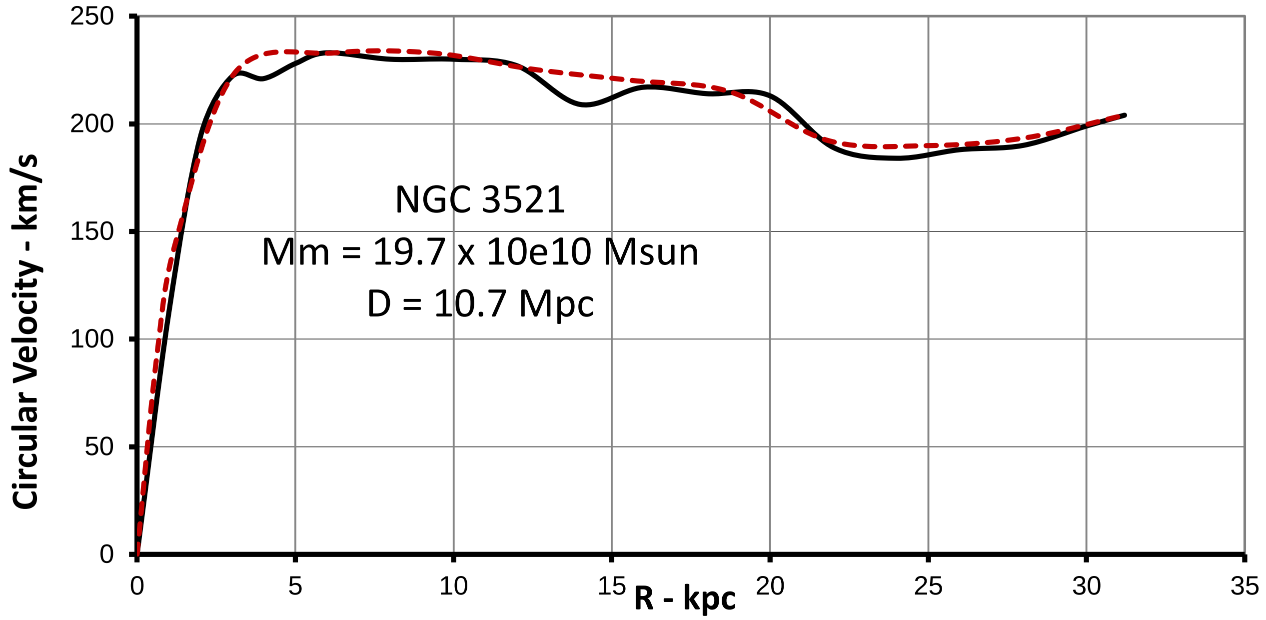

**Figure 8a. Velocity Profiles for NGC 3521 and Model**

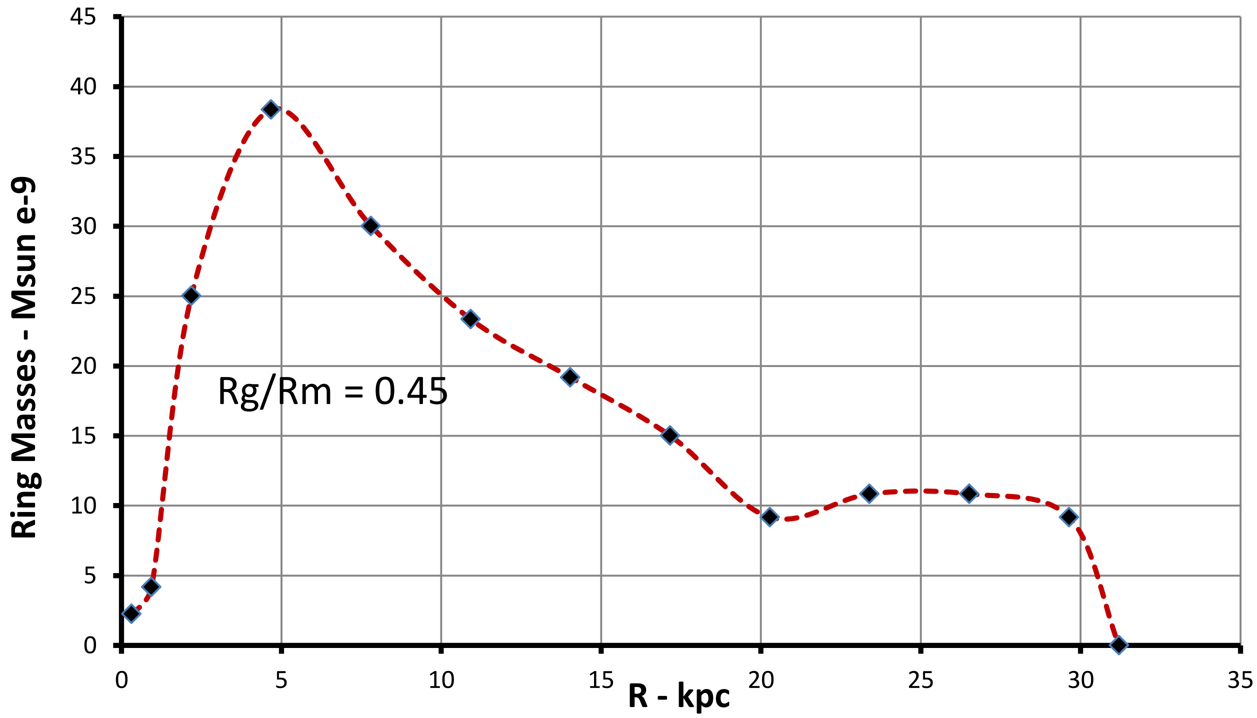

**Figure 8b. Ring Mass Distribution for NGC 3521 Model**

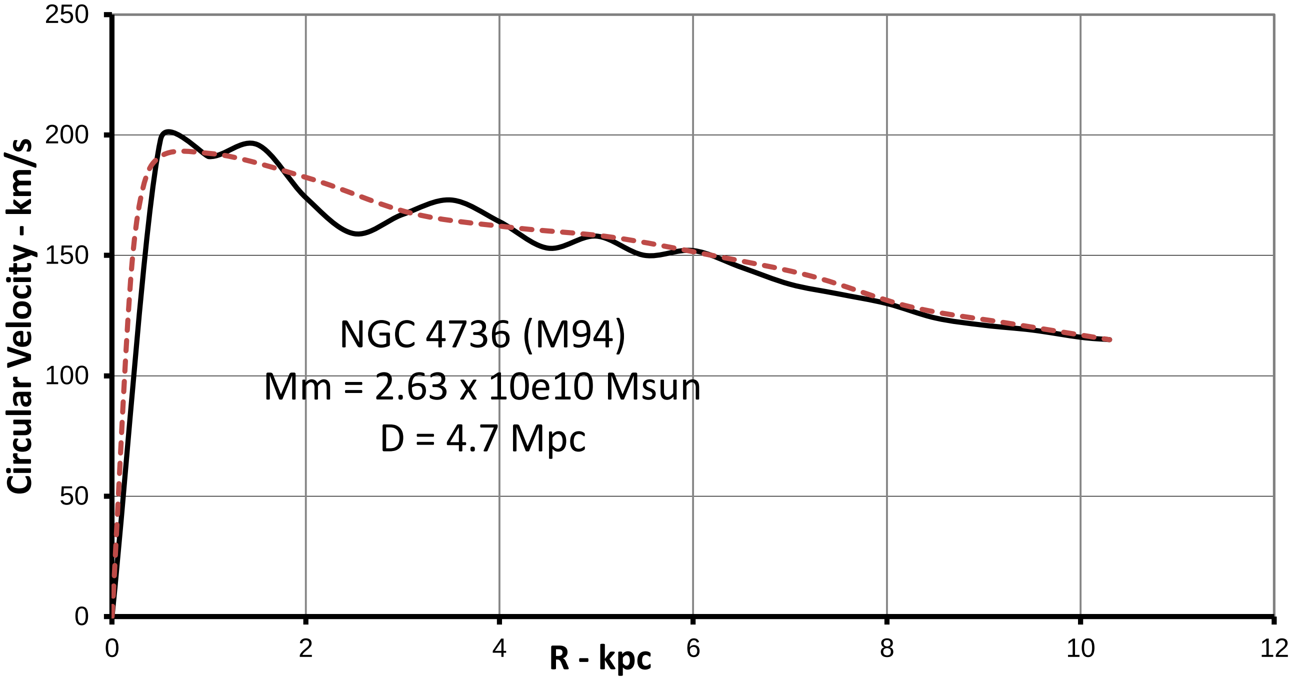

Figure 9a. Velocity Profiles of NGC 4736 and Model

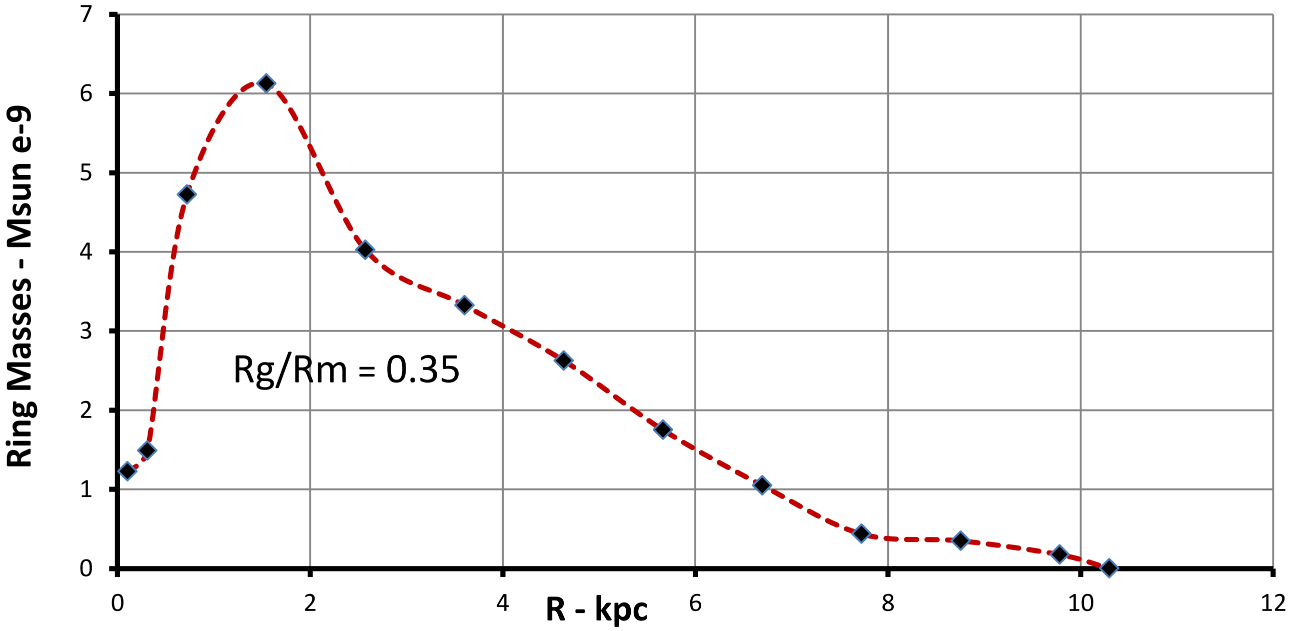

Figure 9b. Ring Mass Distribution for NGC 4736 Model

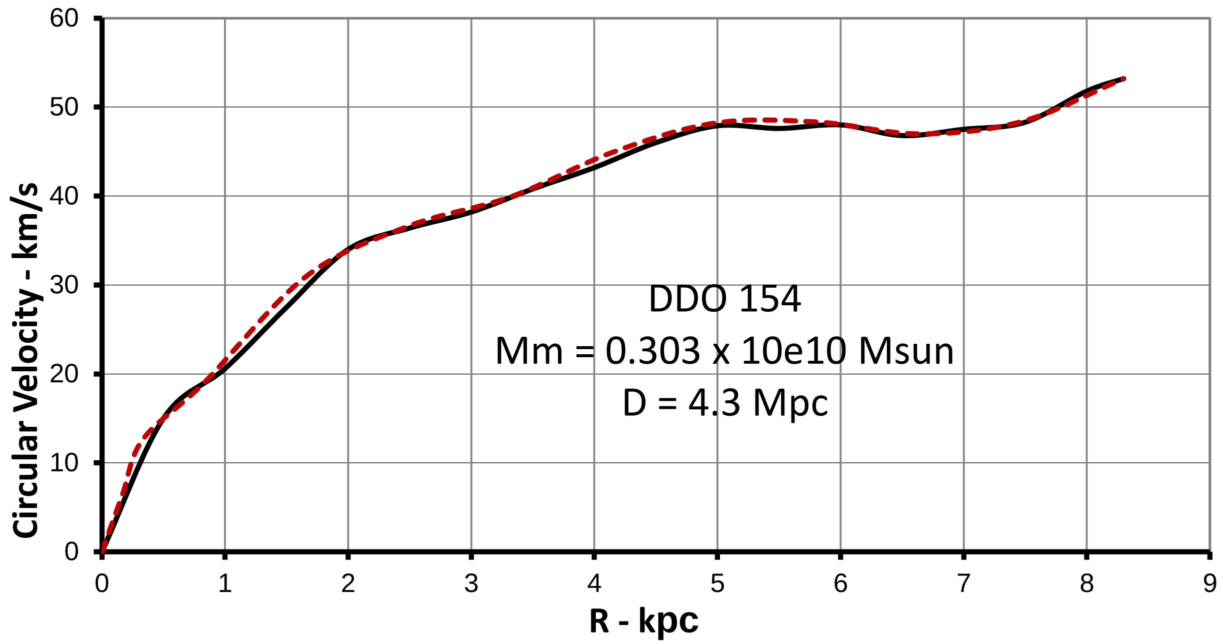

**Figure 10a. Velocity Profiles for DDO 154 and Model**

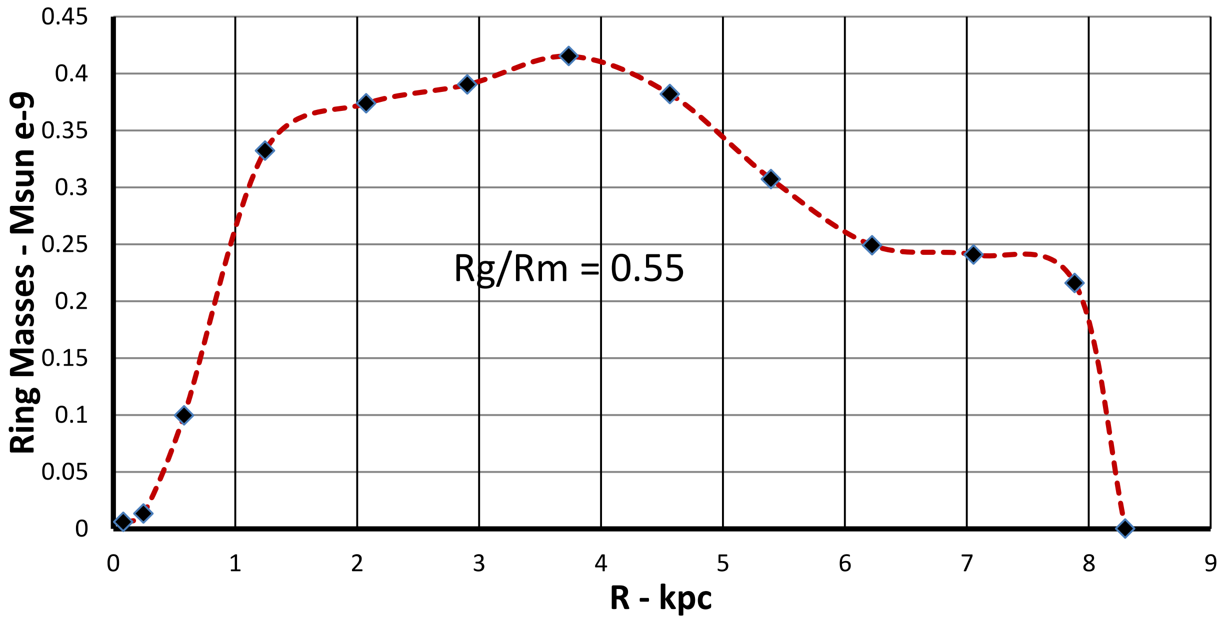

**Figure 10b. Ring Mass Distribution for DDO 154 Model**

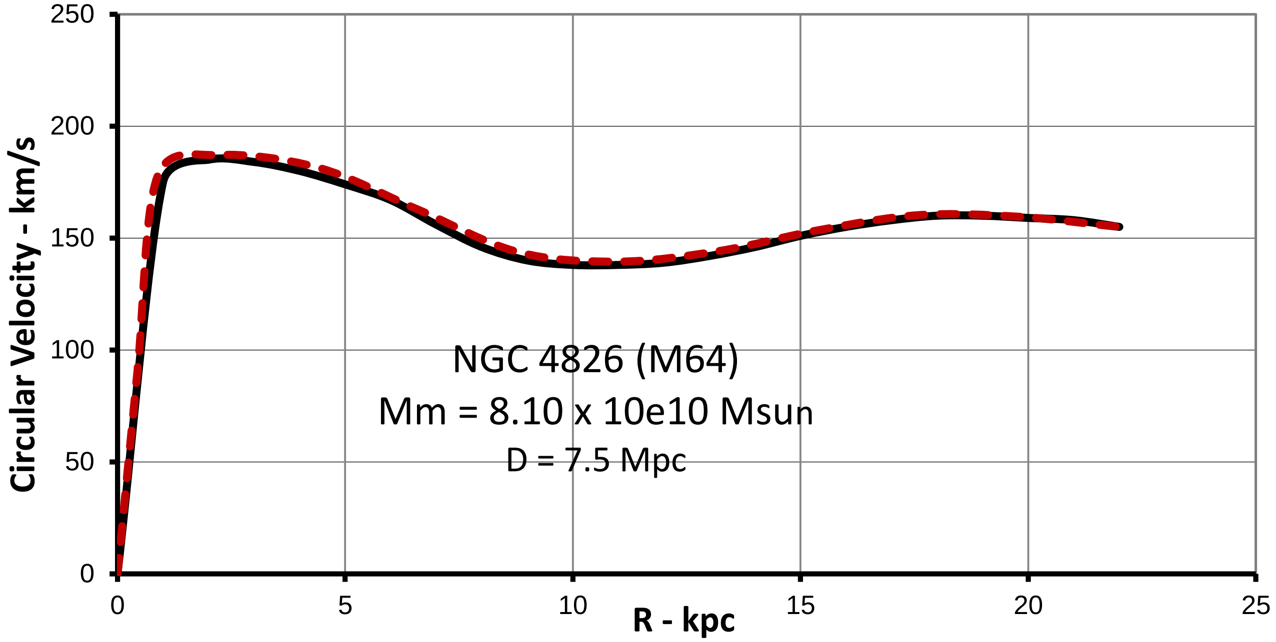

Figure 11a. Velocity Profiles of NGC 4826 and Model

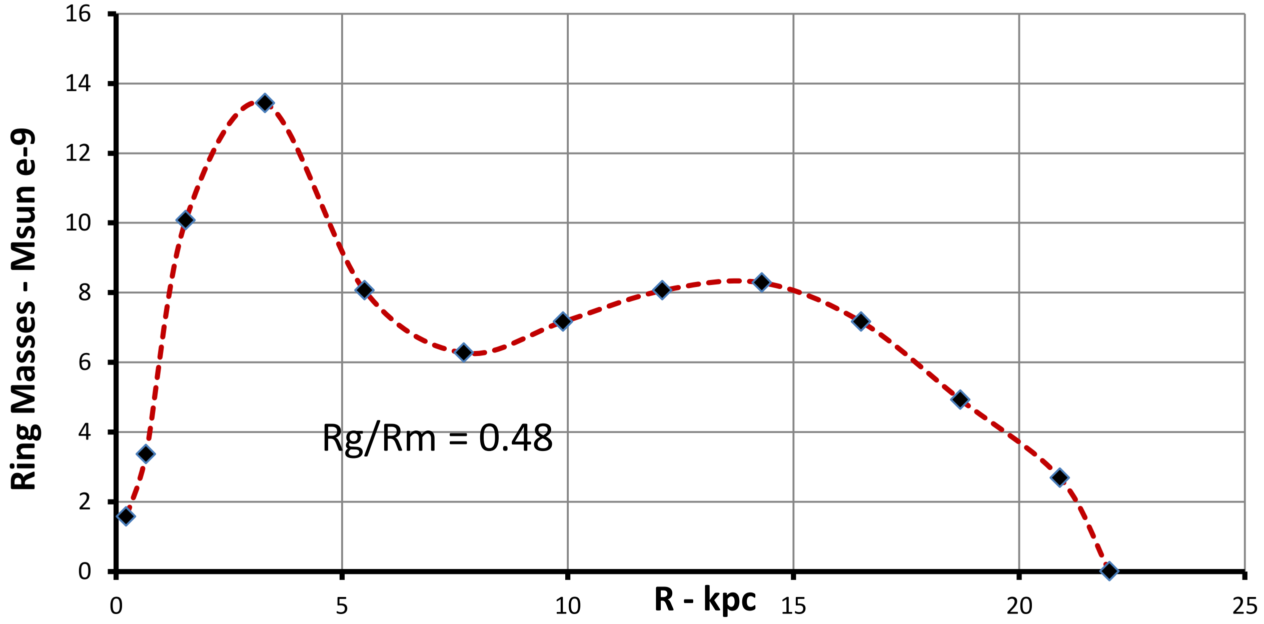

Figure 11b. Ring Mass Distribution for NGC 4826 Model

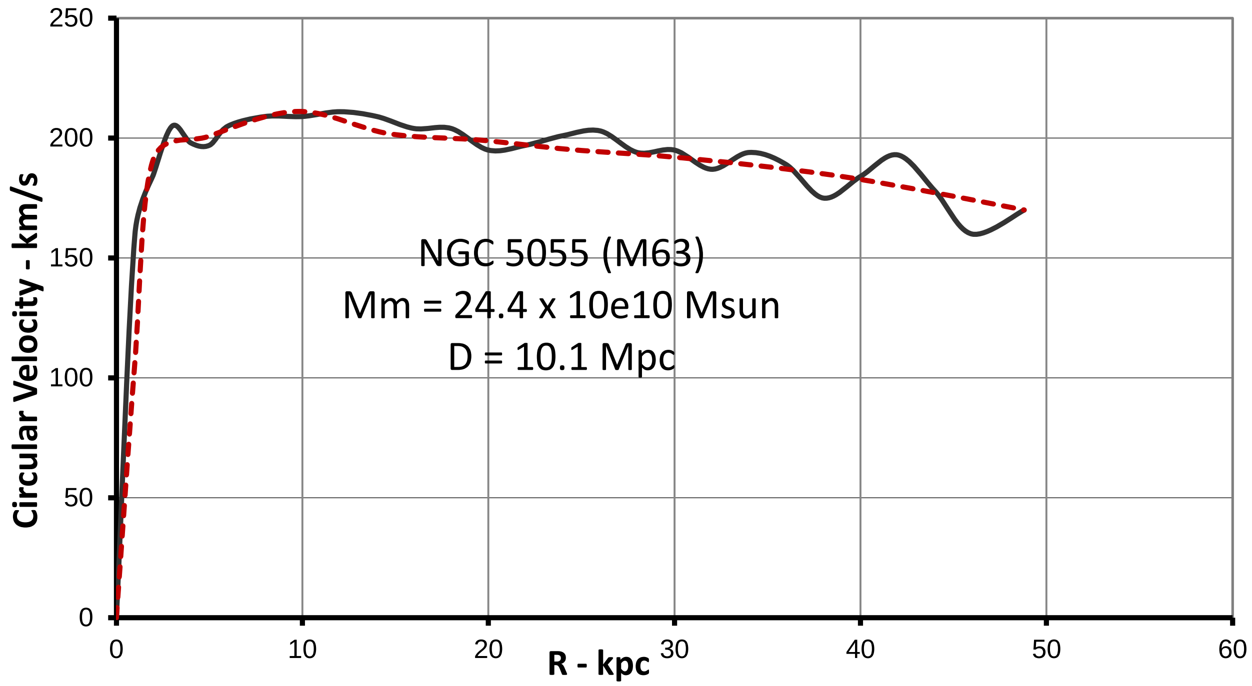

**Figure 12a. Velocity Profiles of NGC 5055 and Model**

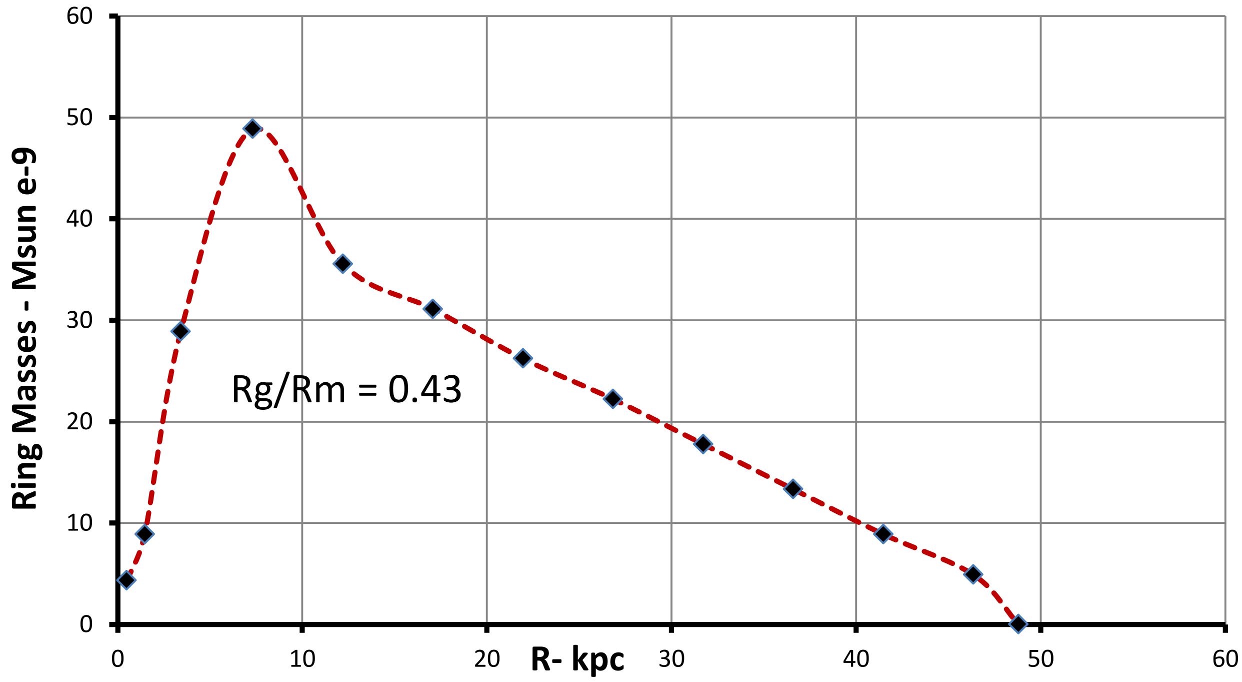

**Figure 12b. Ring Mass Distribution for NGC 5055 Model**

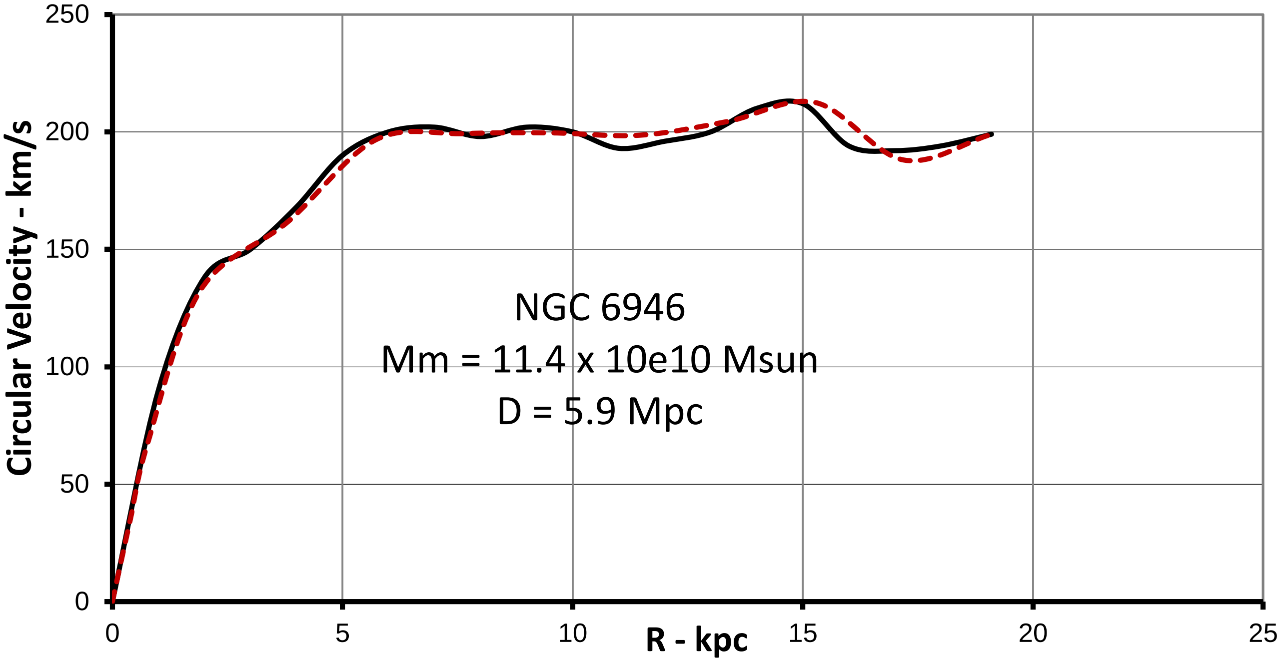

Figure 13a. Velocity Profiles of NGC 6946 and Model

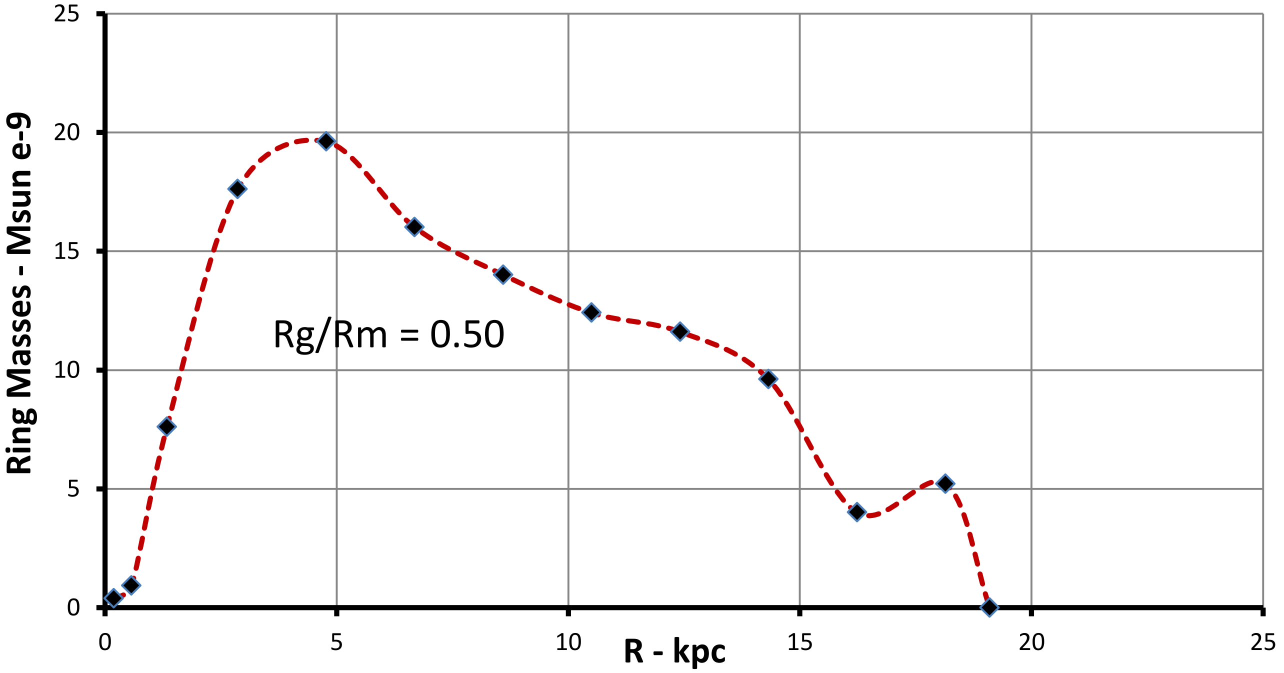

Figure 13b. Ring Mass Distribution for NGC 6946 Model

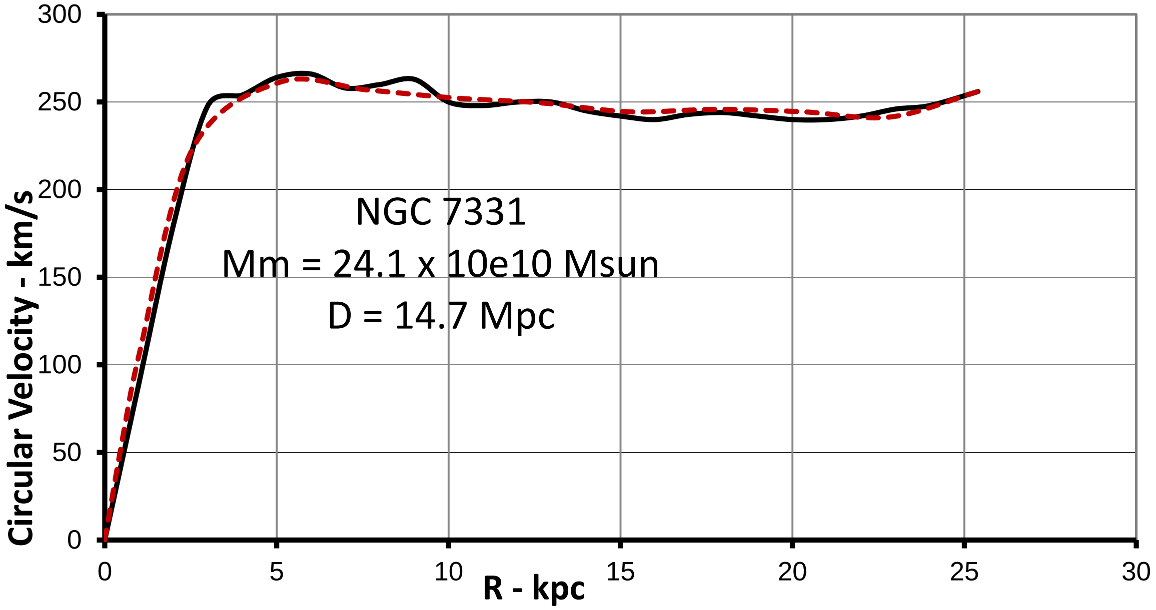

Figure 14a. Velocity Profiles for NGC 7331 and Model

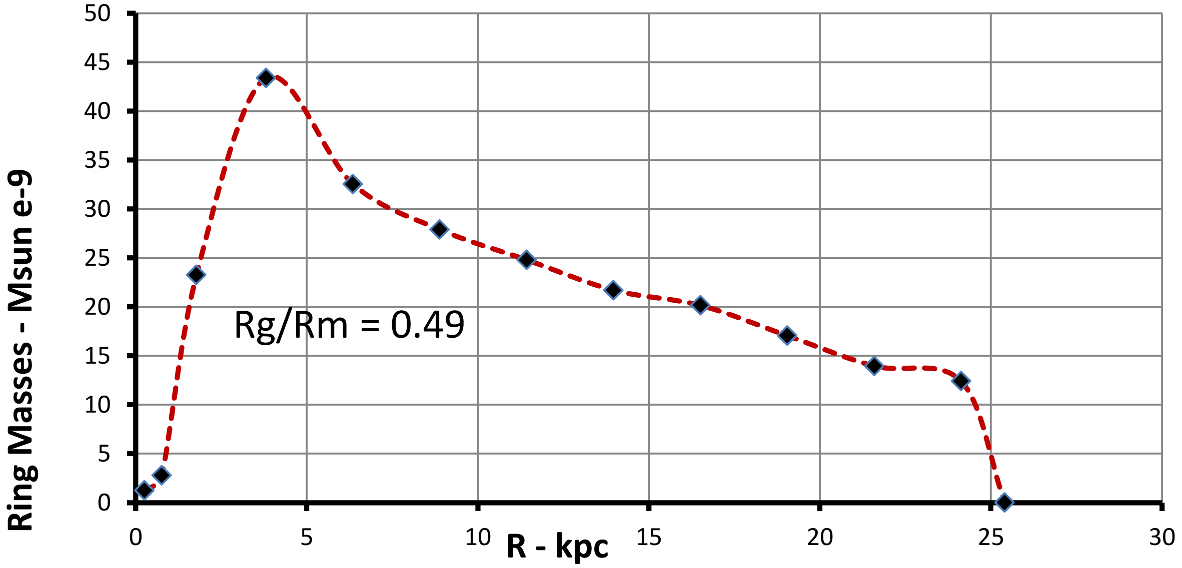

Figure 14b. Ring Mass Distribution for NGC 7331 Model

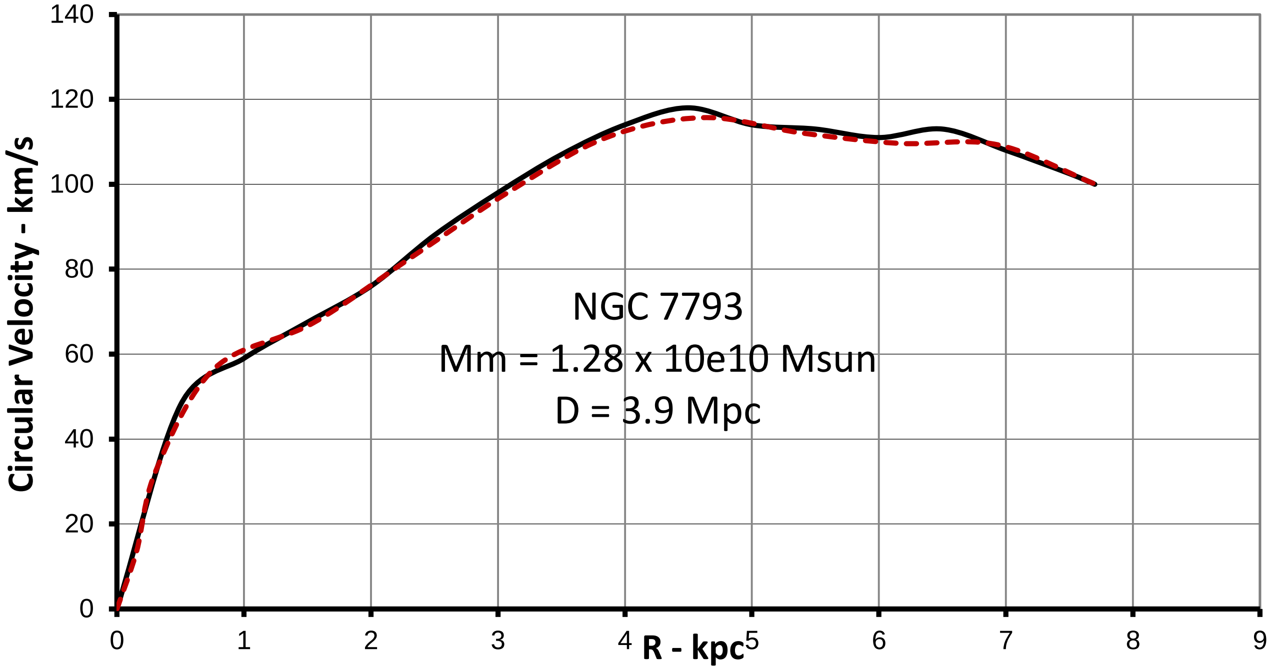

Figure 15a. Velocity Profiles for NGC 7793 and Model

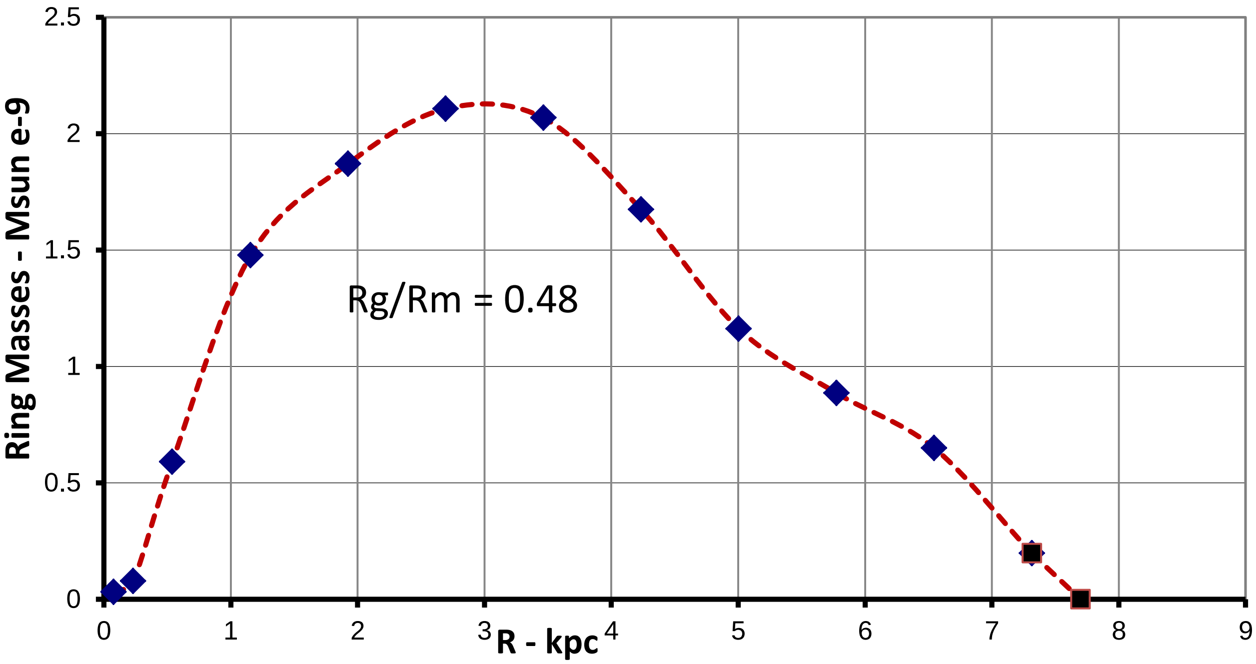

Figure 15b. Ring Mass Distribution for NGC 7793 Model

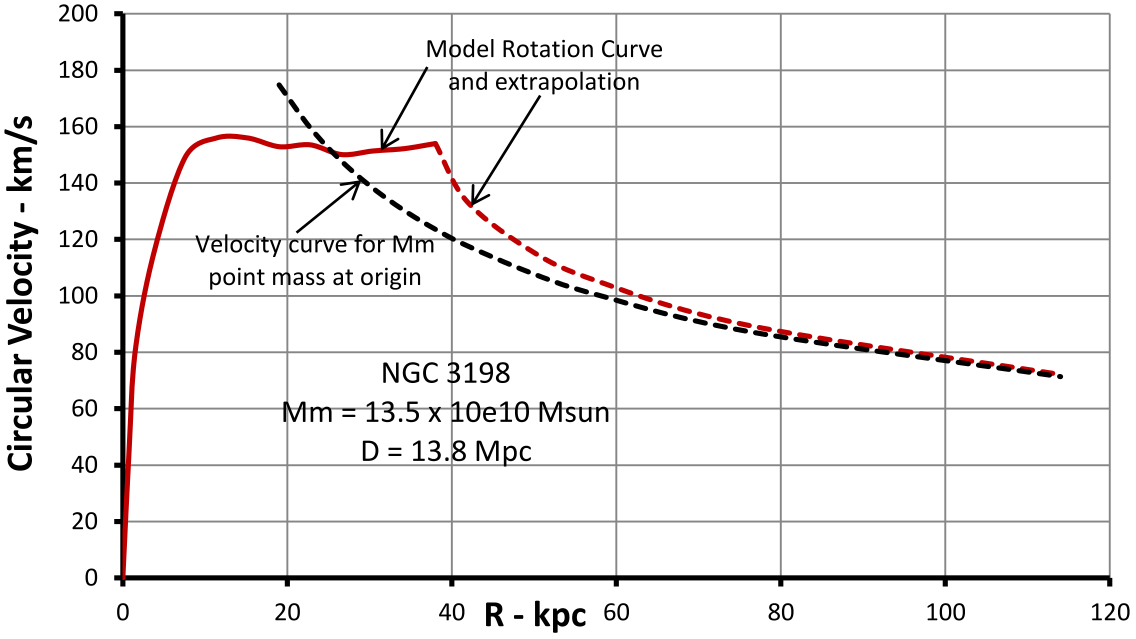

**Figure 16. Velocity Profile for NGC 3198 Model**

NGC 3198 (Figure 7a,7b) is often noted as the prototypical "flat" rotation curve galaxy, and has been extensively studied by professional astronomers. (e.g. References 3,4). One of the advantages of the Newtonian model is that the gravitational force in the galaxy disk plane can be calculated beyond the outer edge of the disk, Rm. This allows appropriate extrapolation of the model velocity curve as shown in the dashed-red line in Figure 16. This would be the velocity of a satellite in a circular orbit in the galaxy disk plane. Also shown in Figure 16 is the (black-dash) Keplerian circular velocity, which assumes the galaxy mass (Mm) is concentrated at the disk center. It can be seen that the model extrapolation curve and the Keplerian curve effectively merge at about 80 kpc (i.e. about 2 Rm, and this would be typical for "flat" rotation curves.)

NGC 3198 is an Sc type spiral galaxy with no noticeable nuclear bulge, so the three components of galaxy mass are stellar disc, HI gas and Dark Matter. Figure 17 displays the mass distribution of the stellar disc and Dark Matter components. (The HI gas component, which is typically a small percentage of the galaxy mass, is ignored in this comparison.) The stellar component is modeled with a thin exponential disk that fits the luminosity profile as described in Reference 3. The Dark Matter component is determined by subtracting the stellar component from the total galaxy mass distribution. The model assumes that all of the galaxy mass is contained as a "flattened" mass within the disk limit of Rm. (Reference 5 (Jalocha et al, 2011) examines the vertical gradients of azimuthal velocity in a series of spiral galaxies, and concludes that the mass distribution appears to be dominated by a "flattened disc"-like component, rather than by a spheroidal dark-matter halo.)

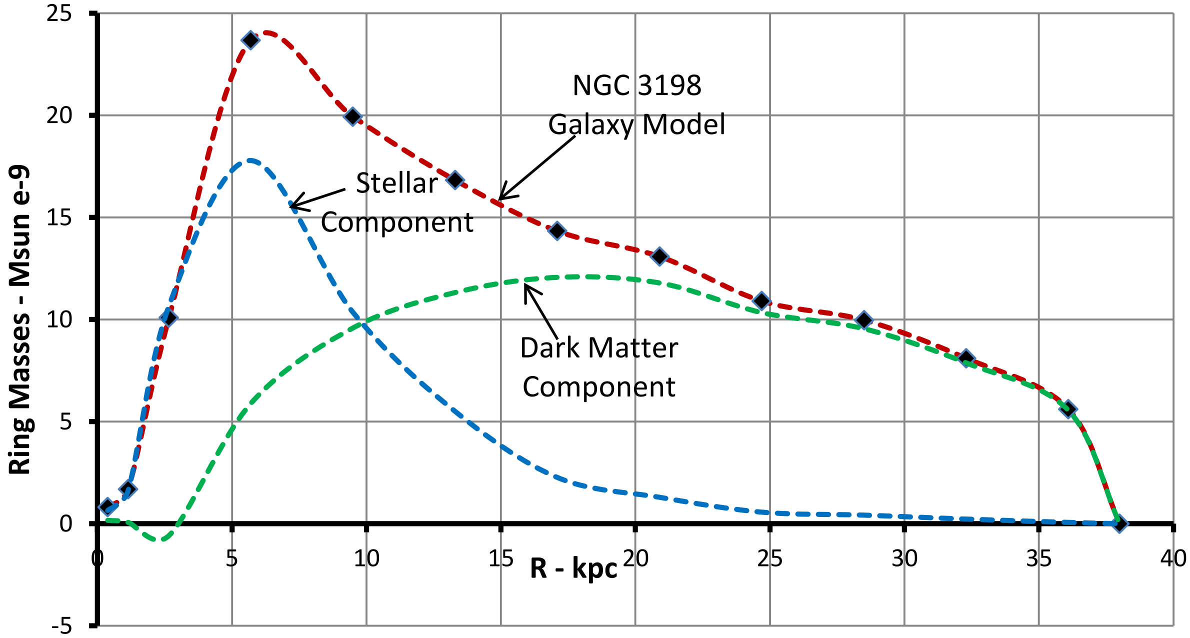

**Figure 17. Ring Mass Distribution of NGC 3198 Model and Stellar and Dark Matter Components**

## Summary

The Newtonian model of Reference 2 has been useful in estimating the enclosed galaxy mass and describing the mass distribution of 15 spiral galaxies, using the rotation curves from THINGS (Reference 1.) The NGC 3198 model also demonstrated the extrapolation of the rotation curve beyond Rm, and indicated the mass distribution of the stellar and dark-matter components.

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3. *Distribution of Dark matter in the Spiral Galaxy NGC 3198.*
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