Interference Between Distinguishable States

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Interference effects are known to have a dependence upon indistinguishability of path. For this reason, it is accepted that different sources of a system cannot interfere if they have distinguishable states. We present an analysis of interference that predicts the allowance of interference between distinguishable states under the strict condition that the preparation has no distinguishability of path that results from the distinguishability of the states of the possible sources. This leads us to a new analysis of the quantum eraser experiment.

In Principle Indistinguishability of paths

In the history of interference effects in quantum mechanics it is dogmatically accepted in the literature that interference will only occur if there is no distinguishing information of path. For the classic double slit interference effect it is often argued that the knowledge of the system's passage through one of the slits will lead to the result of non-interference, and only in the preparation where there is no such knowledge will the interference be measured. The reason for which interference effects require that the preparation be one where there is no implied knowledge of path is because any knowledge of path would imply a different state description. Knowledge of path could only be possible if the state of the system was not the superposition of states which allows for both paths. This can be loosely stated by acknowledging that the measurement outcome of interference corresponds to a calculation with "adding amplitudes before squaring" and non-interference is calculated by "squaring before adding". If the preparation is one where there is no distinguishability of path then the appropriate state description is the desired superposition which leads to the calculation of interference. If the preparation has distinguishability of path then there is a modification to the state description of the system so that it has evolved to a pure state that has orthogonal terms so the "adding before squaring" leads to null valued crossover terms, and this does not lead to a calculation of interference.

The specific argument related to indistinguishability of path may be stated in the form of a principle; *Interference only occurs when the preparation is such that there is in principle no possible way of determining the interfering system's path of origin*. This principle was decisively proven in the historical ZWM study[1,2,3] of quantum coherence where the distinguishing information of path in a single photon interference effect was made *in principle* possible by measurement of the entangled partner photon. This study was decisive in proving that the observations of distinguishing information of path need only be possible in principle, as the measurement of the entangled partner photon didn't have to actually be performed.

Interference Effects

In quantum mechanics the simplest approach to understanding the effect of interference is to explain it in the context of the "bra-ket" formalism. In an interaction of measurement, such as absorption, if one possible final system state is F(x) and a possible initial system state is G(x) then we may calculate the bra-ket between them as representing the probability of the system starting and ending in these states, $\langle F(x)|G(x)\rangle$. We assume that the absorption transformation of the state is a unitary one-to-one mapping of all eigenstates of the initial system to the eigenstates of the final system.

In general, if we wish to get interference between two sources of a system then we would prepare the two sources in the same state, G(x), so that the total initial state is the combination of the two possible states, $2^{-1/2}\{|G^1(x)\rangle + |G^2(x)\rangle\}$ where the superscripted 1 & 2 indicate the different possible sources. It is assumed that the final state has an eigenstate for every possible initial eigenstate of the absorbed system, so if $g_i(x)$ and $f_i(x)$ are the possible eigenstates of G(x) and F(x) respectively then there is a $g_i(x)$ for every $f_i(x)$ in a one-to-one manner. The final state F(x) is always a complete superposition of all possible states $f_i(x)$,

$$|F(x)\rangle = \sum_i c_i |f_i(x)\rangle$$

In order for the calculation to yield an interference term the bra-ket must have an initial state, G(x), which is a superposition of the two states that represent *identical* sources or else the distinguishability of sources will be translated to the final state of the absorbing system. As an example, we may refer to the prepared state of the initial system as a single mode, $G(x) = g_n(x)$. Because the states $G^1(x)$ and $G^2(x)$ are identical, $G^1(x) = G^2(x) = g_n(x)$, then we will calculate interference because the only relevant final state is $f_n(x)$. The bra-ket becomes,

$$\begin{aligned}\langle F(x)|G(x)\rangle &= \langle f_n(x)|2^{-1/2}\{|G^1(x)\rangle + |G^2(x)\rangle\} \\ &= 2^{-1/2}\{\langle f_n(x)|g_n^{\,1}(x)\rangle + \langle f_n(x)|g_n^{\,2}(x)\rangle\}\end{aligned}$$

The final state $f_n(x)$ may also be expanded about the superposition of the two possible initial states superscripted with 1 & 2 because these are spatial path states that overlap upon combination,

$$\langle f_n(x)| = 2^{-1/2}\{\langle f_n^{\,1}(x)| + \langle f_n^{\,2}(x)|\}$$

so that we get the bra-ket as,

$$\begin{aligned}\langle F(x)|G(x)\rangle &= \tfrac{1}{2}\{\langle f_n^{\,1}(x)| + \langle f_n^{\,2}(x)|\}\{|g_n^{\,1}(x)\rangle + |g_n^{\,2}(x)\rangle\} \\ &= \tfrac{1}{2}\{\langle f_n^{\,1}(x)|g_n^{\,1}(x)\rangle + \langle f_n^{\,1}(x)|g_n^{\,2}(x)\rangle + \langle f_n^{\,2}(x)|g_n^{\,1}(x)\rangle + \langle f_n^{\,2}(x)|g_n^{\,2}(x)\rangle\}\end{aligned}$$

The first and last terms are simple probabilities of spontaneous emission, but the second and third terms are what we commonly call cross-over terms. Because the cross-over terms have similar subscripts, n, they will be non-zero valued and they will show an interference term that is a varying function of the phase of the source. If the states superscripted with 1 & 2 (which indicates the possible sources) are distinguishable (orthogonal, with different subscripts) then the cross-over terms vanish and we do not get a calculation of interference.

In optics, the relevant bra-ket is the detection probability which is calculated using the electric dipole approximation. The interaction Hamiltonian between the absorbing atom and the absorbed photon is a scalar product of the dipole moment operator of the atom and the electric field operator of the photon. This is ideal to separate the bra-ket into two time dependent terms, one is a bra-ket using the states of the atom with the electric dipole operator of the atom as the operator, the sensitivity function, and the other is a bra-ket using the states of the photon with the negative and positive frequency components of the electric field as the operator, the correlation function. The detection probability is a product of the two. This separation of the states of the atom and the states of the field are misleading when we consider the theory of interference in optics, we tend to only consider interference to be possible if the calculation of the correlation function leads to interference. The calculation of the correlation function will never yield interference terms if the sources have distinguishable states.

So how do we get around this? How do we get interference between distinguishable sources that are in orthogonal states, and not just in optics but in all species of systems? We consider the option of having an absorbing system which does not *discriminate* between distinguishable states of the absorbed system. Our aim is for the preparation of the absorbing system to be such that it is not possible in principle to measure the absorbing system after absorption in a way that it might indicate the prior state of the absorbed system (which would indicate the path of the absorbed system). If the two interfering sources are in orthogonal states initially then the final state of the absorbing system must only allow the superposition of the appropriate orthogonal states. You could say that we wish for the absorbing system to be coherent and one-to-one with the absorbed system. If our initial distinguishable sources have orthogonal states of $g_n(x)$ and $g_m(x)$ then we prepare the system with coherent overlap at the absorption so that the initial state is

$$|G(x)\rangle = 2^{-½}\{|g_n^1(x)\rangle + |g_m^2(x)\rangle\}$$

With this type of preparation we would usually only be able to measure a projection onto the superposition state and we would not be able to measure interference because we would be using a detector that discriminates between the states of $g_n(x)$ and $g_m(x)$. For this combination we must use an absorbing system which only allows the final state of the absorbed system as

$$|F(x)\rangle = 2^{-½}\{|f_n(x)\rangle + |f_m(x)\rangle\}$$

In this preparation our final state of the absorbing system is one-to-one with our initial state of the absorbed system. The calculation of the bra-ket is the same and we would get non-zero valued cross-over terms.

If we use the example of optics, we could consider a detector with an absorbing atom which was specifically prepared to absorb photons in a superposition of polarization eigenstates. If the atom is prepared to absorb only photons in the 45° polarized state, as is the case for a polarizer with its optical axis at 135° relative to horizontal, a 135°/-45° polarizer, then we would expect that the detection will allow interference between sources that have distinguishable states of vertical (V) and horizontal (H) polarization. If two coherently prepared beams of H and V polarization are combined at a polarizing beam splitter we would usually only be able to measure its projection onto a state of 45° polarization, as could be done with a 45° polarizer and a normal detector. However if we use a detector that was made of a polarizing material with the optical axis at 135°/-45° that only absorbs 45° polarized photons, then we would also be able to see spatial interference between the two sources of H and V polarization. This is because the final state of the detection atom is one-to-one with the input state of the photon.

The Quantum Eraser

With the above stated interpretation of interference we may now consider what is happening in the preparation of the quantum eraser effect[4]. Specifically we consider the case of a quantum eraser which is based upon a single photon interference effect[5,6,7]. In reference 5 the type of quantum eraser described uses an established single photon interference effect (that of reference 1) with a specific action of choice that may be used to destroy the interference effect. In references 6 & 7 the quantum eraser is built upon the typical protocol of having an established interference effect, introducing an action that marks one path and destroys interference, and then introducing another action (erasure) that allows interference again. For the purpose of presenting our argument in this writing we assume the latter quantum eraser protocol of references 6 & 7, but our argument equally applies to any protocol of quantum erasing that includes a “choice” that may be exploited to modulate between states of interference and non-interference where there is a fringe/anti-fringe interference effect that is dependent upon the state of the path marker.

We specifically consider the simple form of quantum eraser in Figure 1 which is also discussed in reference 7, that of the two slit interference effect with horizontally polarized (H) light. In this quantum eraser the double slit produces interference on a distant detector screen where the central maxima is level with the slits. A half wave plate (HWP) is placed in front of one of the slits to mark the upper path by rotating the polarization of the light that traverses this slit to vertical polarization (V). This has the effect of destroying the interference pattern at the detector screen. With this state the detector would display a pattern that drops off in intensity with the inverse square of the distance from the slits, a Gaussian shape. The interference is recovered by placing a ±45° polarizer in between the slits and the screen such that all light that is incident on the detector screen is transmitted through the polarizer. The interference that results is dependent upon the state of the 45°polarizer, if the polarizer is at +45° polarization then we observe the fringe pattern of the original interference pattern with a reduced intensity, and if the polarizer is at -45° polarization then we observe the anti-fringe pattern (where the constructive and destructive interference are interchanged) of a similarly reduced intensity. The intensities of the fringe and anti-fringe pattern are such that they add to produce the non-interference pattern with no ±45° polarizer. An important point to note in this experiment is that the 45° polarizer can be placed arbitrarily close to the detection screen without modification of the effect.

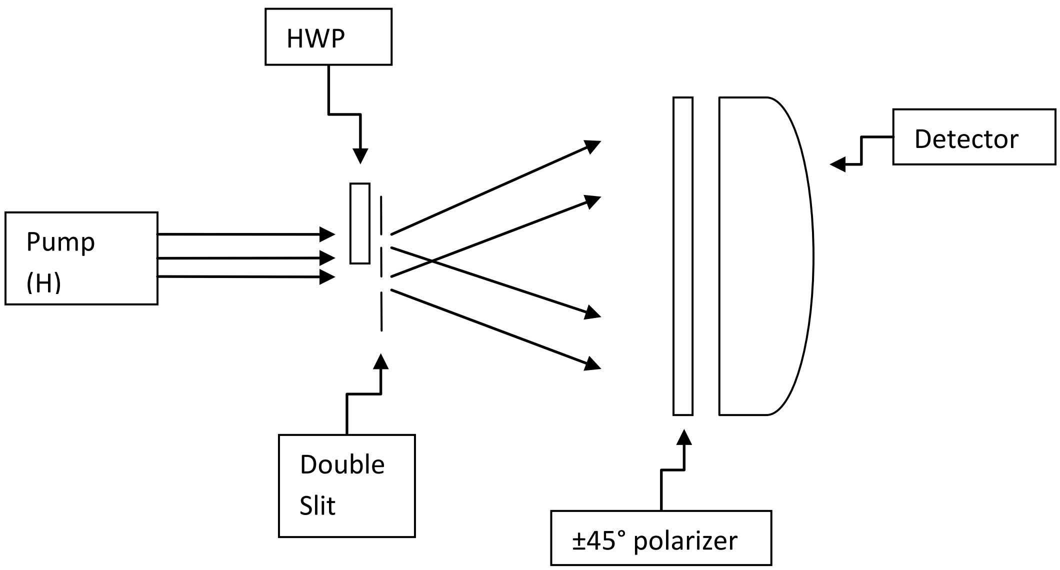

Figure 1 : Horizontally polarized pump light is incident upon a double slit with a half wave plate (HWP) placed in front of the upper slit. The scattered light is incident upon a ±45° polarizer and then a detector screen. The HWP is the "path marker" and the polarizer is the "path information eraser". We ask, is the ±45° polarizer the *real* detector?

For purposes of discussion, we assume that we have chosen the +45° polarizer and that it is placed very close to the detection screen. There is two possible ways to interpret what is happening here, either the polarizer is absorbing the photons in a manner that it acts like a detector that displays interference between distinguishable sources, or the light is in a mixed state of interference and non-interference. We consider the two possible perspectives.

Perspective A : The polarizer is absorbing photons (the -45° polarized photons that would otherwise be detected by the screen as the anti-fringe pattern if we had chosen the -45° polarizer) and it absorbs them in a spatial distribution that is identical to the anti-fringe pattern. Assuming this, we would be forced to admit that the 45° polarizer is effectively acting as a detector which is displaying *interference between distinguishable sources* of H and V polarization states.

Perspective B : The polarizer is absorbing photons (the -45° polarized photons) in a manner which is of an even spatial distribution across the polarizer, but the transmitted photons (the 45° polarized photons) are in a state which has them propagating from the slits to the detection screen in a distribution which matches the interference fringe pattern. This would require that the beam be in a specific state after the HWP with one state of 45° polarization in the spatial distribution of the fringes and a second state with -45° polarization in a spatial distribution of non-interference.

Of the two above stated perspectives only perspective A is consistent with our interpretation of interference. The 45° polarizer is absorbing the photons in a manner that is consistent with it being a detector of the -45° polarized light, and it is detecting it in a manner that it displays interference between distinguishable sources of H and V polarization states. The detector only displays the "interference fringe pattern" because it acts as a detector would in absorption spectrometry, it detects the light that does not become absorbed.

The second perspective, that the polarizer has an absorption with spatial distribution that is non-interfering, we will assume is the conventional interpretation of the quantum eraser. If we consider that the experiment has two distinct measurements, the absorption of the polarizer and the absorption of the detector, the only way the two detections could display such a drastic difference in statistics is if there was a mixed state of the light after the HWP. The argument usually applied to the quantum eraser is that there is an entanglement between the path degree of freedom and the polarization degree of freedom of the light that can be described by the state,

$$|f^1(x)> \times |V> + |f^2(x)> \times |H>$$

Where the superscripted state, $f^i(x)$, is the spatial state that labels the slit and the (H,V) basis is the polarization state. But this type of entanglement alone would not explain how the absorption at the polarizer has a different distribution from the detector. We would need a mixed state where the first part of the mixture would have a density matrix with a state of +45° polarization and a spatial state which is distributed over both slit paths.

$$|f^1(x) + f^2(x) > \times |+45°><+45°| \times <f^1(x) + f^2(x)|$$

The second density matrix would have the polarization states entangled with the spatial states on well defined slit paths as in the above entangled state.

$$\{|f^1(x)> \times |V> + |f^2(x)> \times |H>\}\{<V| \times <f^1(x)| + <H| \times < f^2(x) |\}$$

This would be a long expression with four contributing terms. Adding this to the other density matrix would give us a state description which still couldn't explain the observations assumed by perspective B. Why? Because this state does not explain how the photons of the second density matrix are -45° polarized only. There simply is no acceptable state which could explain the observations assumed in perspective B. The proper explanation of the effect is perspective A, and the appropriate state description is that of the simple pure state description of entanglement that is usually chosen to describe this effect. This entangled state is sufficient because we must remember that the state may still "project onto" a state of ±45° polarization upon absorption. The polarizer absorbs the appropriate state and displays its interference and the detector simply displays the left-over portion that wasn't absorbed and is in the opposite polarization state. This is exactly how the quantum eraser is usually explained but only in the context of the light which *passes* the absorption. The conventional analysis doesn't include a description of interference occurring in the absorption of the polarizer.

To be fair, we have specifically highlighted references 6 and 7 in our criticism of the quantum eraser analysis, when in truth these two references are only a small example of the scores of quantum eraser experiments that have a similar analysis. In all cases there is no mention of the interference occurring at the absorption.

We have only taken into account the cases of quantum erasers that use a single photon interference effect, but our analysis applies equally to all quantum erasers that use a two-photon interference effect that would produce the mysterious fringe/anti-fringe patterns. However, there are quantum eraser protocols that our analysis does not apply to, such as reference 5, which do not have the modern protocol of interference +choice + erasure but rather interference + choice. Of the many quantum erasers that do have the interference +choice + erasure protocol, we would mention that there are some which our analysis does not apply to such as reference 8, but you will note that this study does not have the mysterious fringe/anti-fringe patterns either.

Summary

When we consider that the whole history of interference effects assumes the use of detectors which can discriminate the states of the absorbed system then this confusion is all perfectly understandable. The absorbing system is always able to be measured itself in a manner that would indicate the state of the absorbed system. Specifically for the case of optics, the detectors used will always utilize an atomic absorption for the photon measurement. Because the absorption is unitary, after the absorption the electron's state could be measured to specifically indicate the state of the absorbed photon. If the absorbed photon was in a state of mixed polarization, the electron could be measured to indicate the state of polarization of the photon. It is always possible *in principle* to measure the distinguishable states of the absorbed photons when one uses a discriminatory detection system. In the case of the +45° polarizer acting as a detector, it is not discriminatory because it absorbs -45° polarized light. The light that transmits the slits and coherently recombines at the polarizer is -45° polarized. So the one-to-one nature of the states of the absorbing and absorbed systems allows for interference between the distinguishable sources.

References

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