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Pretty Algebra

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Abstract

The sum of the interior angles of a number triangles were transformed into linear algebraic equations. The analysis of these equations without assuming the fifth Euclidean postulate established the following theorem: There exists a triangle whose interior angle sum is equal to two right angles

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Introduction

It's hard to add to the fame and glory of Euclid who managed to write an all-time bestseller, a classic book read and scrutinized for the last 23 centuries. However insignificant the following point might be, I'd like to give him additional credit for just stating the Fifth Postulate without trying to prove it. For attempts to prove it were many and all had failed. By the end of the last century, it was also shown that the fifth postulate is *independent* of the remaining axioms, i.e., all the attempts at proving it had been doomed from the outset. Did Euclid sense that the task was impossible?

The earliest source of information on attempts to prove the fifth postulate is the commentary of Proclus on Euclid's *Elements*. Proclus, who taught at the Neoplatonic Academy in Athens in the fifth century, lived more than 700 years after Euclid. Although an invaluable source for the history of mathematics, the *Commentary* is unlikely to be complete. Proclus mentions Ptolemy's (2nd century) attempts to prove the postulate and demonstrates that Ptolemy had unwittingly assumed what in later years became known as Playfair's axiom. Proclus left a proof of his own, but the latter rests on the assumption that parallel lines are always a bounded distance apart, and this assumption can be shown to be equivalent to the fifth postulate.

al-Gauhary (9th century) deduced the fifth postulate from the proposition that through any point interior to an angle it is possible to draw a line that intersects both sides of the angle. He deduced the proposition from an implicit assumption that if the alternating angles determined by a line cutting two other lines are equal, then the same will be true for all lines cutting the given two. The proposition was implicitly used by A.M.Legendre (1800) in his proof of the fifth postulate.

al-Haytham's (10th century) kinematic method was criticized by Omar Khayyam (11th century) whose own proof was published for the first time in 1936. To Omar's credit he thought up a figure that was later named after Gerolamo Saccheri (1667-1733). Nasir ad-Din at-Tusi (13th century) was more fortunate. A Latin edition of his work appeared in Europe in 1657. at-Tusi critically analyzed the works of al-Gauhary, al-Haytham and Omar Khayyam. In one of his own attempts, at-Tusi tried to prove the postulate by a *reductio ad absurdum*. This appears to be the first attempt to prove the postulate by deriving a contradiction from the assumption that the fifth postulate is wrong.

John Wallis has been inspired by the work of at-Tusi and delivered a lecture at Oxford on July 11, 1663. To prove the postulate he made an explicit assumption that for every figure there is a similar one of arbitrary size. Unlike many (even later) mathematicians, John Wallis realized that his proof was based on an assumption (more natural in his view but still) equivalent to the postulate.

The line of reasoning of at-Tusi had been taken up by a professor of rhetoric, theology and philosophy at a Jesuit college in Milan, Girolamo Saccheri. In 1733, Saccheri published a two-volume work titled *Euclid Freed of Every Flaw*. Given a line and a point not on the line, there are exactly three possibilities with regard of the number of lines through the point:

A. there is exactly one parallel;
B. there are no parallels;
C. there are more than one parallel.

The three hypotheses are known as hypotheses of the *right*, *obtuse*, and, respectively, *acute* angles. The first one is Playfair's axiom and, thus, is equivalent to Euclid's fifth postulate. Assuming that Euclid's second postulate (*A piece of straight line may be extended indefinitely*.) requires straight lines to be infinitely long, he showed that (B) indeed leads to a contradiction. Based on (C), he proved several counterintuitive statements but couldn't formally obtain a logical contradiction. Probably to justify the title of the work he stated

The hypothesis of acute angle is absolutely false; because repugnant to the nature of the straight line.

Saccheri's work attracted little attention and was virtually unknown until 1899 when it was republished by his compatriot, Eugenio Beltrami (1835-1900). In 1766, Heinrich Lambert (1728-1777) published a similar investigation. He also observed that results derived under the hypothesis (B) resemble those known for spherical geometry and suggested that, geometry following from (C) might be visualized on a sphere of imaginary radius.

Adrien-Marie Legendre (1752-1833) was preoccupied with the fifth postulate for decades. His work appeared in successive additions of his very popular *Éléments de Géométry* (1794-1823). The small book was translated into English first in 1819 and, then, by Thomas Carlyle, in 1822. Carlyle's translation ran through 33 American editions (from H.Eves, MAA, 1983). Legendre succeeded in popularizing geometry and the question of the fifth postulate but, of course, failed to prove it. His last article on parallels saw light in 1833, the year of Legendre's death, four years after publication by the

Russian mathematician N.Lobachevsky of his paper on non-Euclidean geometry and a year after a similar publication by the Hungarian János Bolyai**.**

Construction

In the Euclidean construction as shown in figure 1,x,y,z and m denote the sum of the interior angles of triangles AOB,AOC,DOC and DOBE respectively.Also let a,b,c and d respectively refer to the sum of the interior angles in triangles ABC,ADC,BEC and AED.

Sides OB and OC are equal (1)

Side OA is greater than side OD (2)

The angles AOD and BOC are straight angles and so their measures are equal to 180 degrees.Let v be the value of this 180 degree (3)

Using (3), $x + y = v + a$ (4)

$y + z = v + b$ (5)

$z + m = 2v + c$ (6)

$m + x = 2v + d$ (7)

(4) - (7) gives,$m + a = y + v + d$ (8)

(5) - (6) gives,$m + b = y + v + c$ (9)

Squaring (8), $m^2+a^2+2ma = y^2+v^2+d^2+2yv+2yd+2vd$ (8a)

Squaring(9), $m^2+b^2+2mb = y^2+v^2+c^2+2yv+2yc+2vc$ (9a)

(4) + (6) = (5) + (7) = $a+c = b+d$ (10)

Squaring (7), $a^2+c^2+2ca = b^2+d^2+2bd$ (10a)

(8a) - (9a) given, $a^2-b^2+2ma - 2mb = d^2-c^2 + 2v(d-c)+2y(d-c)$

$a^2-b^2+2ma - 2mb = d^2-c^2 + 2(d-c)[y + v]$

Putting(9) in RHS, $a^2-b^2+2ma - 2mb = d^2-c^2 + 2(d-c)[m + b - c]$

i.e $a^2 - d^2 + m [2a - 2b - 2d + 2c] - b [b + 2c - 2d] + c [2d +c -2c]$

Applying (10) in the second factor of LHS, $(a + d) (a - d)- b [b + 2c - 2d]+ c [2d - c]$

From (10), $a - d = b - c$ and $b - a = c - d$

.Putting these in the above equation, $b [2a - 3b + a + d]+ c [2d - c -a - d]$

Applying (10) in the second factor, $b [2a - 3b + a + d] - bc = 0$

i.e $b [3a - 3b +d - c] = 0$

From (10) we get that d - c = a - b.Applying this b[4a-4b] =0

i.e $a = b$ (11)

Analysing (4) , (5) and (11) we have $z = x$ (12)

By construction Sides OB and OC are equal (1)

and side OD is greater than side OA (2)

Now look at figure 2.On OA, cut off F such that OD = OF.

By SAS correspondence, triangles FOB and COD are congruent.

But from (12) the sum of the interior angles of triangles AOB and COD are equal.

From this we obtain that the sum of the interior angles of triangles ABO and ABF are equal. Consequently, we get that the sum of the interior angles of triangle ABF is equal to two right angles (13)

Discussion

Since we have derived (13) without assuming Euclid's fifth postulate which is an unsolved classical problem for more than 2300 years, beyond each and every mathematical doubt, (13) establishes the parallel postulate. [1 - 4] .Figure 1 can be extended to both hyperbolic and elliptic spaces. So, (13) will hold even in non- Euclidean spaces. But the mere existence of consistent models of non-Euclidean geometries demonstrate that Euclid V can NOT be deduced from Euclid I to IV. But our result canNOT be challenged. Questioning (13) will force us to doubt the fundamental operations of number theory and algebra.

The famous unsolved classical problems such as squaring the circle, duplicating the cube, trisection of a general angle and to draw a regular septagon are not merely difficult but IMpossible to solve. In this study, application of classical algebra explored a masterpiece result in geometry. There is something hidden treasure of mathematics. Further probes will unlock this mystery.

References

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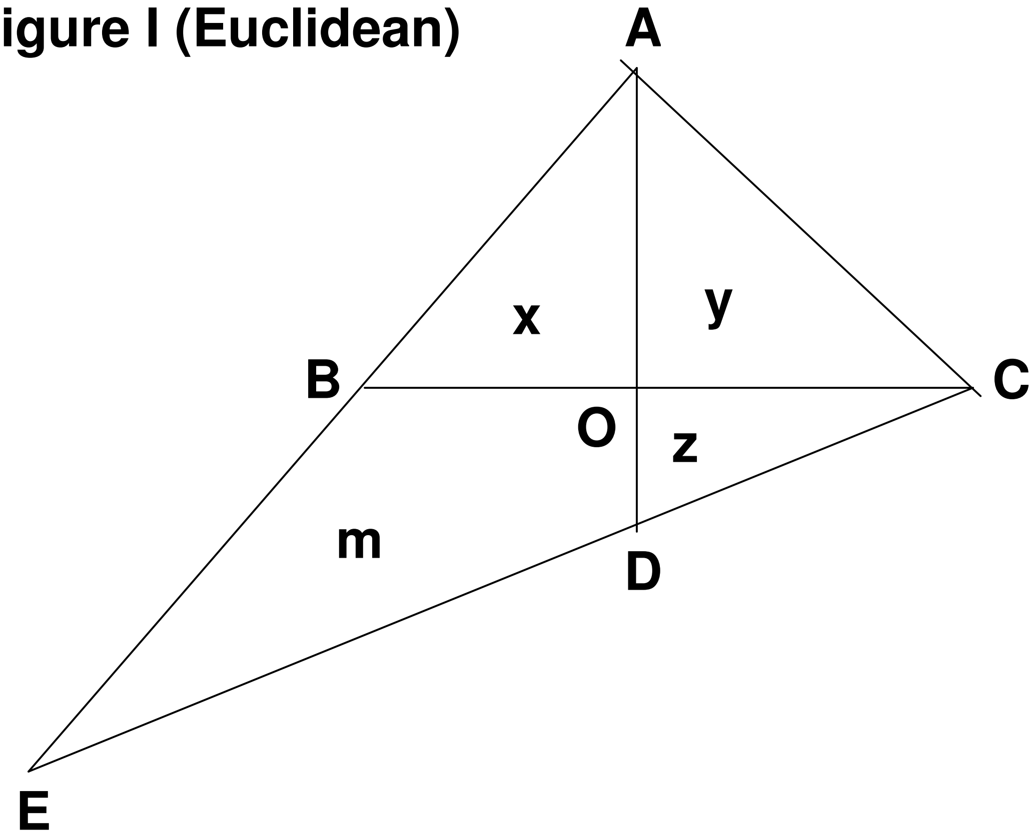
Figure I (Euclidean)



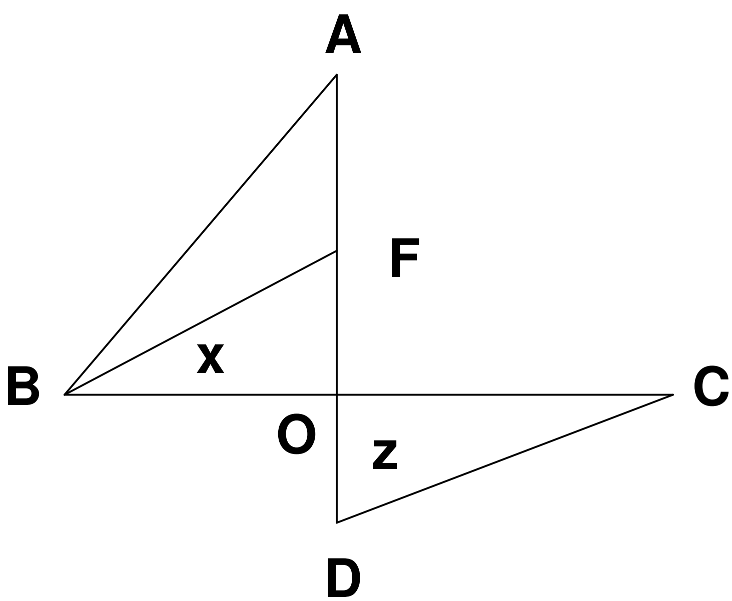


Figure II (Euclidean)