Four sequences of integers regarding balanced primes and Poulet numbers

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Abstract. A simple list of sequences of integers that reveal interesting properties of few subsets of balanced primes.

I.
Balanced primes B that can be written as B = P ± 24, where P is a Fermat pseudoprime to base two (a Poulet number):

1747, 2677, 4657, 41017, 188437, 195997 (...).

Comments:
B that can be written as P + 24: 1747;
B that can be written as P – 24: 2677, 4657, 41017, 188437, 195997.

Note that all these balanced primes are of the form 10*k + 7!

Note: For a list of Poulet numbers see the sequence A001567 in OEIS. For a list of balanced primes see the sequence A006562 in OEIS.

II.
*Balanced primes B2 that can be written as B1 + 330*n – 6, where B1 is also a balanced prime and n is non-negative integer:*

257, 977, 1367, 1511, 1747, 1907, 2417, 2677 (...).

Comments:
B1 corresponding to the least n for that B2 can be written this way and the least n: (263,0), (653,1), (53,4), (1187,1), (1753,0), (593,4), (1103,4), (373,7).

Note that 7 from the first 12 balanced primes of the form 10*k + 7 can be written this way!

Note: Seems that the formula p + 330*n produces many primes when p is a balanced prime of the form 10*k + 3 or 10*k + 7; for instance the number 257 + 330*n is prime for n = 0, 1, 5, 6, 8, 10, 12, 13, 14, 17, 18, 20, 21, 22, 26, 28, 31, 35, 39, 40, 43, 45, 47, 48, 49, 52, 53, 54, 59, 62, 64, 66, 67, 68, 69, 70, 71, 74, 77, 78, 81, 83, 85, 88, 94, 95, that means for 46 values of n from the first 99. I also noticed that the same formula produces many primes and squares of primes when p is a square of prime; for instance the number 361 + 330*n is prime or square of prime for n = 0, 1, 2, 4, 5, 6, 7, 8, 9, 13, 16, 18, 20, 22, 23, 26, 28, 29, 33, 37, 42, 43, 46, 51, 53, 54, 58, 60, 64, 68, 69, 74, 75, 77, 79, 81, 83, 84, 85, 88, 90, 91, 93, 96, 97, that means for the first 45 values of n from the first 99.

III.

*Balanced primes B2 that can be written as B1 + 330*n + 6, where B1 is also a balanced prime and n is non-negative integer:*

263, 593, 1753, 2903, 2963, 4013 (...).

Comments:

B1 corresponding to the least n for that B2 can be written this way and the least n: (257,0), (257,1), (1747,0), (257,8), (977,6), (1367,8).

Note that 5 from the first 14 balanced primes of the form 10*k + 3 can be written this way!

IV.

*Balanced primes B2 that can be written as B1 + 1980*n, where B1 is also a balanced prime and n is positive integer:*

3733, 4013, 4657, 6863, 11411, 11807, 11933, 13463, 15193, 15767, 16097, 16787, 16987, 17483, 19463, 19477, 20107, 20123, 22447, 23333, 23893, 24413, 25621, 26177, 26393, 26693, 26723, 27067 (...).

Comments:

The corresponding (B1,n): (1753,1), (53,2), (2677,1), (2903,2), (1511,5), (1907,5), (4013,4), (7523,3), (3313,6), (11807,2), (257,8), (947,8), (5107,6), (7583,5), (7583,6), (3637,8), (2287,9), (6263,7), (12547,5), (9473,6), (6073,9), (653,12), (21661,2), (2417,12), (24413,1), (10853,8), (2963,12), (3307,12).

Comments:
B2 may sometimes be written this way for more than one set of values of B1 and n (for instance 11933 = 4013 + 4*1980 = 53 + 6*1980); we refered through the corresponding (B1,n) to the least value of n.

Note that 32 from the first 171 balanced primes can be written as B + 1980*n, where B is a smaller balanced prime.

Conjecture: Any balanced prime B beside the first one, 5, generates an infinity of balanced primes of the form B + 1980*n (e.g. the second balanced prime, 53, generates for n = 2, 6, 14, 56 the balanced primes 4013, 11933, 27773, 110933).

Conjecture: Any balanced prime B beside the first one, 5, generates through the formula B – 1980*n an infinity of balanced primes in absolute value (e.g. 5807 – 6*1980 = -6073, where 5807 and 6073 are balanced primes).