Creator's Standard Equation, General Solution and Naked Barred Galaxies

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Abstract We have not found the general solution to the Creator's equation system. However, we have outlined the strategy for determining the solution. Firstly, we should study the stretch equation which is the first order linear and homogeneous partial differential equation, and find its all stretches which correspond to the given vector field (i.e., the gradient of the logarithmic stellar density). Our solution G(x,y), however, must be simultaneously the modulus of some analytic complex function. It is called the modulus stretch. Secondly, among all possible modulus stretches, we find the right solution (i.e., the orthogonal net of curves) which satisfies the Creator's standard equation.

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The Creator's standard equation system without composite functions is the following which is presented as the formula (67) in the following Section 5

$$\begin{cases} G(x,y)=\alpha^2+\beta^2,\ \ \phi(x,y)=\alpha\beta,\ \ \psi(x,y)=\alpha^2-\beta^2, \\ \alpha'_y=\beta'_x,\ \ \alpha'_x=-\beta'_y, \\ G'_x=H(x,y)f'_x,\ \ G'_y=H(x,y)f'_y, \\ 2G(\phi(f''_{xx}-f''_{yy})-\psi f''_{xy})=H(\phi(f'^2_x-f'^2_y)-\psi f'_x f'_y) \end{cases}$$

Now we derive it step by step.

1 Rational Structure

Rational structure in two dimension

$$\rho(x,y) \tag{1}$$

means that not only there exists an orthogonal net of curves in the plane

$$x=x(\lambda,\mu),\ y=y(\lambda,\mu) \tag{2}$$

but also, for each curve, the matter density on one side of the curve is in constant ratio to the density on the other side of the curve. Such a curve is called a proportion curve or a Darwin curve. Such a distribution of matter is called a rational structure.

Because the ratio of density $\rho(x, y)$ is proportional to the derivative to the logarithm of the density

$$f(x, y) = \ln \rho(x, y) \tag{3}$$

we, from now on, are only concerned with the logarithmic density $f(x, y)$. We know that, given the two partial derivatives

$$\frac{\partial f}{\partial x}, \quad \frac{\partial f}{\partial y} \tag{4}$$

the structure $f(x, y)$ is determined provided that the Green's theorem is satisfied

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} \tag{5}$$

Now we are interested in rational structure and ignore the partial derivatives (4). Instead, we calculate the directional derivatives along the tangent direction to the above curves (2)

$$\frac{\partial f}{\partial l_\lambda}, \quad \frac{\partial f}{\partial l_\mu} \tag{6}$$

where l_λ is the linear length on the (x, y) plane and is along the row curves whose parameter is λ while l_μ is the linear length along the column curves whose parameter is μ. Given the two partial derivatives (6), however, the structure $f(x, y)$ may not be determined. A similar Green's theorem must be satisfied

$$\frac{\partial}{\partial \mu}\left(P\,\frac{\partial f}{\partial l_\lambda}\right) - \frac{\partial}{\partial \lambda}\left(Q\,\frac{\partial f}{\partial l_\mu}\right) = 0 \tag{7}$$

where

$$P(\lambda, \mu) = \sqrt{x_\lambda'^2 + y_\lambda'^2}, \quad Q(\lambda, \mu) = \sqrt{x_\mu'^2 + y_\mu'^2} \tag{8}$$

are the lengths or magnitudes of the vectors (x'_λ, y'_λ) and (x'_μ, y'_μ) on the (x, y) plane, respectively. Note that we have used the simple notation $x'_\lambda = \frac{\partial x}{\partial \lambda}$. From now on, we always use the similar simple notation. To simplify the expression of our equations, we introduce one more notation

$$u(\lambda, \mu) = \frac{\partial f}{\partial l_\lambda}, \quad v(\lambda, \mu) = \frac{\partial f}{\partial l_\mu} \tag{9}$$

As you might know, the above notation is very useful. However, our articles on rational structure which employ the notations were rejected hundreds of times by the editors of over fifty scientific journals. We are afraid that the editors could not understand the notation at all.

The condition of rational structure is that u depends only on λ and v depends only on μ

$$u = u(\lambda), \ v = v(\mu) \tag{10}$$

Now we prove the condition. Assume you walk along a row curve. The logarithmic ratio of the density on your left side to the immediate density on your right side is approximately the directional derivative of $f(x, y)$ along the column direction. That is, the logarithmic ratio is approximately the directional derivative $v(\lambda, \mu)$. Because $v(\lambda, \mu)$ is constant along the row curve (rational), $v(\lambda, \mu)$ is independent of λ: $v = v(\mu)$. Similarly, we can prove that $u(\lambda, \mu) = u(\lambda)$.

2 Rational Structure Equation

In the case of rational structure, the directional derivatives, $u = \frac{\partial f}{\partial l_\lambda}$ and $v = \frac{\partial f}{\partial l_\mu}$, are the functions of the single variables λ and μ, respectively (see the formula (10)). Therefore, the Green's theorem (7) turns out to be much simpler that is called the rational structure equation [1,2]

$$u(\lambda)P'_\mu = v(\mu)Q'_\lambda \tag{11}$$

To transform the equation and find its geometric meaning, we calculate

$$\begin{aligned} P'_\mu &= (x'_\lambda x''_{\lambda\mu} + y'_\lambda y''_{\lambda\mu})/P, \\ Q'_\lambda &= (x'_\mu x''_{\lambda\mu} + y'_\mu y''_{\lambda\mu})/Q \end{aligned} \tag{12}$$

That is,

$$\begin{aligned} P'_\mu &= \hat{\mathbf{x}}'_\lambda \cdot \mathbf{x}''_{\lambda\mu}, \\ Q'_\lambda &= \hat{\mathbf{x}}'_\mu \cdot \mathbf{x}''_{\lambda\mu} \end{aligned} \tag{13}$$

where boldface letters are the notations of vectors

$$\mathbf{x} = (x, y), \quad \mathbf{x}'_\lambda = (x'_\lambda, y'_\lambda), \quad \mathbf{x}''_{\lambda\mu} = (x''_{\lambda\mu}, y''_{\lambda\mu}), \quad \text{etc.} \tag{14}$$

The hats above letters mean that the corresponding vectors are unit ones. The dot symbol is the inner product of vectors. The geometric meaning of the formula (13) is that P'_μ is the projection of the vector $\mathbf{x}''_{\lambda\mu}$ in the direction of the vector $\mathbf{x}'_\lambda$ and Q'_λ is the projection of the same vector in the direction of the vector $\mathbf{x}'_\mu$. They are also the geometric meaning of our final rational structure equation (16) (see also the paper [3]).

Finally, our rational structure equation becomes

$$u(\lambda)\hat{\mathbf{x}}'_\lambda \cdot \hat{\mathbf{x}}''_{\lambda\mu} = v(\mu)\hat{\mathbf{x}}'_\mu \cdot \hat{\mathbf{x}}''_{\lambda\mu} \tag{15}$$

However, the solution $f(x, y)$ of the equation may not be rational structure because the net of curves may not be orthogonal. The solution of the following equation system must be rational

$$\begin{cases} u(\lambda)\hat{\mathbf{x}}'_\lambda \cdot \hat{\mathbf{x}}''_{\lambda\mu} = v(\mu)\hat{\mathbf{x}}'_\mu \cdot \hat{\mathbf{x}}''_{\lambda\mu}, \\ \mathbf{x}'_\lambda \cdot \mathbf{x}'_\mu = 0 \end{cases} \tag{16}$$

where the second equation is the orthogonal condition.

3 The Creator's Equation

Now we want to change the rational structure equation into the Creator's equation. It is true that the Creator's equation is a different form of the rational structure equation but the new form is more elegant and should be easier for its solution.

We know that $f(x, y)$ is the function of Cartesian coordinates x, y. However, the equation system (2) suggests that it is also the composite function of the parameters λ, μ. We use the same symbol f to denote the composite function. For example, we denote its partial derivatives by f'_λ, f'_μ. It is straightforward to show that the partial derivatives are

$$\begin{aligned} f'_\lambda &= u(\lambda)P(\lambda, \mu), \\ f'_\mu &= v(\mu)Q(\lambda, \mu) \end{aligned} \tag{17}$$

That is,

$$\begin{array}{l} u(\lambda) = f'_\lambda/P(\lambda,\mu), \\ v(\mu) = f'_\mu/Q(\lambda,\mu) \end{array} \tag{18}$$

Therefore,

$$\begin{array}{l} (u(\lambda))'_\mu = (f'_\lambda/P(\lambda,\mu))'_\mu = 0, \\ (v(\mu))'_\lambda = \left(f'_\mu/Q(\lambda,\mu)\right)'_\lambda = 0 \end{array} \tag{19}$$

That is,

$$\begin{array}{l} f''_{\lambda\mu} = \frac{f'_\lambda}{P} P'_\mu, \\ f''_{\mu\lambda} = \frac{f'_\mu}{Q} Q'_\lambda \end{array} \tag{20}$$

Finally we have our Creator's equation (see also the paper [4])

$$\begin{cases} \frac{f'_\lambda}{P} P'_\mu = \frac{f'_\mu}{Q} Q'_\lambda, \\ f''_{\lambda\mu} = \frac{f'_\lambda}{P} P'_\mu, \\ \mathbf{x}'_\lambda \cdot \mathbf{x}'_\mu = 0 \end{cases} \tag{21}$$

Which is the necessary and sufficient condition for rational structure. That is, the equation system is the iff (if and only if) condition for rational structure. Noticing the formulas (18), we find out that the first equation of the Creator's equation system is nothing but an alternative form of the rational structure equation (11).

Using the derivative rule of composite functions for f, we have the second form of the Creator's equation

$$\begin{cases} \frac{(f'_x x'_\lambda + f'_y y'_\lambda)(x'_\lambda x''_{\lambda\mu} + y'_\lambda y''_{\lambda\mu})}{x'^2_\lambda + y'^2_\lambda} = \frac{(f'_x x'_\mu + f'_y y'_\mu)(x'_\mu x''_{\lambda\mu} + y'_\mu y''_{\lambda\mu})}{x'^2_\mu + y'^2_\mu}, \\ f''_{xx} x'_\lambda x'_\mu + f''_{xy}(x'_\lambda y'_\mu + x'_\mu y'_\lambda) + f''_{yy} y'_\lambda y'_\mu + f'_x x''_{\lambda\mu} + f'_y y''_{\lambda\mu} = \frac{(f'_x x'_\lambda + f'_y y'_\lambda)(x'_\lambda x''_{\lambda\mu} + y'_\lambda y''_{\lambda\mu})}{x'^2_\lambda + y'^2_\lambda}, \\ \mathbf{x}'_\lambda \cdot \mathbf{x}'_\mu = 0 \end{cases} \tag{22}$$

Using the same vector notation in the last Section, we have the third form of the Creator's equation

$$\begin{cases} \nabla f \cdot \hat{\mathbf{x}}'_\lambda \, \hat{\mathbf{x}}'_\lambda \cdot \mathbf{x}''_{\lambda\mu} = \nabla f \cdot \hat{\mathbf{x}}'_\mu \, \hat{\mathbf{x}}'_\mu \cdot \mathbf{x}''_{\lambda\mu}, \\ (x'_\lambda \; y'_\lambda) \begin{pmatrix} f''_{xx} & f''_{xy} \\ f''_{xy} & f''_{yy} \end{pmatrix} \begin{pmatrix} x'_\mu \\ y'_\mu \end{pmatrix} + (f'_x \; f'_y) \begin{pmatrix} x''_{\lambda\mu} \\ y''_{\lambda\mu} \end{pmatrix} = \nabla f \cdot \hat{\mathbf{x}}'_\lambda \, \hat{\mathbf{x}}'_\lambda \cdot \mathbf{x}''_{\lambda\mu}, \\ \mathbf{x}'_\lambda \cdot \mathbf{x}'_\mu = 0 \end{cases} \tag{23}$$

Partial geometric meaning of the Creator's equation is that the two vectors

$$\begin{array}{l} \mathbf{A} = \begin{pmatrix} \hat{\mathbf{x}}'_\lambda \cdot \mathbf{x}''_{\lambda\mu} \\ \hat{\mathbf{x}}'_\mu \cdot \mathbf{x}''_{\lambda\mu} \end{pmatrix} \\ \mathbf{B} = \begin{pmatrix} \nabla f \cdot \hat{\mathbf{x}}'_\mu \\ \nabla f \cdot \hat{\mathbf{x}}'_\lambda \end{pmatrix} \end{array} \tag{24}$$

are parallel to each other. Therefore, we have the fourth form of the Creator's equation

$$\begin{cases} \mathbf{A} = h(\lambda,\mu)\mathbf{B}, \\ (x'_\lambda \; y'_\lambda) \begin{pmatrix} f''_{xx} & f''_{xy} \\ f''_{xy} & f''_{yy} \end{pmatrix} \begin{pmatrix} x'_\mu \\ y'_\mu \end{pmatrix} + (f'_x \; f'_y) \begin{pmatrix} x''_{\lambda\mu} \\ y''_{\lambda\mu} \end{pmatrix} = \nabla f \cdot \hat{\mathbf{x}}'_\lambda \, \hat{\mathbf{x}}'_\lambda \cdot \mathbf{x}''_{\lambda\mu}, \\ \mathbf{x}'_\lambda \cdot \mathbf{x}'_\mu = 0 \end{cases} \tag{25}$$

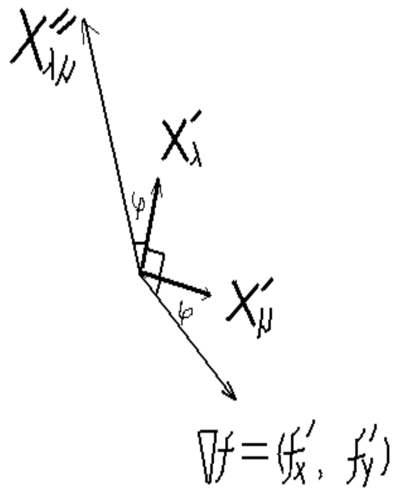

Figure 1: The directions of the two vectors $\mathbf{x}''_{\lambda\mu}$ and $\nabla f = (f'_x, f'_y)$ are symmetric with respect to the bisector line between the two vectors $\mathbf{x}'_\lambda$ and $\mathbf{x}'_\mu$.

where h is an unknown function. The geometric meaning of the formula is shown in the Figure 1. It is that the directions of the two vectors $\mathbf{x}''_{\lambda\mu}$ and $\nabla f = (f'_x, f'_y)$ are symmetric with respect to the bisector line between the two vectors $\mathbf{x}'_\lambda$ and $\mathbf{x}'_\mu$.

4 The Creator's Equation System without Composite Functions

The above equation systems may not be easy for their solutions because they are involved with composite functions

$$f \circ \mathbf{x} \tag{26}$$

The chain rule for its first partial derivatives is well known

$$f'_\lambda = \nabla f \cdot \mathbf{x}'_\lambda \tag{27}$$

However, the rule for its higher-order derivatives may not be familiar,

$$(f \circ \mathbf{x})''_{\lambda\mu} = \sum_i f'_{x_i} x''_{i\lambda\mu} + \sum_{ij} f''_{x_i x_j} x'_{i\lambda} x'_{j\mu} \tag{28}$$

where x_i or x_j is x and y.

Therefore, we want to find the Creator's equation system without composite functions. To do so, we need the inverse functions of the functions (2)

$$\lambda = \lambda(x, y), \;\; \mu = \mu(x, y) \tag{29}$$

The inverse functions satisfy an orthogonal condition

$$\lambda'_x \mu'_x + \lambda'_y \mu'_y = 0 \tag{30}$$

which is resulted from the original orthogonal condition, i. e., the second equation of the equation system (16). Because our model of barred spiral galaxy structure is

$$\rho(x,y) = \rho_0(x,y) + \rho_1(x,y) + \rho_2(x,y) \tag{31}$$

where $\rho_0(x,y)$ is the exponential disk, $\rho_1(x,y)$ and $\rho_2(x,y)$ are the double breasts structures, and we are only concerned with barred galaxy structure in our current business, the function $f(x,y) = \ln\rho(x,y)$ is considered to be given. Therefore, the only unknowns are the functions $\lambda = \lambda(x,y)$ and $\mu = \mu(x,y)$ whose independent variables are the direct Cartesian coordinates x, y. Their equation system is accordingly got rid of composite functions. However, to derive the equation system from the old one which is shown in the above Section 3, we need the chain rules for composite functions (formulas (27) and (28)).

The first chain rule is

$$\begin{pmatrix} x'_\lambda & x'_\mu \\ y'_\lambda & y'_\mu \end{pmatrix} \begin{pmatrix} \lambda'_x & \lambda'_y \\ \mu'_x & \mu'_y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{32}$$

With the introduction of a new symbol

$$\phi = \lambda'_x\mu'_y - \lambda'_y\mu'_x \tag{33}$$

we have

$$\begin{array}{ll} x'_\lambda = \mu'_y/\phi, & x'_\mu = -\lambda'_y/\phi, \\ y'_\lambda = -\mu'_x/\phi, & y'_\mu = \lambda'_x/\phi \end{array} \tag{34}$$

Now we consider the second chain rule (28) for the composite relation $\mathbf{x} \to (\lambda,\mu) \to \mathbf{x}$. The corresponding result for the second derivative $''_{xx}$ is

$$0 = x'_\lambda\lambda''_{xx} + x'_\mu\mu''_{xx} + x''_{\lambda\lambda}\lambda'_x\lambda'_x + 2x''_{\lambda\mu}\lambda'_x\mu'_x + x''_{\mu\mu}\mu'_x\mu'_x \tag{35}$$

That is,

$$\begin{aligned} x''_{\lambda\lambda}\lambda'_x\lambda'_x + x''_{\lambda\mu}2\lambda'_x\mu'_x + x''_{\mu\mu}\mu'_x\mu'_x &= -x'_\lambda\lambda''_{xx} - x'_\mu\mu''_{xx} \\ &= \mu'_y\tfrac{-\lambda''_{xx}}{\phi} + (-\lambda'_y)\tfrac{-\mu''_{xx}}{\phi} \end{aligned} \tag{36}$$

Other second derivatives are

$$\begin{aligned} x''_{\lambda\lambda}\lambda'_x\lambda'_y + x''_{\lambda\mu}(\lambda'_x\mu'_y + \lambda'_y\mu'_x) + x''_{\mu\mu}\mu'_x\mu'_y &= -x'_\lambda\lambda''_{xy} - x'_\mu\mu''_{xy} \\ x''_{\lambda\lambda}\lambda'_y\lambda'_y + x''_{\lambda\mu}2\lambda'_y\mu'_y + x''_{\mu\mu}\mu'_y\mu'_y &= -x'_\lambda\lambda''_{yy} - x'_\mu\mu''_{yy} \\ y''_{\lambda\lambda}\lambda'_x\lambda'_x + y''_{\lambda\mu}2\lambda'_x\mu'_x + y''_{\mu\mu}\mu'_x\mu'_x &= -y'_\lambda\lambda''_{xx} - y'_\mu\mu''_{xx} \\ y''_{\lambda\lambda}\lambda'_x\lambda'_y + y''_{\lambda\mu}(\lambda'_x\mu'_y + \lambda'_y\mu'_x) + y''_{\mu\mu}\mu'_x\mu'_y &= -y'_\lambda\lambda''_{xy} - y'_\mu\mu''_{xy} \\ y''_{\lambda\lambda}\lambda'_y\lambda'_y + y''_{\lambda\mu}2\lambda'_y\mu'_y + y''_{\mu\mu}\mu'_y\mu'_y &= -y'_\lambda\lambda''_{yy} - y'_\mu\mu''_{yy} \end{aligned} \tag{37}$$

These (36) and (37) are the linear algebraic equation system for the six variables $x''_{\lambda\lambda}, \cdots, y''_{\mu\mu}$. However, the righthand sides have the derivatives $'_\lambda$ and $'_\mu$. What we want is the derivative $'_x$ or $'_y$. Fortunately, we have the formulas (34) to replace the derivatives $'_\lambda$ and $'_\mu$. We have already done this in the formula (36). Because of linearity, the solution of our algebraic equation system depends solely on the two other equation systems whose constant terms (i. e., righthand-side terms) are $\frac{-\lambda''_{xx}}{\phi}, \frac{-\lambda''_{xy}}{\phi}, \cdots$ and $\frac{-\mu''_{xx}}{\phi}, \frac{-\mu''_{xy}}{\phi}, \cdots$, respectively (please take a look at the righthand side of the equation (36)). We use the symbols Λ and Ω to denote the corresponding solutions, respectively. For example,

$$\Lambda_{01} = \frac{\begin{vmatrix} \lambda'_x\lambda'_x & -\lambda''_{xx}/\phi & \mu'_x\mu'_x \\ \lambda'_x\lambda'_y & -\lambda''_{xy}/\phi & \mu'_x\mu'_y \\ \lambda'_y\lambda'_y & -\lambda''_{yy}/\phi & \mu'_y\mu'_y \end{vmatrix}}{\Phi} \tag{38}$$

$$\Omega_{01} = \frac{\begin{vmatrix} \lambda'_x\lambda'_x & -\mu''_{xx}/\phi & \mu'_x\mu'_x \\ \lambda'_x\lambda'_y & -\mu''_{xy}/\phi & \mu'_x\mu'_y \\ \lambda'_y\lambda'_y & -\mu''_{yy}/\phi & \mu'_y\mu'_y \end{vmatrix}}{\Phi} \tag{39}$$

where

$$\Phi = \begin{vmatrix} \lambda'_x\lambda'_x & 2\lambda'_x\mu'_x & \mu'_x\mu'_x \\ \lambda'_x\lambda'_y & \lambda'_x\mu'_y + \lambda'_y\mu'_x & \mu'_x\mu'_y \\ \lambda'_y\lambda'_y & 2\lambda'_y\mu'_y & \mu'_y\mu'_y \end{vmatrix} \tag{40}$$

It is straightforward to show that

$$\begin{aligned} \Lambda_{01} &= \tfrac{1}{\Phi}(\lambda'_y\mu'_y\lambda''_{xx} - (\lambda'_x\mu'_y + \lambda'_y\mu'_x)\lambda''_{xy} + \lambda'_x\mu'_x\lambda''_{yy}) \\ &= \tfrac{1}{\Phi}(\lambda'_x\mu'_x(\lambda''_{yy} - \lambda''_{xx}) - (\lambda'_x\mu'_y + \lambda'_y\mu'_x)\lambda''_{xy}), \\ \Omega_{01} &= \tfrac{1}{\Phi}(\lambda'_y\mu'_y\mu''_{xx} - (\lambda'_x\mu'_y + \lambda'_y\mu'_x)\mu''_{xy} + \lambda'_x\mu'_x\mu''_{yy}) \\ &= \tfrac{1}{\Phi}(\lambda'_x\mu'_x(\mu''_{yy} - \mu''_{xx}) - (\lambda'_x\mu'_y + \lambda'_y\mu'_x)\mu''_{xy}), \\ \Phi &= (\lambda'_x\mu'_y - \lambda'_y\mu'_x)^3 = \phi^3 \end{aligned} \tag{41}$$

where we have used the orthogonal condition (30).

Finally we have our solution to the algebraic equation

$$\begin{array}{lll} x''_{\lambda\lambda} = \mu'_y\Lambda_{00} - \lambda'_y\Omega_{00}, & x''_{\lambda\mu} = \mu'_y\Lambda_{01} - \lambda'_y\Omega_{01}, & x''_{\mu\mu} = \mu'_y\Lambda_{11} - \lambda'_y\Omega_{11}, \\ y''_{\lambda\lambda} = -\mu'_x\Lambda_{00} + \lambda'_x\Omega_{00}, & y''_{\lambda\mu} = -\mu'_x\Lambda_{01} + \lambda'_x\Omega_{01}, & y''_{\mu\mu} = -\mu'_x\Lambda_{11} + \lambda'_x\Omega_{11} \end{array} \tag{42}$$

The formulas (34) and (42) are all we need to be substituted into the Creator's equation system (21) or (22) to achieve a different form without composite functions. Let us do it step by step

$$\begin{aligned} \tfrac{f'_\lambda}{P}P'_\mu &= \tfrac{f'_x x'_\lambda + f'_y y'_\lambda}{x'^2_\lambda + y'^2_\lambda}(x'_\lambda x''_{\lambda\mu} + y'_\lambda y''_{\lambda\mu}) \\ &= \tfrac{f'_x\mu'_y - f'_y\mu'_x}{\mu'^2_x + \mu'^2_y}(\mu'_y(\mu'_y\Lambda_{01} - \lambda'_y\Omega_{01}) - \mu'_x(-\mu'_x\Lambda_{01} + \lambda'_x\Omega_{01})) \\ &= (f'_x\mu'_y - f'_y\mu'_x)\Lambda_{01} \end{aligned} \tag{43}$$

where we have used the orthogonal condition (30). Similarly we have

$$\begin{aligned} \tfrac{f'_\mu}{Q}Q'_\lambda &= \tfrac{(f'_x x'_\mu + f'_y y'_\mu)(x'_\mu x''_{\lambda\mu} + y'_\mu y''_{\lambda\mu})}{x'^2_\mu + y'^2_\mu} \\ &= (-f'_x\lambda'_y + f'_y\lambda'_x)\Omega_{01} \end{aligned} \tag{44}$$

Then we have

$$
\begin{aligned}
f''_{\lambda\mu} &= \tfrac{1}{\phi^2}(-f''_{xx}\lambda'_y\mu'_y + f''_{xy}(\lambda'_x\mu'_y + \lambda'_y\mu'_x) - f''_{yy}\lambda'_x\mu'_x) \\
&+ f'_x(\mu'_y\Lambda_{01} - \lambda'_y\Omega_{01}) + f'_y(-\mu'_x\Lambda_{01} + \lambda'_x\Omega_{01})
\end{aligned} \tag{45}
$$

Finally, the Creator's equation system (21) or (22) is transformed into the one without composite functions

$$
\begin{cases}
(f'_x\mu'_y - f'_y\mu'_x)\Lambda_{01} = (-f'_x\lambda'_y + f'_y\lambda'_x)\Omega_{01}, \\
\\
\frac{1}{\phi^2}(-f''_{xx}\lambda'_y\mu'_y + f''_{xy}(\lambda'_x\mu'_y + \lambda'_y\mu'_x) - f''_{yy}\lambda'_x\mu'_x) \\
+f'_x(\mu'_y\Lambda_{01} - \lambda'_y\Omega_{01}) + f'_y(-\mu'_x\Lambda_{01} + \lambda'_x\Omega_{01}) = (f'_x\mu'_y - f'_y\mu'_x)\Lambda_{01}, \\
\\
\lambda'_x\mu'_x + \lambda'_y\mu'_y = 0
\end{cases} \tag{46}
$$

That is,

$$
\begin{cases}
(f'_x\mu'_y - f'_y\mu'_x)\Lambda_{01} = (-f'_x\lambda'_y + f'_y\lambda'_x)\Omega_{01}, \\
\frac{1}{\phi^2}(f''_{xx}\lambda'_y\mu'_y - f''_{xy}(\lambda'_x\mu'_y + \lambda'_y\mu'_x) + f''_{yy}\lambda'_x\mu'_x) = (-f'_x\lambda'_y + f'_y\lambda'_x)\Omega_{01}, \\
\lambda'_x\mu'_x + \lambda'_y\mu'_y = 0
\end{cases} \tag{47}
$$

or

$$
\begin{cases}
(f'_x\mu'_y - f'_y\mu'_x)(\lambda'_x\mu'_x(\lambda''_{yy} - \lambda''_{xx}) - (\lambda'_x\mu'_y + \lambda'_y\mu'_x)\lambda''_{xy}) \\
= (-f'_x\lambda'_y + f'_y\lambda'_x)(\lambda'_x\mu'_x(\mu''_{yy} - \mu''_{xx}) - (\lambda'_x\mu'_y + \lambda'_y\mu'_x)\mu''_{xy}), \\
\\
(\lambda'_x\mu'_y - \lambda'_y\mu'_x)(\lambda'_x\mu'_x(f''_{yy} - f''_{xx}) - (\lambda'_x\mu'_y + \lambda'_y\mu'_x)f''_{xy}) \\
= (\lambda'_x f'_y - \lambda'_y f'_x)(\lambda'_x\mu'_x(\mu''_{yy} - \mu''_{xx}) - (\lambda'_x\mu'_y + \lambda'_y\mu'_x)\mu''_{xy}), \\
\\
\lambda'_x\mu'_x + \lambda'_y\mu'_y = 0
\end{cases} \tag{48}
$$

where we have used the orthogonal condition (30).

If we express the first equation in the Creator's equation system (48) into a form of array multiplication and further require that the two functions (29) satisfy Cauchy-Riemann equations,

$$
\begin{aligned}
\lambda'_x &= \mu'_y, \\
\lambda'_y &= -\mu'_x
\end{aligned} \tag{49}
$$

then the equation reduces to

$$
f'_x(\lambda'_y\lambda''_{xx} - \lambda'_x\lambda''_{xy}) + f'_y(\lambda'_x\lambda''_{xx} + \lambda'_y\lambda''_{xy}) = 0 \tag{50}
$$

The Creator's equation system (48) becomes

$$
\begin{cases}
f'_x(\lambda'_y\lambda''_{xx} - \lambda'_x\lambda''_{xy}) + f'_y(\lambda'_x\lambda''_{xx} + \lambda'_y\lambda''_{xy}) = 0, \\
\\
(\lambda'^2_x + \lambda'^2_y)(-\lambda'_x\lambda'_y(f''_{yy} - f''_{xx}) + (\lambda'^2_y - \lambda'^2_x)f''_{xy}) \\
= (\lambda'_x f'_y - \lambda'_y f'_x)(-2\lambda'_x\lambda'_y\lambda''_{xy} + (\lambda'^2_y - \lambda'^2_x)\lambda''_{xx}), \\
\\
\lambda''_{xx} + \lambda''_{yy} = 0
\end{cases} \tag{51}
$$

5 The Creator's Standard Equation

Now we want to change the above equation system (51) into the so-called standard equation system which looks simpler and more symmetric and should be easier for its general solution. To do so, we need introduce new variables

$$\begin{cases} \alpha(x,y) = \lambda'_x, \\ \beta(x,y) = \lambda'_y \end{cases} \tag{52}$$

It means that

$$\alpha'_y = \beta'_x \tag{53}$$

In the meantime, the third equation of the Creator's equation system (51) becomes

$$\alpha'_x = -\beta'_y \tag{54}$$

They together mean that the complex function

$$\beta(x,y) + \mathrm{i}\alpha(x,y) \tag{55}$$

is an analytic function, and the formulas (53) and (54) are its Cauchy-Riemann equations.

Now we turn to the first equation of the Creator's equation system (51). With the new variables, it is

$$f'_x(\beta\alpha'_x - \alpha\beta'_x) + f'_y(\alpha\alpha'_x + \beta\beta'_x) = 0 \tag{56}$$

Using the Cauchy-Riemann equations (53) and (54), we have

$$f'_x((\alpha^2)'_y + (\beta^2)'_y) = f'_y((\alpha^2)'_x + (\beta^2)'_x) \tag{57}$$

Now we introduce another variable

$$G(x,y) = \alpha^2 + \beta^2, \tag{58}$$

and the first equation of the Creator's equation system (51) becomes finally

$$f'_x G'_y = f'_y G'_x \tag{59}$$

This is called the stretch equation for the given vector field (f'_x, f'_y). It is the simplest first order partial differential equation because it is linear and homogeneous. Its solution is completely determined by the function $H(x,y)$

$$\begin{cases} G'_x = H(x,y)f'_x, \\ G'_y = H(x,y)f'_y \end{cases} \tag{60}$$

We call $H(x,y)$ a stretch of the stretch equation (59). It is straightforward to show that $H(x,y)$ itself must be another solution of the stretch equation and must be determined by another stretch $I(x,y)$. $I(x,y)$ must be some other solution of the stretch equation and must be determined by some other stretch $J(x,y)$, and so forth. Therefore, the general solution of the stretch equation (59) corresponds to all stretches. However, our solution $G(x,y)$ must be simultaneously the modulus of some analytic complex function (55).

Now we turn to the second equation of the Creator's equation system (51). With the new variables, it is

$$\begin{aligned}&(\alpha^2+\beta^2)(-\alpha\beta(f''_{yy}-f''_{xx})+(\beta^2-\alpha^2)f''_{xy})\\&=(\alpha f'_y-\beta f'_x)(-2\alpha\beta\alpha'_y+(\beta^2-\alpha^2)\alpha'_x)\end{aligned}\tag{61}$$

That is,

$$\begin{aligned}&G(-\alpha\beta(f''_{xx}-f''_{yy})+(\alpha^2-\beta^2)f''_{xy})\\&+(\beta f'_x-\alpha f'_y)(2\alpha\beta\alpha'_y+(\alpha^2-\beta^2)\alpha'_x)=0\end{aligned}\tag{62}$$

The final factor of the above equation can be further transformed with the Cauchy-Riemann equations (53) and (54),

$$\begin{aligned}&2\alpha\beta\alpha'_y+(\alpha^2-\beta^2)\alpha'_x\\&=\alpha\beta\alpha'_y+\alpha\beta\alpha'_y+\tfrac{1}{2}\alpha(\alpha^2)'_x+\beta^2\beta'_y\\&=\tfrac{1}{2}\beta(\alpha^2)'_y+\alpha\beta\beta'_x+\tfrac{1}{2}\alpha(\alpha^2)'_x+\tfrac{1}{2}\beta(\beta^2)'_y\\&=\tfrac{1}{2}\beta(\alpha^2)'_y+\tfrac{1}{2}\alpha(\beta^2)'_x+\tfrac{1}{2}\alpha(\alpha^2)'_x+\tfrac{1}{2}\beta(\beta^2)'_y\\&=\tfrac{1}{2}\beta(\alpha^2+\beta^2)'_y+\tfrac{1}{2}\alpha(\alpha^2+\beta^2)'_x\\&=\tfrac{1}{2}\alpha G'_x+\tfrac{1}{2}\beta G'_y\\&=\tfrac{1}{2}\alpha Hf'_x+\tfrac{1}{2}\beta Hf'_y\end{aligned}\tag{63}$$

Substituting the result (63) into the equation (62), we transform the second equation of the Creator's equation system (51) into

$$\begin{aligned}&G(-\alpha\beta(f''_{xx}-f''_{yy})+(\alpha^2-\beta^2)f''_{xy})\\&+\tfrac{1}{2}H(\alpha\beta(f'^2_x-f'^2_y)-(\alpha^2-\beta^2)f'_xf'_y)=0\end{aligned}\tag{64}$$

Now we introduce two other variables

$$\phi(x,y)=\alpha\beta,\ \ \psi(x,y)=\alpha^2-\beta^2,\tag{65}$$

and the equation (64) becomes

$$2G(\phi(f''_{xx}-f''_{yy})-\psi f''_{xy})=H(\phi(f'^2_x-f'^2_y)-\psi f'_xf'_y)\tag{66}$$

which is called the Creator's standard equation.

Finally the Creator's equation system (51) becomes

$$\begin{cases}G(x,y)=\alpha^2+\beta^2,\ \ \phi(x,y)=\alpha\beta,\ \ \psi(x,y)=\alpha^2-\beta^2,\\ \alpha'_y=\beta'_x,\ \ \alpha'_x=-\beta'_y,\\ G'_x=H(x,y)f'_x,\ \ G'_y=H(x,y)f'_y,\\ 2G(\phi(f''_{xx}-f''_{yy})-\psi f''_{xy})=H(\phi(f'^2_x-f'^2_y)-\psi f'_xf'_y)\end{cases}\tag{67}$$

If we substitute the formulas (52) into the above equation system (67) then it must return to the Creator's original equation system (51).

6 General Solution

We have not found the general solution to the Creator's standard equation system (67). However, we have outlined the strategy for determining the solution in the above Section. Firstly, we should study the stretch equation (59) which is the simplest first order partial differential equation, and find its all stretches which correspond to the given vector field (f'_x, f'_y). Our solution $G(x, y)$, however, must be simultaneously the modulus of some analytic complex function (55). It is called the modulus stretch.

Secondly, among all possible modulus stretches, we find the right solution $\alpha(x, y), \beta(x, y)$ which satisfies the Creator's standard equation (66), i. e., the equation on the third line of the equation system (67).

7 Exponential Disk

Exponential disk is rational structure. Its logarithmic density is

$$f(x, y) = d_1 r \tag{68}$$

where d_1 is a constant and $r = \sqrt{x^2 + y^2}$. Its orthogonal net of curves is

$$\begin{cases} x = e^\lambda \cos(\mu), \;\; y = e^\lambda \sin(\mu) \\ \lambda > -\infty \end{cases} \tag{69}$$

Its inverse is

$$\begin{cases} \lambda = (1/2) \ln(x^2 + y^2), \\ \mu = \tan^{-1} \frac{y}{x} \end{cases} \tag{70}$$

8 Heaven Breasts Structure

Heaven breasts structure is rational [5,6]. From now on, we change our convention so that the double breasts are paired horizontally. Previously we always present the bars of barred spiral galaxy pattern vertically, and the double breasts are demonstrated vertically accordingly. To be consistent with the Cauchy-Riemann equations of complex analytic functions (formulas (49)), we, from now on, present the bars of barred spiral galaxy pattern horizontally. Accordingly, the double breasts are demonstrated horizontally and its logarithmic density is

$$f(x, y) = (b_2/3) \left((r^2 - b_1^2)^2 + 4b_1^2 y^2\right)^{3/4} \tag{71}$$

where b_1, b_2 are constants. Its orthogonal net of curves is

$$\begin{cases} x = b_1 \cosh(\lambda) \cos(\mu), \;\; y = b_1 \sinh(\lambda) \sin(\mu), \\ \lambda \geq 0 \end{cases} \tag{72}$$

Its inverse is

$$\begin{cases} \lambda = \sinh^{-1}(p(x, y)/b_1), \\ \mu = \sin^{-1}(y/p(x, y)) \end{cases} \tag{73}$$

where

$$p(x,y) = \sqrt{\left(r^2 - b_1^2 + \sqrt{(r^2-b_1^2)^2 + 4b_1^2y^2}\right)/2} \tag{74}$$

9 Application to Naked Galaxy Structure

If we ignore those strongly interacting galaxies or some extremely small galaxies, there exist only two types of galaxies: spiral galaxies and elliptical galaxies. There are two kinds of spiral galaxies. A spiral galaxy with a bar is called a barred spiral, and a spiral galaxy without a bar is called an ordinary spiral. Astronomers found out that the stellar density distribution of ordinary spiral galaxies, i. e., the naked ordinary spiral galaxies without much gas or dust, is basically an axi-symmetric disk described by the exponential disk (see the Section 7)

$$\rho_0(x,y) = d_0 \exp(d_1 r) \tag{75}$$

where d_0 is a constant. Its logarithmic density is

$$f_0(x,y) = d_1 r \tag{76}$$

Therefore, ordinary spiral galaxies are rational structure.

Astronomers have found out that the main structure of barred spiral galaxies is also the exponential disk. Therefore, we subtract the fitted exponential disk from a barred spiral galaxy image. What is left over? Jin He discovered that the left-over resembles human breasts [5,6]

$$\rho_i(x,y) = b_{i0} \exp\left((b_{i2}/3)\ \left((r^2 - b_{i1}^2)^2 + 4b_{i1}^2y^2\right)^{3/4}\right) \tag{77}$$

where b_{i0}, b_{i1}, b_{i2} are constants. Its logarithmic density is

$$f_i(x,y) = (b_{i2}/3)\ \left((r^2 - b_{i1}^2)^2 + 4b_{i1}^2y^2\right)^{3/4} \tag{78}$$

Jin He calls it Heaven Breasts structure (see the above Section 8). Barred spiral galaxies, however, generally have more than a pair of breasts. The bar of barred spiral galaxies is composed of two or three pairs of breasts which are usually aligned. The addition of the two or three pairs of breasts to the major structure of exponential disk becomes a bar-shaped pattern which crosses galaxy center.

Therefore, our model of barred spiral galaxy structure is

$$\rho(x,y) = \rho_0(x,y) + \rho_1(x,y) + \rho_2(x,y) \tag{79}$$

The logarithmic density of barred spiral galaxies is

$$f(x,y) = \ln\rho(x,y) = \ln(\rho_0(x,y) + \rho_1(x,y) + \rho_2(x,y)) \tag{80}$$

It is straightforward to show that

$$\nabla f = (\rho_0 \nabla f_0 + \rho_1 \nabla f_1 + \rho_2 \nabla f_2)/(\rho_0 + \rho_1 + \rho_2) \tag{81}$$

That is, the gradient of sum is the sum of weighted gradients. It is some kind of averaging which is usually denoted by a bar above the corresponding symbol of the variable. We follow the notation. Here is an example

$$f'_x = (\rho_0 f'_{0x} + \rho_1 f'_{1x} + \rho_2 f'_{2x})/(\rho_0 + \rho_1 + \rho_2) = \overline{f'_{ix}} \tag{82}$$

Here comes the big question for humans.

10 The Creator's Question for Humans

Is the sum of rational structures also a rational structure (see the formula (79) in the above Section)? It is called the Creator's big question for humans. Numerical calculation suggests that it is approximately true for the fitted galaxy values of the parameters $d_0, d_1, b_{i0}, b_{i1}, \cdots$ (see the paper [6]). However, we need mathematical justification. The authors are very old and are not experts in mathematics. Please help humans resolve the question.

The Creator's Question: If we substitute the expression (81) into the equation system (67), does the solution (29) exist?

In case you need them, we present some useful formulas in the Appendix.

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11 Appendix

In the following formulas, $\alpha, \beta, \gamma, \delta$ may be the Cartesian coordinates x or y, or may be the parameters λ or μ.

$$f'_\alpha = \overline{f'_{i\alpha}}, \tag{83}$$

$$f''_{\alpha\beta} = \overline{f'_{i\alpha} f'_{i\beta}} + \overline{f''_{i\alpha\beta}} - \overline{f'_{i\alpha}}\ \overline{f'_{i\beta}}, \tag{84}$$

$$\begin{aligned} f'''_{\alpha\beta\gamma} \quad &= \overline{f'_{i\alpha} f'_{i\beta} f'_{i\gamma}} + \overline{f''_{i\alpha\beta} f'_{i\gamma}} + \overline{f''_{i\beta\gamma} f'_{i\alpha}} + \overline{f''_{i\alpha\gamma} f'_{i\beta}} + \overline{f'''_{i\alpha\beta\gamma}} \\ &- \overline{f'_{i\alpha}}\ \overline{f'_{i\beta}}\ \overline{f'_{i\gamma}} - \overline{f''_{i\alpha\beta}}\ \overline{f'_{i\gamma}} - \overline{f''_{i\beta\gamma}}\ \overline{f'_{i\alpha}} - \overline{f''_{i\alpha\gamma}}\ \overline{f'_{i\beta}}, \end{aligned} \tag{85}$$

$$
\begin{aligned}
&f''''_{\alpha\beta\gamma\delta} \\
&= \overline{f'_{i\alpha} f'_{i\beta} f'_{i\gamma} f'_{i\delta}} \\
&+ \overline{f''_{i\alpha\beta} f'_{i\gamma} f'_{i\delta}} + \overline{f''_{i\alpha\gamma} f'_{i\beta} f'_{i\delta}} + \overline{f''_{i\alpha\delta} f'_{i\gamma} f'_{i\beta}} + \overline{f''_{i\beta\gamma} f'_{i\alpha} f'_{i\delta}} + \overline{f''_{i\beta\delta} f'_{i\gamma} f'_{i\alpha}} + \overline{f''_{i\gamma\delta} f'_{i\alpha} f'_{i\beta}} \\
&+ \overline{f''_{i\alpha\beta} f''_{i\gamma\delta}} + \overline{f''_{i\alpha\gamma} f''_{i\beta\delta}} + \overline{f''_{i\alpha\delta} f''_{i\beta\gamma}} \\
&+ \overline{f'''_{i\alpha\beta\gamma} f'_{i\delta}} + \overline{f'''_{i\alpha\beta\delta} f'_{i\gamma}} + \overline{f'''_{i\alpha\gamma\delta} f'_{i\beta}} + \overline{f'''_{i\beta\gamma\delta} f'_{i\alpha}} + \overline{f''''_{i\alpha\beta\gamma\delta}} \\
&- \overline{f'_{i\alpha}}\ \overline{f'_{i\beta}}\ \overline{f'_{i\gamma}}\ \overline{f'_{i\delta}} \\
&- \overline{f''_{i\alpha\beta}}\ \overline{f'_{i\gamma}}\ \overline{f'_{i\delta}} - \overline{f''_{i\alpha\gamma}}\ \overline{f'_{i\beta}}\ \overline{f'_{i\delta}} - \overline{f''_{i\alpha\delta}}\ \overline{f'_{i\gamma}}\ \overline{f'_{i\beta}} - \overline{f''_{i\beta\gamma}}\ \overline{f'_{i\alpha}}\ \overline{f'_{i\delta}} - \overline{f''_{i\beta\delta}}\ \overline{f'_{i\gamma}}\ \overline{f'_{i\alpha}} - \overline{f''_{i\gamma\delta}}\ \overline{f'_{i\alpha}}\ \overline{f'_{i\beta}} \\
&- \overline{f''_{i\alpha\beta}}\ \overline{f''_{i\gamma\delta}} - \overline{f''_{i\alpha\gamma}}\ \overline{f''_{i\beta\delta}} - \overline{f''_{i\alpha\delta}}\ \overline{f''_{i\beta\gamma}} \\
&- \overline{f'''_{i\alpha\beta\gamma}}\ \overline{f'_{i\delta}} - \overline{f'''_{i\alpha\beta\delta}}\ \overline{f'_{i\gamma}} - \overline{f'''_{i\alpha\gamma\delta}}\ \overline{f'_{i\beta}} - \overline{f'''_{i\beta\gamma\delta}}\ \overline{f'_{i\alpha}}
\end{aligned}
\tag{86}
$$