Dimensional Reduction and Physics of the Standard Model

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Abstract

The Standard Model of particle physics (SM) is a theoretical framework that integrates our current knowledge of the subatomic world and its fundamental interactions. A key program built in the structure of the SM is the Renormalization Group (RG), whose function is to preserve self-consistency and describe how parameters of the theory evolve with the energy scale. Despite being overwhelmingly supported by experimental data, the SM has many puzzling aspects, such as the large number of parameters, a triplication of chiral families and the existence of three gauge interactions. In contrast with the majority of mainstream proposals advanced over the years, the basic premise of our study is that a satisfactory resolution of challenges confronting the SM requires further advancing the RG program. In particular, understanding the nonlinear dynamics of RG equations and the unavoidable transition from smooth to fractal dimensionality of space-time are critically important for the success of this endeavor. Here we show how the onset of fractal space-time near or above the electroweak scale is likely to settle at least some of these challenges.

Key words: Standard Model, Renormalization Group, Landau-Ginzburg-Wilson model, fractal space-time, scale invariance, dimensional reduction.

1. Introduction

As coherent synthesis of Quantum Mechanics and Special Relativity, Quantum Field Theory (QFT) provides a compelling description of phenomena up to the energy scales probed by present accelerators. Nevertheless, there are plausible reasons to suspect that QFT breaks down at some high energy scale (Λ_{UV}), above which it needs to be replaced by a more fundamental theory. Although there is no consensus among theorists on this issue, the underlying reasons may be summarized as follows:

a) New interactions, hidden symmetries or any other exotic extensions of QFT could likely unfold near Λ_{UV},

b) Reconciling classical gravity with QFT appears to be an insurmountable challenge. A major obstacle is that perturbative quantization of classical gravity cannot be extrapolated at energies close to the Planck scale, $\Lambda_{UV} = O(M_{Pl})$. As a result, the theory is said to be "non-renormalizable", meaning that it lacks any predictive power at scales comparable with Λ_{UV}. A number of non-perturbative models of quantum gravity have been proposed as alternative solutions, but it is presently unclear if they yield a truly consistent integration scheme of General Relativity and QFT [1, 2].

c) The current accelerator technology probes energies moderately above the range defining the Standard Model (SM) of particle physics ($\mu_{SM} = O$ (TeV)). The prevalent view is that M_{Pl} is the only genuine scale in QFT and stems from the assumption that no dramatic change in physics develops between μ_{SM} and M_{Pl}. But if this assumption is true, there is at present no compelling explanation for the *mass hierarchy*, stemming from the observation that fermion masses are scattered over thirteen orders of magnitude and are confined around $\mu_{SM} << M_{Pl}$. In addition, quantum corrections applied to the Higgs vacuum can shift the Higgs mass close to M_{Pl}, leading to the so-called "*fine-tuning problem*" [3].

d) The dynamics of QFT may undergo transition to classical behavior as a result of *decoherence* [4] or become *unstable* near or below Λ_{UV}. The instability can arise from unbalanced quantum corrections or from the transition to chaos in nonlinear evolution of interacting fields [5, 6, 39, 41].

Due to its limitations in dealing with phenomena on scales nearing Λ_{UV}, the conventional interpretation of QFT is that it represents an "effective" field theory (EFT), which is likely to be amended by new physics emerging above Λ_{UV}. EFT describes phenomena occurring exclusively on low-energy scales $\mu_{SM} << \Lambda_{UV}$, in the same way the continuum theory of elasticity describes the long wavelength excitations of a crystal [7, 8]. In the case of a crystal, the continuum theory breaks down at the scale of the lattice spacing. Likewise, EFT breaks down as the energy scale is ramped up close to Λ_{UV}.

It is instructive to briefly review at this point two examples of successful EFT's:

1) In the Wilson treatment of critical phenomena using the *Renormalization Group* program (RG) [7, 9, 52], quantum fields present in the theory (Φ_μ) depend on the running scale μ and are separated into two components

$$\Phi_\mu^{(l)} : 0 \le \mu \le \frac{\Lambda_{UV}}{s} \tag{1.1}$$

$$\Phi_\mu^{(s)} : \frac{\Lambda_{UV}}{s} \le \mu \le \Lambda_{UV} \tag{1.2}$$

Here, the parameter "s" is an arbitrary scaling factor ($s > 1$), $\Phi_\mu^{(l)}$ and $\Phi_\mu^{(s)}$ are the long and short wavelength excitations and correspond, respectively, to the light and heavy particles carried by Φ_μ. Starting with an EFT defined at Λ_{UV}, the core idea of Wilson's approach is to integrate out all heavy particles contained in the "momentum shell" (1.1, 1.2) and form a new EFT with the remaining fields below the separation scale Λ_{UV}/s. Since μ is considered a running parameter, iterating this process yields a flow of EFT's from Λ_{UV} toward their low-energy limit. It is customary to refer to this iterative process as a *RG flow* (or *RG trajectory*). A key property of local EFT's is that the low-energy endpoint of the RG flow must describe phenomena that are fully decoupled from physical processes occurring near the high-energy limit Λ_{UV}. This property conveys the basic idea behind the concept of *scale invariance* [7, 9, 52].

2) The second example is the SM itself, a robust EFT that has been in place for more than three decades. It includes the $SU(3)\otimes SU(2)\otimes U(1)$ gauge model of strong and electroweak interactions along with the Higgs mechanism that spontaneously breaks the electroweak $SU(2)\otimes U(1)$ group down to the $U(1)$ group of electrodynamics. The SM has been confirmed countless times in all accelerator experiments, including the latest runs of the Large Hadron Collider (LHC). Despite this convincing body of evidence, the SM is confronted with many unsolved challenges [10-12]. Over the years, this has led to an overflow of theoretical extensions targeting the physics beyond the SM scale ($m > \mu_{SM}$). The majority of these proposals center on solving some unsatisfactory aspects of the theory while introducing new unknowns. Experiments are expected to provide guidance in pointing to the correct theory yet, so far, LHC searches show no credible hint for physics beyond the SM up to a center-of-mass energy of $\sqrt{s}$ = 8 TeV [13]. These results, albeit entirely preliminary, suggest two possible scenarios, namely:

a) SM fields are either decoupled or ultra-weakly coupled to new dynamic structures emerging in the low or intermediate TeV scale,

b) There is an undiscovered and possibly non-trivial connection between the SM and TeV phenomena.

More importantly, this discussion raises a key question: What should be the principles guiding model-building efforts beyond the SM? In contrast with many mainstream proposals on how to tackle this question, our basic premise is that moving beyond the SM requires further advancing the RG program. As we shall argue below, understanding the *nonlinear dynamics of RG flow equations* and the transition from smooth to *fractal dimensionality* of space-time are essential steps for the success of this endeavor.

The article is organized in the following way: section two surveys the principles of the RG program, with emphasis on phase space trajectories and their fixed points. The idea of dimensional regularization and its implications on the emergence of fractal space-time in QFT form the topic of section three. Section four and five describe the asymptotic approach to scale invariance of RG trajectories and presents a natural solution for the so-called *mass hierarchy problem* of the SM. The emergence of a Higgs-like resonance as Bose-Einstein condensate on fractal space-time is introduced in section six. Concluding remarks are gathered in section seven. To facilitate reading, frequently used text abbreviations are also listed at the end of the article.

This work represents a development of ideas published by the author in [5-6, 14-22, 35, 46]. To make its content fully transparent, we have opted for minimal mathematics but adequate clarity and level of detail. Since many of these ideas are under development, concurrent analysis is needed to confirm, expand or falsify our provisional findings.

Before going into details, it is essential to point out that in our work the terms "scale invariance" and "conformal invariance" are considered identical, although they are not synonymous (a field theory can be scale invariant without being conformal invariant [23, 38]).

2. Renormalization Group trajectories

We begin by briefly reviewing the conventional construction of Lagrangian field theory. Consider a field theory whose action in D - dimensional space-time is given by

$$S[\Phi,\overline{\Phi}]=\int L[\Phi,\overline{\Phi}]d^{D}x \qquad (2.1)$$

$$L[\Phi,\overline{\Phi}]=u_{\alpha}P_{\alpha}[\Phi,\overline{\Phi}] \qquad (2.2)$$

The theory is fully specified by three primary inputs, namely: a) the field content of the Lagrangian (2.2), b) the set of symmetry constraints imposed on (2.2) and c) the dimension of space-time (D). The basis functionals $P_{\alpha}[\Phi,\overline{\Phi}]$ with $\alpha=1,2,...N$ represent a sum of local products of fields Φ, their conjugates $\overline{\Phi}$ and/or their derivatives and u_{α} are a set of coupling parameters. Here, coupling parameters mean the coefficients describing *interaction strengths* as well as *particle masses*. The summation convention is assumed throughout.

As alluded to in the first section, the RG program posits that the description of the physics embodied in (1.1) can be done in terms of a family of "*effective actions*", each valid at a certain energy scale μ [24]. A key concept of this program is the RG flow, whose role is to define how effective formulations of the theory evolve with μ. According to this program, parameters u_{α} run with μ following the general system of non-autonomous equations of order "n" [7, 24]

$$\mu \frac{\partial u_\alpha(\mu)}{\partial \mu} = \beta_\alpha(u_1, u_2, ..., u_\alpha, ..., u_N, \mu) \quad (2.3)$$

Using the standard field-theoretic jargon, $u_\alpha(\infty)$ are called "*bare*" parameters of the theory whereas $u_\alpha(0)$ are referred to as the "*renormalized*" parameters. In this picture, the flow (2.3) describes the evolution of u_α from the ultraviolet region of arbitrarily large energies (UV) to the infrared limit of low energies (IR). If d_α represents the canonical mass dimension of u_α, the substitution

$$g_\alpha = \mu^{-d_\alpha} u_\alpha \quad (2.4)$$

transforms (2.3) into a coupled system of autonomous differential equations of order "n" relating *dimensionless* couplings g_α, that is,

$$\mu \frac{\partial g_\alpha(\mu)}{\partial \mu} = \beta_\alpha(g_1, g_2, ..., g_\alpha, ..., g_N) \quad (2.5)$$

A fixed point (FP) is invariant under (2.5) and corresponds to the stationary condition

$$\beta(g_1^*, g_2^*, ..., g_N^*) = \beta(g^*) = 0 \quad (2.6)$$

By definition, integral curves along the vector field (2.5) are called *RG trajectories* parameterized by the running scale μ. The goal of the RG theory consists in constructing trajectories which do not develop divergences in either UV or IR limits. Each such trajectory defines a possible field theory characterized by the continuous mapping

$$\mu \to g(\mu) \to S_\mu[\Phi(\mu), \overline{\Phi}(\mu)] \quad (2.7)$$

RG trajectories encode in a universal way the natural flow of (2.1) towards FP's. It follows that, since FP's are independent from μ, (2.7) unveils the asymptotic approach of field theories to *scale invariance*. A relativistic QFT that is scale invariant and contains massless particles is necessarily a free theory [25].

The fixed-point structure of quantum field theories is typically difficult to extract analytically. In particular, Feynman diagrams fail in the neighborhood of any strong coupling point ($g^* >> 0$) and the behavior of the theory is ill-defined near such points. By contrast, perturbation methods are applicable near any trivial FP ($g^* = 0$) and (2.5) can be evaluated there using the series expansion

$$\mu \frac{\partial g_\alpha(\mu)}{\partial \mu} = a_\alpha^{ij} g_i g_j + b_\alpha^{ijk} g_i g_j g_k + ... \qquad (2.8)$$

It is customary to assume that the perturbative flow (2.8) of typical QFT's evolves toward isolated sets of stable FP's [7, 15]. A typical example is the strong FP of Quantum Chromodynamics (QCD), which characterizes the passage to asymptotic freedom of quarks and gluons in the UV limit. Another example is the IR "conformal" FP describing the approach to scale invariance in the low-energy limit of QFT. But in addition to FP's, RG flows can display *limit cycles* (isolated closed trajectories) and *chaotic behavior*. If either (2.5) or (2.8) are regarded as nonlinear systems of coupled differential equations, the behavior of these systems is characterized by sensitivity to initial conditions, the loss of stability and the emergence of local or global bifurcations [26-30, 34-35, 38, 56].

At first sight, this observation appears to stand in conflict with one foundational postulates of relativistic field theory, *the cluster decomposition principle*. This principle asserts that local processes in any relativistic field theory must be insensitive to distant environment in space-time or energy scale and guarantees the factorization of the S-matrix of scattering amplitudes [7]. A direct consequence of this principle is that transitions amplitudes measured in the laboratory must be insensitive to the physics of short-distance scales. Rather than contradicting the clustering principle, the nonlinear behavior of the RG flow imparts a new twist to it. As we show below, abandoning the notion that the RG flow typically evolves towards isolated and stable FP's, is likely to bring closure to some of the open challenges facing the SM.

3. Dimensional regularization and the onset of fractal space-time

The previous section has touched upon the fundamentals of Wilson's RG program. Closely related to this program in QFT is the concept of *dimensional regularization*, which we now briefly outline.

A well-known difficulty of QFT is that perturbative calculations using momentum integrals do not converge [7, 31]. The root cause is that integrands fall off too slowly at large momenta. Infinities arising from the short-wavelength region of the integrals are called *ultraviolet* (UV) divergences. For massive fields, this type of singular behavior in the UV limit is the only anomaly of such integrals. In the zero-mass limit, further singularities show up at small momenta and are called *infrared* (IR) divergences. The zero-mass limit is relevant to critical behavior in statistical and condensed matter physics, as it relates to phenomena that exhibit unbounded correlation length and scale invariance.

Renormalization is a powerful technique for removing both UV and IR divergences and it consists in a two-step program: *regularization* and *subtraction*. One first controls the divergence present in momentum integrals by inserting a suitable regulator, and then brings in a set of counter-terms to cancel out the divergence. Momentum integrals in QFT have the generic form

$$I = \int_0^\infty d^4 q\, F(q) \tag{3.1}$$

Two regularization techniques are frequently employed to manage (3.1), namely "momentum cutoff" and "dimensional regularization". When the momentum cutoff scheme is applied for regularization in the UV region, the upper limit of (3.1) is replaced by a finite cutoff Λ_{UV},

$$I \to I_{\Lambda_{UV}} = \int_0^{\Lambda_{UV}} d^4 q F(q) \tag{3.2}$$

Explicit calculation of the convergent integral (3.2) amounts to a sum of three polynomial terms

$$I_{\Lambda_{UV}} = A(\Lambda_{UV}) + B + C(1/\Lambda_{UV}) \tag{3.3}$$

Dimensional regularization proceeds instead by shifting the momentum integral (3.1) from a four-dimensional space to a continuous D - dimensional space

$$I \to I_D = \int_0^\infty d^D q F(q) \tag{3.4}$$

Introducing the dimensional parameter $\varepsilon = 4 - D$ leads to

$$I_D \to I_\varepsilon = A'(\varepsilon) + B' + C'(1/\varepsilon) \tag{3.5}$$

Historically, the idea of continuous dimension was introduced by Wilson and Fisher [52] and initially used to compute physical quantities of interest as expansions in powers of ε. Later on, Veltman and ’t Hooft have shown how this idea can be incorporated in QFT and developed into a reliable renormalization technique [31].

Regularization techniques are not independent from each other. For example, the connection between dimensional and cutoff regularizations is given by [6, 32]

$$\log\frac{\Lambda_{UV}^2}{\mu^2} = \frac{2}{\varepsilon} - \gamma_E + \log 4\pi + \frac{5}{6} \tag{3.7}$$

We find it convenient to present (3.7) is a slightly different form, that is,

$$\varepsilon \sim \frac{1}{\log(\frac{\Lambda_{UV}^2}{\mu^2})} \tag{3.8}$$

It is apparent from (3.8) that the four-dimensional space-time is recovered in either one of these limits:

a) $\Lambda_{UV} \to \infty$ and $0 < \mu << \Lambda_{UV}$,

b) $\Lambda_{UV} < \infty$ and $\mu \to 0$.

However, both limits are in conflict with our current understanding of the far UV and the far IR boundaries of field theory. Theory and experimental observations alike tell us that the notions of infinite *or* zero energy are, strictly speaking, meaningless. This is to say that either infinite energies (point-like objects) or zero energy (infinite distance scales) lead to divergences whose removal requires the machinery of the RG program. Indeed, there is always a finite cutoff at both ends of either energy or energy density scale (far UV = Planck scale, far IR = finite radius of the observable Universe or the non-vanishing energy density of the vacuum set by cosmological constant). It follows from these considerations that the limit $\varepsilon \to 0$ works as a highly accurate approximation and realistic models near or beyond the SM scale must account for space-time geometries having continuous dimensionality. Fractal space-time defined by the continuous dimension $D = 4 - \varepsilon$ asymptotically approaches ordinary space-time near or below the SM scale, that is, for $\mu \le \mu_{SM}$.

4. The asymptotic approach to scale invariance

Section two has surveyed how RG trajectories describe the asymptotic behavior of field theory and the universal approach to scale invariance. One finds it quite natural to also demand that RG trajectories themselves maintain scale invariance as they evolve toward FP's.

To clarify this point, let us return to the system of differential equations (2.5). Taking advantage of the large numerical disparity between μ and Λ_{UV}, we may use (3.8) to rewrite (2.5) as

$$\frac{\partial g_\alpha(\mu)}{\partial[\log(\frac{\mu}{\Lambda_{UV}})]} \sim \frac{\partial g_\alpha(\mu)}{\partial(\frac{\mu}{\Lambda_{UV}})} \Rightarrow \frac{\partial g_\alpha(\varepsilon)}{\partial \varepsilon} = \beta_\alpha(g_1, g_2, ..., g_\alpha, ..., g_N) \tag{4.1}$$

or

$$F(g_\alpha, \frac{\partial g_\alpha(\varepsilon)}{\partial \varepsilon}) = \frac{\partial g_\alpha(\varepsilon)}{\partial \varepsilon} - \beta_\alpha(g_1, g_2, ..., g_\alpha, ..., g_N) = 0 \tag{4.2}$$

As briefly alluded in section two, realistic RG flows evolve in the presence of *weak perturbations* which affect stability of their FP's [28]. The minimal way to include the effect of weak perturbations of the linear form $\eta(\varepsilon) \sim \varepsilon$ is to modify (4.2) as follows

$$F(g_\alpha, \frac{\partial g_\alpha}{\partial \varepsilon}) = 0 \Rightarrow F[\eta(\varepsilon), g_\alpha, \frac{\partial g_\alpha}{\partial \varepsilon}] \sim F(\varepsilon, g_\alpha, \frac{\partial g_\alpha}{\partial \varepsilon}) = 0 \tag{4.3}$$

By definition, (4.3) represents a system of *autonomous differential equations* if it remains unchanged under the substitutions $\varepsilon \to \varepsilon + c$ and $g_\alpha(\varepsilon) \to g_\alpha(\varepsilon)$, where c is a fixed but arbitrary parameter and $\alpha = 1, 2, ..., N$. Also by definition, (4.3) represents a system of *scale invariant differential equations* if it remains unchanged under the substitutions $\varepsilon \to \lambda\varepsilon$ and $g_\alpha(\varepsilon) \to \lambda g_\alpha(\lambda\varepsilon)$, where λ is a fixed but arbitrary parameter. It is always possible to transform scale invariant equations into autonomous equations [33]. In short,

$$F(\varepsilon + c, g_\alpha, \frac{\partial g_\alpha}{\partial \varepsilon}) = 0 \Leftrightarrow F[\lambda\varepsilon, \lambda g_\alpha(\lambda\varepsilon), \frac{\partial g_\alpha(\lambda\varepsilon)}{\partial \varepsilon}] = 0 \tag{4.4}$$

It can be shown that, any system of ordinary differential equations (4.3) that is scale invariant and whose associated autonomous equation possesses a FP, has a power-series solution in the basin of attraction of this FP. This power-series solution can be presented as [33]

$$g_\alpha(\varepsilon) = \varepsilon(d_\alpha + e_{\alpha,i}\,\varepsilon^{h_i}) \sim O(\varepsilon) \tag{4.5}$$

Here, d_α and $e_{\alpha,i}$ are finite numerical coefficients and the index $i = 1, 2, ..., M$, with $M = N \times n$. For $h_i > 0$, all couplings and masses vanish in the four-dimensional space-time limit ($\varepsilon = 0$). This confirms what we concluded in section two, namely that massless particles in a scale-invariant QFT residing at the trivial FP ($g_\alpha^*(0) = 0$) are necessarily free. [25]. But more importantly, what (4.5) also reveals is that set of couplings $g_\alpha(\varepsilon)$ assume non-vanishing values on *fractal space-time*, that is, on a space-time with continuous dimensionality ($D = 4 - \varepsilon < 4$).

Building on this finding, next section recovers the pattern of massive SM particles and interaction couplings from the transition to chaos of RG equations (4.3).

In closing this section we note that this conclusion is consistent with the Landau-Ginzburg-Wilson (LGW) theory of critical behavior for the so-called Φ^4 model. This model describes the attributes of many statistical systems and field theories approaching criticality in the IR limit. An essential feature of the LGW theory is that the trivial FP $g_\alpha^* = 0$ is unstable and it cannot characterize critical behavior in less than four dimensions ($D < 4$). There is another non-trivial fixed point, referred to as the *Wilson-Fisher* FP, defined on fractal space-time of arbitrary dimension $D = 4 - \varepsilon$. The Wilson-Fisher point accounts for the onset of critical behavior and scale invariance in less than four dimensions [9]. One is necessarily led to conclude from this analysis that the couplings of any interacting QFT arise from the fractional dimension of space-time $\varepsilon \neq 0$, as deviations from *trivial* scale invariance at $\varepsilon = 0$ [5-6].

5. Toward a solution for the mass hierarchy problem

Let us now return to (4.3) and (4.4). Under the reasonable assumptions that the solution (4.5) drifts toward a limit cycle $g_\alpha^0(\varepsilon)$ which becomes unstable near $\varepsilon = 0$, it can be shown that (4.3) and (4.4) undergo transition to chaos driven by the dimensional parameter $\varepsilon \to 0$ [14-15, 35]. The first stage of this transition is a *Feigenbaum cascade* of period-doubling bifurcations of $g_\alpha^0(\varepsilon)$. Numerous examples of this scenario show that the sequence of critical values $\varepsilon_n,\ n = 1, 2, ...$ driving the transition to chaos in (4.3) and (4.4) satisfies the geometric progression [36-37]

$$\varepsilon_n - \varepsilon_\infty = \varepsilon_n - 0 \sim k_n\, \overline{\delta}^{\,-n} \tag{5.1}$$

Here, $n >> 1$ is the index counting the number of cycles created through the period-doubling cascade, $\bar{\delta}$ is the rate of convergence and k_n is a coefficient that becomes asymptotically independent of n as $n \to \infty$. Period-doubling cycles are characterized by $n = 2^p$, for $p >> 1$. Substituting (5.1) in (4.5) yields the following ladder-like progression of critical couplings

$$\boxed{g_\alpha^*(p) \sim (\bar{\delta})^{-2^p}} \qquad (5.2)$$

It can be shown that (5.2) recovers the full mass and flavor content of the SM, including neutrinos, together with the coupling strengths of gauge interactions [6, 15, 35]. Specifically,

- The *trivial FP* of the RG flow consists of the massless photon (γ) and the massless UV gluon (g).
- The *non-trivial FP* of the RG flow is degenerate and consists of massive quarks (q), massive charged leptons and their neutrinos (l, ν) and massive EW bosons (W, Z).
- *Gauge interactions* develop near the non-trivial FP and include electrodynamics (e), the weak interaction (g_W^*) and the strong interaction (g_s^*).

It is instructive to note that a similar treatment applied to QCD is able to retrieve the spectrum of hadron masses [16, 39]. Along the same lines of thought, it can be also shown that the number of fermion generations follows from the stability analysis of RG trajectories [19]. These findings reinforce the point made earlier about the many unexplored implications of the RG program on the SM physics.

6. Higgs-like scalar as Bose-Einstein condensate on fractal space-time

It is widely accepted that the SM embodies our current knowledge of the strong and EW interactions. SM is a self-contained framework of remarkable predictive power whose fundamental degrees of freedom are the spin one-half quarks and leptons, the spin one gauge bosons and the spin-zero Higgs doublet. Symmetry constraints play a key role in fixing the dynamical structure of SM, which exhibits invariance under the combined $SU(3)_L \times SU(2)_Y \times U(1)_{EM}$ gauge group. Despite being confirmed in many independent tests, SM is an *incomplete* framework as it leaves many basic questions unanswered [55]. The reported narrow resonance seen by the LHC, whose mass is centered on $m = 126$ GeV, is strongly consistent with a CP even *Higgs-like* boson [10-11]. However, at the time of writing, no consensus has yet been reached on two important points, namely, a) that the Higgs-like boson is the simplest possible type predicted by the SM and b) that the Higgs mechanism based on the Weinberg-Salam potential is the actual source of EWSB [10, 54].

It was shown in [40] that the transition from order to chaos in classical and quantum systems of gauge and Higgs fields is prone to occur somewhere in the low to mid TeV scale. The inability of the Higgs vacuum to survive not too far above the LHC scale explains away the fine-tuning problem and signals the breakdown of the SM in this region [40]. The likely instability of the vacuum in the low to intermediate TeV scale brings up an intriguing speculation on the nature of the Higgs scalar. In particular, what we interpret as the Higgs scalar may actually be a Bose-Einstein condensate of gauge fields on fractal space of dimension $D = 4 - \varepsilon$. The goal of this section is to elaborate on this idea.

First, recall that in relativistic QFT, pure scalar fields are a peculiar class of operators due to the following reasons [7, 42]:

a) Since canonical mass dimension of fields is linearly dependent on their spin j,

$$d_{\Phi} = j + 1 \tag{6.1}$$

scalar fields have the minimal mass dimension, $d_{\Phi} = 1$.

b) Self-interacting scalars carry dimensionless coupling parameters in four-dimensional space-time ($d_{u_{\Phi}} = 0$). As it is known, dimensionless couplings ensure consistency of quantum field theory, in particular compliance with conformal symmetry [42].

c) Scalar fields do not carry any gauge charges (electric, weak hypercharge or color) and are free from chirality.

d) The mapping theorem states that non-abelian gauge fields are indistinguishable from scalars in the infrared limit of field theory [43].

Secondly, fractal space-time has the ability to confine quantum fields in a similar manner with the phenomenon of Anderson localization in condensed matter physics [18, 44, 45].

It follows from these considerations that scalars are the simplest embodiment of quantum fields. They are the most likely to form a *Higgs-like* condensate of gauge bosons on space-times endowed with low level fractionality ($\varepsilon \ll 1$), that is,

$$\boxed{\Phi_C = \tfrac{1}{4}[(W^+ + W^- + Z^0 + \gamma + g) + (W^+ + W^- + Z^0 + \gamma + g)]} \quad (6.2)$$

As further explained in the Appendix, a remarkable feature of (6.2) is that it is a weakly coupled cluster of gauge fields having *zero topological charge*. Compliance with this requirement motivates the duplicate construction of (6.2), which contains individual *WW*, *ZZ*, photon and gluon doublets. Stated differently, (6.2) is the only inclusive combination of gauge field doublets that is free from all gauge and topological charges. Table 1 shows a comparative display of properties carried by the SM Higgs and the Higgs-like condensate.

Scalar field	**Original form**	**Composition**	**Mass (GeV)**	**Weak hypercharge**	**Electric charge**	**Color**	**Topological charge**
SM Higgs	$\begin{pmatrix}\varphi^+ \\ \varphi^0\end{pmatrix}$	none	~ 126	$\begin{pmatrix}+1 \\ +1\end{pmatrix}$	$\begin{pmatrix}+1 \\ 0\end{pmatrix}$	0	0
Higgs-like condensate	Φ_C	(6.2)	~ 126	0	0	0	0

Tab. 1: SM Higgs doublet versus the Higgs-like condensate.

7. Concluding remarks

There is a vast body of proposed extensions of the SM offering solutions to its open questions or ideas on how to pursue model-building beyond its boundaries. Most proposals add new layers of complexity to the structure of the SM under the tacit assumption that these must come into play at large energy scales. The widespread belief is that new physics is prone to show up in the form of hidden particles or extended symmetry groups (examples include supersymmetric partners, sterile neutrinos, axions, Kaluza-Klein particles, WIMP's, dark photons and so on [55]).

By contrast, ideas developed here hint that an unexplored range of phenomena emerge from the nontrivial topology of space-time as the dimensional parameter $\varepsilon = 4 - D$ deviates slightly from zero. Our study shows that nonlinear dynamics of RG equations, along with the unavoidable transition to fractal space-time above the EW scale, can settle some of the puzzles surrounding the SM. In particular, the low fractality texture of space-time naturally explains the mass hierarchy problem and suggests the emergence of the Higgs-like resonance as condensate of gauge bosons with a vanishing topological charge. In addition, as detailed in [5, 6, 51], the onset of space-time of low level fractality clarifies the fermion chirality and the violation of CP symmetry in weak interactions, the gauge hierarchy and cosmological constant problems as well as the possible content of non-baryonic dark matter. On this last point and by analogy with the Higgs-like structure (6.2), dark matter may surface as low-energy condensates of gauge bosons on fractal space-time that are likely to quickly annihilate into lepton-antilepton or quark-antiquark pairs [6, 53].

The concept of fractal space-time helps also set the stage for a unified understanding of symmetries that operate within QFT. To this end, recall section four where autonomous equations were shown to be isomorphic with scale invariant equations (relation 4.4). This observation unveils a tantalizing connection between local symmetries of QFT and the dimensional parameter ε. Local infinitesimal transformation in ordinary space-time ($D = 4$), including translations, rotations and boosts, may be viewed as local scale transformations characterized by infinitesimal changes of dimension, $\varepsilon \rightarrow \varepsilon'$. The same applies to infinitesimal gauge transformations. It follows that *all symmetry groups* of QFT, including the Poincaré and gauge groups, may be deeply related to scale invariance on fractal space-time and the concept of continuous dimension [5-6, 20-21]. Furthermore, this connection brings to the fore two important insights:

a) The concept of *arbitrary spin* may be seen as a topological manifestation of fractal space-time.

b) Classical gravitation defined in ordinary space-time is equivalent to field theory on fractal space-time. At least in principle, this observation opens up an unforeseen path to the long-sought unification of General Relativity and the physics of subatomic scales [5-6, 20-21].

Needless to say, our tentative findings need to be further scrutinized and, most importantly, confronted with the experiment. Unfortunately, as of today, many aspects of the weak and strong interactions still elude us and the accelerator data do not provide clear guidance on where to focus the theory next.

Although intriguing, follow up research is required to consolidate or falsify the body of ideas discussed above. For example, to be compelling, the postulated Higgs-like condensate (6.2) has to duplicate all production/decay cross sections and branching ratios predicted by the SM. It must also be consistent with preservation of unitarity in scattering of polarized *WW* bosons. One must also explain why the chaotic behavior of the RG flow is not directly observable in field theories describing separate gauge sectors of the SM, namely *U(1)*, *SU(2)* and *SU(3)* [56].

It is our hope that our work will inspire further developments on the subtle connection between the dynamics of the SM and the fractal structure of space-time above the EW scale.

List of abbreviations used in the text:

SM = Standard Model, EW = electroweak, EWSB = electroweak symmetry breaking, FP = fixed point, QFT = Quantum Field Theory, EFT = effective field theory, RG = Renormalization Group, IR = infrared, UV = ultraviolet, QG = Quantum Gravity, LGW = Landau-Ginzburg-Wilson, QCD = Quantum Chromodynamics, C.C. = cosmological constant.

Appendix: *Conserved topological charge of field theory on fractal space-time*:

It is known that scale invariance in classical field theory amounts to the condition [23]

$$\partial_{\nu} S^{\nu} = 0 \tag{A.1}$$

where the scale current is related to the energy-momentum tensor of the theory via

$$S^{\nu} = x_{\sigma} T^{\nu\sigma} \tag{A.2}$$

(A.1) and (A.2) imply that the trace of this tensor vanishes, that is,

$$\partial_{\nu} S^{\nu} = T_{\nu}^{\nu} = 0 \tag{A.3}$$

The space-time continuum of both classical and quantum physics represents a smooth four-dimensional manifold. Its geometrical properties are fully specified by the metric tensor, which also determines the geodesics of classical particles and light beams. Consider now an ordinary space-time with metric $\eta_{\sigma\nu} = diag(-,+,...,+)$ and constrain all coordinates $\nu = 0,1,2,3$ to be slightly dependent on ε as in

$$x^{\nu} \to (x^{\nu})^{1-\varepsilon} \tag{A.4}$$

with ε << 1. This space-time has low level fractionality and it can be referred to as a *minimal fractal manifold* (MFM). The measure of the MFM generalizes the familiar definition of classical and quantum physics and is given by [50]

$$d\rho(x) = d^4 x \, \mathrm{v}(x, \varepsilon) \tag{A.5}$$

$$\mathrm{v}(x,\varepsilon)=\frac{(|x^0||x^1||x^2||x^3|)^{-4\varepsilon}}{(\Gamma(1-\varepsilon))^4} \tag{A.6}$$

Likewise, the ordinary differential operator on the MFM is upgraded to

$$\mathrm{D}_\nu=\frac{1}{\sqrt{\mathrm{v}(x,\varepsilon)}}\partial_\nu[\sqrt{\mathrm{v}(x,\varepsilon)}\cdot] \tag{A.7}$$

Let us now introduce on the MFM a classical scalar field $\varphi(x)$ with Lagrangian L and assume that the potential part of the Lagrangian is a polynomial of order r having the form $V(\varphi)\sim\lambda_0\varphi^r$. The energy-momentum tensor of the field can be presented as [50]

$$\overline{T_{\sigma\nu}}=\mathrm{v}(x,\varepsilon)T_{\sigma\nu} \tag{A.8}$$

where $\sigma=0,1,2,3$ and

$$T_{\sigma\nu}=\eta_{\sigma\nu}L+\mathrm{D}_\sigma\varphi\mathrm{D}_\nu\varphi \tag{A.9}$$

The continuity equation reads

$$\partial_\nu\overline{T_\sigma^\nu}=\lambda_0\varphi^r(\frac{r}{2}-1)\mathrm{v}^{-r/2}\partial_\sigma\mathrm{v} \tag{A.10}$$

For $r\neq 2$, the energy-momentum tensor is not conserved due to the non-vanishing right-hand term. Adding to $\overline{T_\sigma^\nu}$ a contribution whose four-divergence amounts to the right-hand term of (A.10),

$$\partial_\nu t_\sigma^\nu=-\lambda_0\varphi^r(\frac{r}{2}-1)\mathrm{v}^{-r/2}\partial_\sigma\mathrm{v} \tag{A.11}$$

turns (A.10) into

$$\partial_\nu(\overline{T_\sigma^\nu} + t_\sigma^\nu) = \partial_\nu \theta_\sigma^\nu = 0 \tag{A.12}$$

Unlike $\overline{T_\sigma^\nu}$, the newly defined energy-momentum tensor θ_σ^ν is conserving. Its component t_σ^ν is clearly linked to the fractal geometry of the MFM via (A.6). As with any other constant currents, there are also conserved charges which arise from integrating the temporal component of θ_σ^ν over the spatial domain. Since these charges follow from the fractal geometry of space-time, they are intrinsically *topological* in nature and characterized by

$$\boxed{\tau_\sigma \sim \int \theta_\sigma^0 d^3x} \tag{A.13}$$

It is instructive to note that (A.13) falls in line with the concept of *space-time polarization* induced by fractal topology [47-49].

Due to the manifestly neutral attributes of the scalar field embodied in (6.2), it is natural to assume that the Higgs-like condensate Φ_C is the sole state with *zero topological charge* that includes all gauge boson flavors. A logical corollary of this assumption is that, in general, particle-antiparticle pairs of gauge bosons or fermions carry *non-zero* topological charges. In light of this interpretation, conservation of the topological charge (A.13) requires that, in all decay or production channels of Φ_C shown in Tab. 2, a fraction of this charge becomes an "effective" polarization of the space-time manifold. This mechanism explains, at least in principle, why the Higgs-like condensate Φ_C couples to both SM gauge and fermion operators without carrying any gauge charge.

Decay	$\Phi_c \to bb$	$\Phi_c \to \tau\tau$	$\Phi_c \to WW\,(l\nu l\nu)$	$\Phi_c \to \gamma\gamma$	$\Phi_c \to ZZ\,(4l)$
Production	$gg \to tb \to \Phi_c$	$W, Z \to \Phi_c$	$gg \to tt \to \Phi_C$	–	–

Tab. 2: Decay and production channels of the Higgs-like condensate.

The entries of this table are, respectively, $l =$ lepton, $t =$ top quark, $b =$ bottom quark, $\nu =$ neutrino, $\tau =$ tau lepton, $\gamma =$ photon, $g =$ gluon.

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