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QUANTUM MECHANICS Solved using only Newtonian Mechanics

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Introduction

In this paper I solve classical quantum mechanics problems without using Schrödinger equation. Problems solved are: particle in a 1, 2, and 3 dimensional box, particle in a ring, Quantum harmonic oscillator, and hydrogen-like atoms. Solutions are very short, and rely on Newtonian Mechanics.
This issue make easy to understand the mysteries of Quantum Theory

Abbreviations

c	speed of light
m, m_0	moving mass/rest mass
V	velocity
$p=mV$	momentum
$\lambda_e = \frac{h}{m \cdot V} = \frac{h}{p}$	De Borglie wavelength
$\lambda_c = \frac{h}{m_0 \cdot c} = \frac{h}{p_0}$	Compton wavelength
f	frequency
$f_c \cdot \lambda_c = c$	
$f_e \cdot \lambda_e = c$	

Introduction
Wave's interference

Wave interference is a phenomenon that occurs when two waves meet while traveling along the same path

1. If two sine wave are in phase the interference is constructive
2. If two sine wave are 180^0 out of phase the interference is destructive
3. In between, the out of phase can be any degree

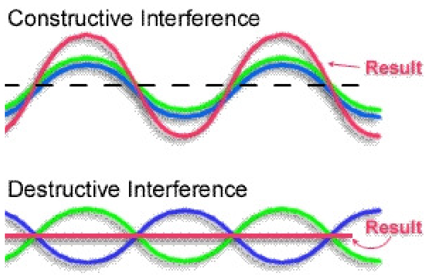
Interference is due to mathematical superposition principles. Physical superposition does not exist.

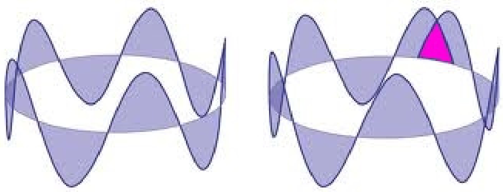
Waves Superposition and wave interference occur only when the wave are in the same volume. The waves do not react with each other. And when the wave continue to travel in different paths they look the same as before they had met

Photon does not react with photons electron does not react with electrons.

According to Compton's effect Photons may react with electrons

So I claim that waves are conserved in a wide range of energies





(a) Standing wave (b) Destructive Interference

Particle in one dimensional box

The simplest form of the "particle in a box" model considers a one-dimensional system. Here, the particle may only move backwards and forwards along a straight line with impenetrable barriers at either end. The walls of a one-dimensional box may be visualized as regions of space with an infinitely large potential $V = \infty$. Conversely, the interior of the box has a constant, zero potential. This means that no forces act upon the particle inside the box and it can move freely in that region. However, infinitely large forces repel the particle if it touches the walls of the box, preventing it from escaping.

The barriers potential in this model is $V = \infty$

And L is the distance between the barriers

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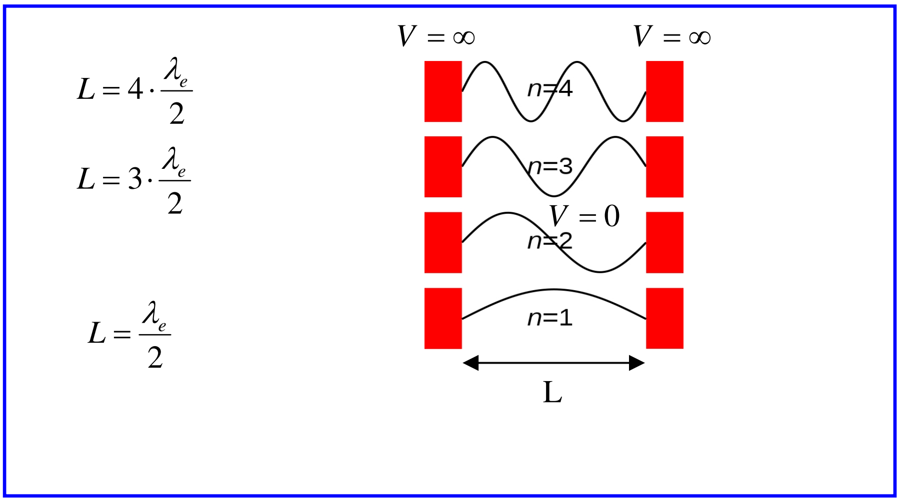

According to De Borglie Hypothesis the particle behaves also as a wave
The length of that wave is given by

1] $$\lambda_e = \frac{h}{mV} = \frac{h}{p}$$

The momentum p depend on the particle mass and velocity

2] $$p = mV$$

Where

m The particle mass
V The particle velocity
h Planck's constant

The wavelength λ_e is not constant and depends on particle velocity

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The box is a kind of resonator and the infinite potential of the barriers create a standing wave. The number of half waves between the walls is a whole number n (n=1, 2, 3…….)
Standing waves exist only when the following condition is met

3] $$L = n \cdot \frac{\lambda_{en}}{2} \qquad n=1,2,3........$$

Combining eq -1 eq -2 and eq -3 we get

4] $$L = \frac{n}{2} \cdot \frac{h}{p_n} = \frac{n}{2} \cdot \frac{h}{mV_n}$$

And from the last eq we find that the momentum p of the particle depends on n

5] $$p = p_n = mV_n = \frac{nh}{2L}$$

The momentum p enables us to find the kinetic energy of that particle since

6] $$E_n = \frac{mV_n^2}{2} = \frac{m^2 V_n^2}{2m} = \frac{p_n^2}{2m}$$

From eq -5 and eq -6 the energy of the particle is found to be

7] $$E_n = \frac{n^2 h^2}{8mL^2} \qquad n = 1,2,3.......$$

From eq- 7 it is well understood that the energy of the particle is quantized because only standing wave can exist between the barriers

But, remember, that standing waves are superposition of many waves. Each wave in the superposition does not change

The lowest energy is when $n = 1$. In that case

8] $$E_1 = \frac{h^2}{8mL^2}$$

This energy is not zero.

To rise up the particle to a higher energy level, a photon is needed

Photons energy is

9] $$E_p = h\nu$$

And the momentum of the photon can be found from following relations

11] $$c = \lambda_p \cdot \nu$$

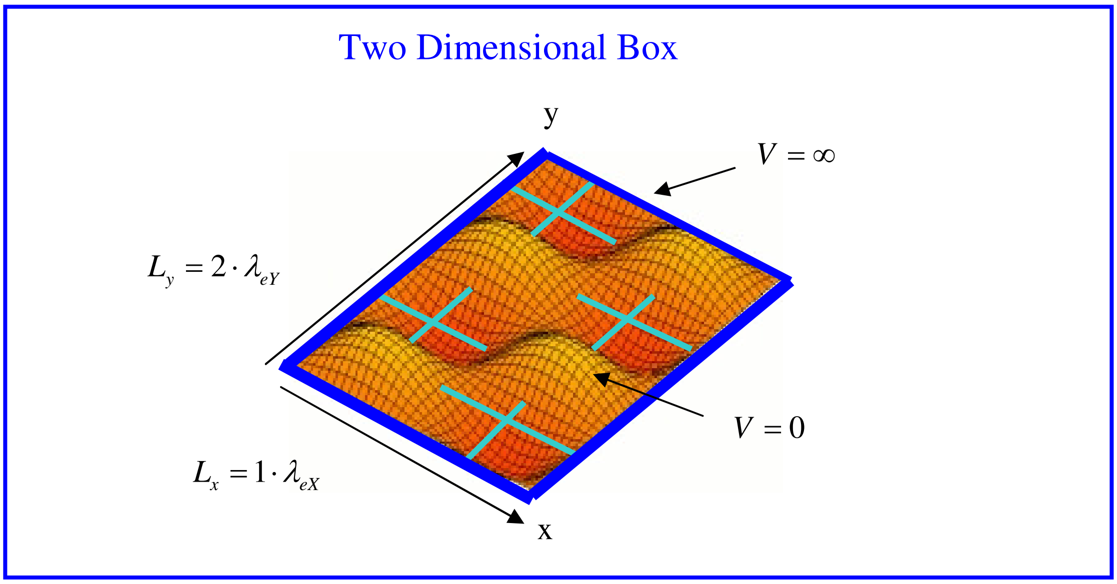
12] $$\lambda_p = \frac{h}{p_p}$$

The energy needed to raise the particle in a 1 dimensional box from state k to state n is given by

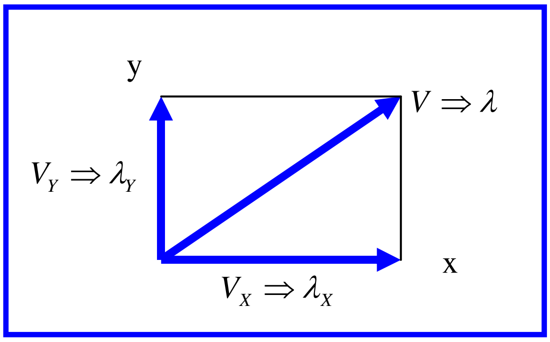
13] $$h\nu = h\frac{c}{\lambda_p} = E_n - E_k = \left(n^2 - k^2\right)\frac{h^2}{8mL^2}$$

We don’t know in detail what happens to the photon in a photon electron collision. What we know that energy and momentum are conserved (Compton Effect).

Particle in 2 &3 dimensional box



In a "two dimensional box" the blue frame is a barrier with an infinite potential $V = \infty$. Inside the frame the potential is zero $V = 0$ and the particle inside the frame can move freely. Particle movement direction is a vector that can be resolved into two separate vectors.



One vector is in the x direction and the second vector in the y direction. The particle movement in the x direction can be considered as a "particle in a one dimensional box" in the x direction. In a similar way, the particle movement in the y direction can be considered as a "particle in a one dimensional box" in the y direction. The two boxes are in depended according to linear superposition principle.

In the picture above the box width in x direction is: $L_x = 1 \cdot \lambda_{eX}$ And the box width in the y direction is $L_y = 2 \cdot \lambda_{eY}$

Notice that $\lambda_{eX} \neq \lambda_{eY}$ *this is because* $V_X \neq V_Y$

Now we can use the results of the "particle in one dimensional box" to solve a particle in 2 dimensional box, or a particle in 3 dimensional box and find the energy levels for the x and y (z) axis separately

For a particle in 2 dimensional box

1] $$E_{nX} = \frac{n_X^2 h^2}{8mL_X^2} \qquad n_X = 1,2,3.......$$

2] $$E_{nY} = \frac{n_Y^2 h^2}{8mL_Y^2} \qquad n_Y = 1,2,3.......$$

The total energy of the two dimensional box is the sum of energies in the x and y direction (energy is a scalar)

3] $$E_{nX,nY} = E_{nX} + E_{nY} = \frac{n_X^2 h^2}{8mL_X^2} + \frac{n_Y^2 h^2}{8mL_Y^2}$$

In the case of a particle in 3 dimensional box,

The last equation can be extended

4] $$E_{nX,nY,nZ} = E_{nX} + E_{nY} + E_{nZ} = \frac{n_X^2 h^2}{8mL_X^2} + \frac{n_Y^2 h^2}{8mL_Y^2} + \frac{n_Z^2 h^2}{8mL_Z^2}$$

In some systems the box is a perfect cube (Some kind of molecules) In that case

$$L_X = L_Y = L_Z$$

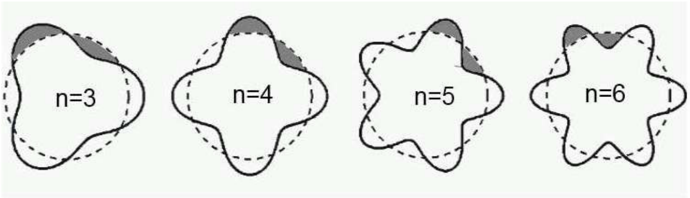
And the total energy for a perfect cube is

$$E_{nX,nY,nZ} = \left(n_X^2 + n_Y^2 + n_Z^2\right)\frac{h^2}{8mL^2}$$

In that case we see again that quantum phenomena can exist only in region with potential barriers that limit the movement of the particle.

Particle in a ring

Suppose now a circular particle trap. The two circular walls of the trap may be visualized as regions of space with an infinitely large potential $V = \infty$ and the potential between the walls is $V = 0$. Here, the particle may only move along a circular line with a radius R



The associated wavelength of the particle is still

1] $$\lambda_e = \frac{h}{mV} = \frac{h}{p}$$

But the wave is not on straight line. So, λ_e is bend and it looks like a rainbow with a radius R. The wave is a standing wave. No destructive interference is allowed

2] $$2\pi R = n \cdot \lambda_e \qquad n = 1,2,3........$$

Let use the last two equations to find the momentum of the particle

3] $$p_n = mV_n = \frac{nh}{2\pi R}$$

The energy of the particle is proportional to the square of its momentum

4] $$E_n = \frac{mV_n^2}{2} = \frac{m^2V_n^2}{2m} = \frac{p_n^2}{2m}$$

If we define I to be the angular moment of inertia

5] $$I = mR^2$$

Then we find that the energy of the particle is

6] $$E_n = \frac{n^2\hbar^2}{2I} \qquad n = 0,\pm1,\pm2,\pm3.......$$

Notice that n can be negative in that case the energy is still positive but for negative n the momentum is in the opposite direction and the particle moves in the opposite direction.

Comment

"The particle in a one dimensional box" and "The particle in a ring" are the same problems. The first problem is solved in Cartesian coordinates, the second in polar coordinates.

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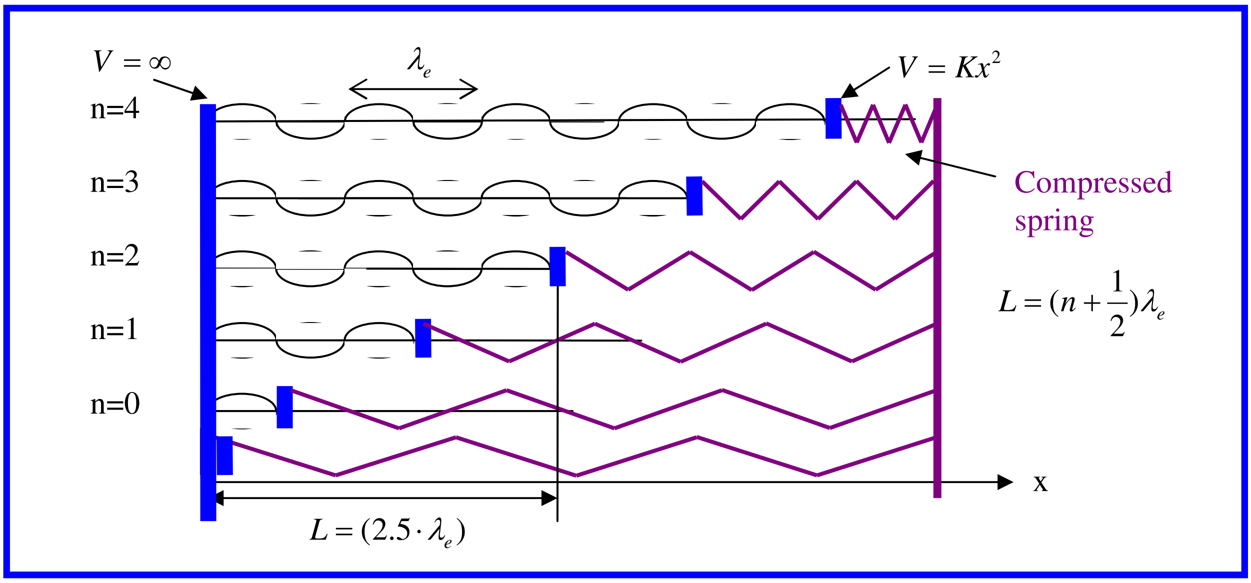
Quantum Harmonic oscillator



In all previous examples the barriers were with an infinite potential. In this example the left barrier is still with an infinite potential. But the right barrier is kept in its x position using an ideal spring.

The force needed to compress the spring is:

1] $$F = k \cdot x$$

And the energy of the compressed spring is

2] $$E = k\frac{x^2}{2}$$

In order to compress the spring a single particle is put between the walls (blue lines)

The particle momentum is:

3] $$p = mV$$

And its wavelength

4] $$\lambda_e = \frac{h}{mV} = \frac{h}{p}$$

The kinetic energy that the particle has to produce in order to balance the pressure of the spring is

5] $$E = m\frac{V^2}{2} = k\frac{x^2}{2}$$

As in previous examples, the particle energy relate to the particle momentum by

6] $$\frac{mV^2}{2} = \frac{m^2V^2}{2m} = \frac{p^2}{2m}$$

Unfortunately, because of the barriers potential and in order to achieve a standing wave with no destructive interference, the particle wavelength must obey that (That's because of the boundary conditions)

7] $$x_n = \left(n + \frac{1}{2}\right)\lambda_e = \left(n + \frac{1}{2}\right) \cdot \frac{h}{p} \qquad \text{n=0,1,2......}$$

Now the spring energy can be easily found to be

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8] $$k\frac{x_n^2}{2}=\frac{k}{2}\left[\left(n+\frac{1}{2}\right)\frac{h}{p_n}\right]^2$$

And the kinetic energy of the particle equals to the kinetic energy of the spring

9] $$\frac{p_n^2}{2m}=k\frac{x_n^2}{2}=k\frac{\left(n+\frac{1}{2}\right)^2}{2}\frac{h^2}{p_n^2}$$

Then we find that

10] $$p^2=\sqrt{km}\left(n+\frac{1}{2}\right)h$$

And the energy is quantized

11] $$E_n=\frac{p_n^2}{2m}=\sqrt{\frac{k}{m}}\left(n+\frac{1}{2}\right)h \qquad n=1,2,3..........$$

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Hydrogen like atom The Bohr atom

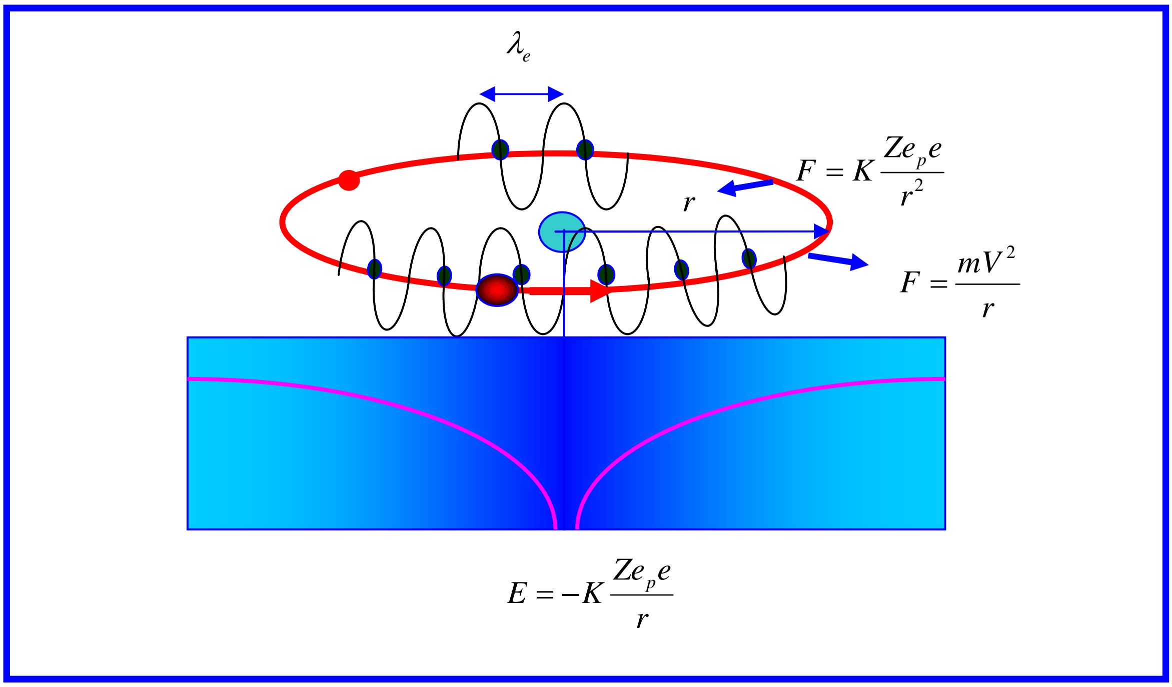
The force maintaining the electron in it circular orbit around the nuclei of the atom is the attractive Coulomb force

1] $$F = K\frac{Ze_p e}{r^2}$$

Were e is the electron and proton charge and Z is an integer that indicates how many protons are in the nuclei of the atom.
The electric field potential is found by integration

2] $$E = \int_{\infty}^{r} K\frac{Ze_p e}{r^2}dr = -K\frac{Ze_p e}{r}$$

Now its seems that this problem is very similar to the Quantum Harmonic Oscillator
The spring force in the quantum harmonic oscillator is replace by Coulombs electrostatic force
But the problems are entirely different



The centripetal (centrifugal) force of the orbiting electron is balance by the Coulomb electric force

3] $$\frac{mV^2}{r} = K\frac{Ze_p e}{r^2}$$

And in order to avoid destructive interference

4] $$2\pi r = n \cdot \lambda_e \qquad n = 1,2,3........$$

After the following mathematical manipulations we find:

5] $$\frac{mV^2}{r}=\frac{(mV)^2}{mr}=\frac{p^2}{mr}=K\frac{Ze_pe}{r^2}$$

6] $$\frac{p^2}{m}=K\frac{Ze_pe}{r}=KZe_pe\cdot\frac{2\pi}{2\pi}\cdot\frac{1}{r}$$

7] $$\frac{p^2}{m}=K2\pi\frac{Ze_pe}{n\lambda_e}=KZe_pe\frac{2\pi}{n\cdot\frac{h}{p}}$$

Let define $\hbar$ to be

8] $$\hbar=\frac{h}{2\pi}$$

The particle momentum is

9] $$p=mKZe_pe\frac{1}{\hbar}\cdot\frac{1}{n}$$

And the particle energy is

10] $$E_n=\frac{p^2}{2m}=\frac{m^2\left[KZe_pe\right]^2}{2m\cdot n^2\hbar^2}=\frac{m\left[KZe_pe\right]^2}{2\cdot\hbar^2}\cdot\frac{1}{n^2}$$

Note that the centripetal force act on the side of that wave and not on the front of the wave

From our experience with waves, the front of the wave can apply power, not the side of it

What is Schrödinger Equation?

Schrödinger Equation is a differential equation, that is hard to solve. The first step in solving Schrödinger Equation is to find a function $\Psi(x,t)$. The second step is to find E_n and p_n. Since I did not used Schrödinger Equation at all, I found E_n and p_n at the first step and did not succeed to find the distribution of the energy and momentum in time and space. So Schrödinger Equation is much better than my solution. My solution can be helpful in order to explain Quantum Mechanics to student with limited knowledge in higher mathematics.

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